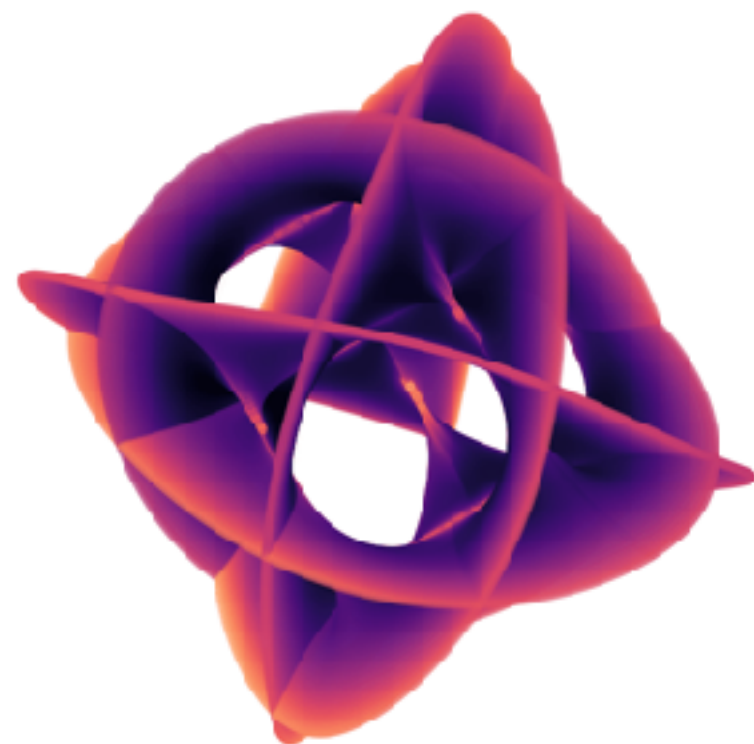


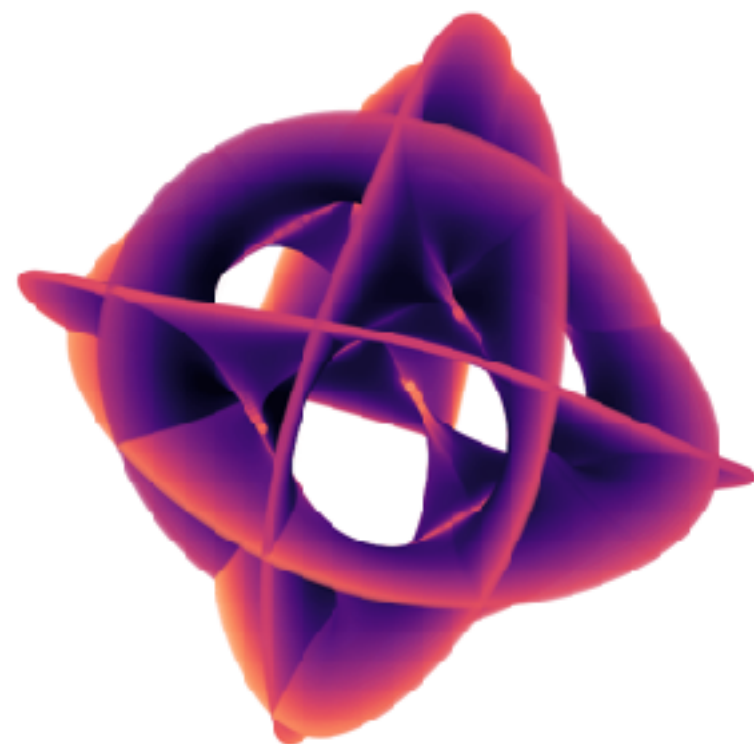
CALABI–YAU METRICS & YUKAWA COUPLINGS



VISHNU JEJALA
UNIVERSITY OF THE WITWATERSRAND

STRINGS 2025
NYU ABU DHABI

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Collaborators



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Challenger Mishra



Justin Tan

String Compactification

- Four-dimensional $\mathcal{N} = 1$ effective field theories with chiral matter from compactification of $E_8 \times E_8$ heterotic string on Calabi–Yau threefold

- Complex, Kähler manifold admits a Ricci-flat metric when $c_1 = 0$

Calabi (1957)
Yau (1977)

- Depending on rank, vector bundle can break E_8 to E_6 or $SO(10)$ or $SU(5)$

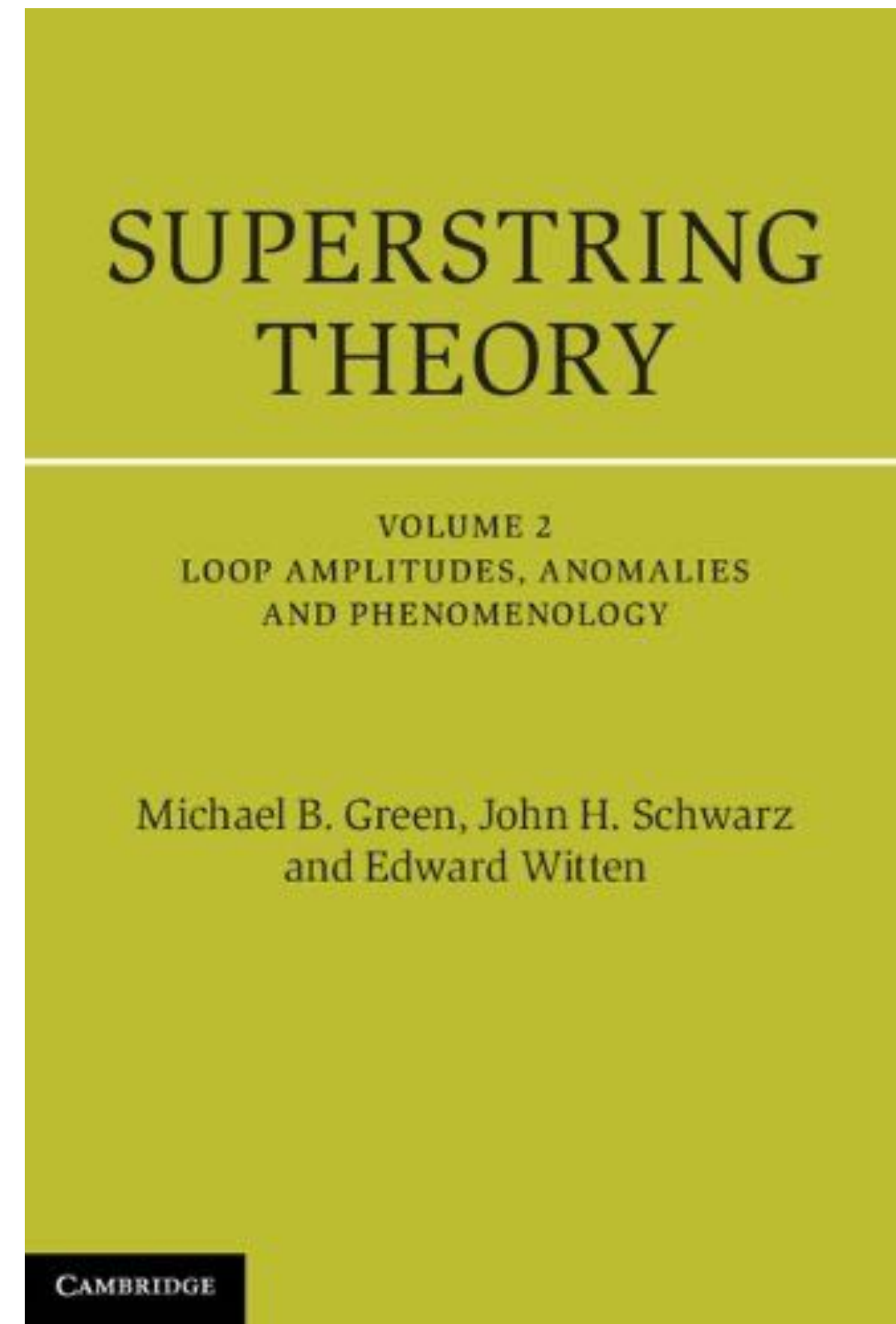
- Break gauge symmetry further by introducing freely acting Wilson lines

- Textbook paradigm for model building in particle physics

Candelas, Horowitz, Strominger, Witten (1985)

- 1/2 billion 4d reflexive polytopes

Kreuzer, Skarke (2000)



Effective Field Theory

- The Lagrangian: $\mathcal{L}_D = \int d^4\theta \, K(\Phi, \bar{\Phi}) \supset N_{i\bar{j}} \Phi^i \bar{\Phi}^{\bar{j}}$

$$\mathcal{L}_F = \int d^2\theta \, W(\Phi) + \text{h.c.} \supset \lambda_{ijk} \Phi^i \Phi^j \Phi^k$$

- Topological data gives us fields in low-energy theory plus the types of interactions that are present
- λ_{ijk} are Yukawa couplings, which tell us how strongly fields interact
- To calculate these in a general construction, need to know Calabi–Yau, X
 - holomorphic vector bundle, V
 - Ricci-flat metric
- Ricci-flat metric is unique for any complex structure, Kähler class

Why a Ricci-flat Metric?

- Ricci-flat metric, $g_{a\bar{b}} = \partial_a \partial_{\bar{b}} K$, $R_{a\bar{b}} = 0$, solves leading order vacuum equations of motion
- For purposes of phenomenology, the Ricci-flat metric needed for the following computations
 - normalize kinetic terms from Kähler potential for matter
 - Yukawa couplings in general heterotic compactifications
 - moduli stabilization
 - α' corrections
 - supersymmetry breaking

- Ricci-flatness is a system of fourth order, non-linear partial differential equations, *viz.*,

$R_{a\bar{b}} = \partial_a \partial_{\bar{b}} \log \det g = 0 \implies$ equivalent to second order Monge–Ampère equation for φ

$$\det (g_{a\bar{b}} + \partial_a \partial_{\bar{b}} \varphi) = e^f \det g_{a\bar{b}}$$

Yau (1977)

- To date, best we can do is a numerical approximation for complex dimension $n \geq 2$

How to Approximate Ricci-flat Metrics

- Solve Monge–Ampère directly

Headrick, Wiseman (2005)

$$K(z, \bar{z}) = \frac{1}{\pi} \log |z|^2, \quad |z|^2 = \sum_{a=1}^{n+1} z^a \bar{z}^a$$

- Donaldson’s algorithm

– start from Fubini–Study metric on ambient projective space, pullback to manifold

– iterative procedure to construct “balanced metric”

– $k \rightarrow \infty$ limit gives Ricci-flat metric

Tian (1990)

Donaldson (2005)

– numerical implementation expensive, slow convergence

– baseline for comparison

Douglas, Karp, Lukic, Reinbacher (2006)

Braun, Brelidze, Douglas, Ovrut (2007)

$$g_{a\bar{b}} = \partial_a \partial_{\bar{b}} K = \frac{|z|^2 \delta^{a\bar{b}} - z^a \bar{z}^b}{\pi |z|^4}$$

$$K^{(k)}(z, \bar{z}) = \frac{1}{k\pi} \log(h^{a\bar{b}} s_a \bar{s}_b)$$

$$H_{a\bar{b}} = \frac{N_k}{\text{Vol}_\Omega} \int_X d\text{Vol}_\Omega \frac{s_a \bar{s}_b}{h^{c\bar{d}} s_c \bar{s}_d}$$

$$h^{a\bar{b}} = (H_{a\bar{b}})^{-1}$$

$$dJ_H^{n-1} = 0$$

- Neural network

– engineer a loss function that vanishes when Ricci-flat metric is realized

Ashmore, He, Ovrut (2019)

Anderson, Gerdes, Gray, Krippendorf, Raghuram, Ruehle (2020)

Douglas, Lakshminarasimhan, Qi (2020)

VJ, Mayorga Peña, Mishra (2020)

Douglas (2021)

Larfors, Lukas, Ruehle, Schneider (2021, 2022)

Berglund, Butbaia, Hübsch, VJ, Mayorga Peña, Mishra, Tan (2022)

Gerdes, Krippendorf (2022)

...

Neural Network Approximation

- Loss function: $\mathcal{L} \sim \sum_{\text{points}} \sum_{\text{conditions}} |\text{LHS} - \text{RHS}|$

- One of the conditions is flatness: $\sum_{a,\bar{b}} |R_{a\bar{b}}|$ $\longleftarrow$ derivatives amplify error

$$\text{Vol}_\Omega = \int_X \Omega \wedge \bar{\Omega} \quad \longleftarrow \text{metric independent}$$

$$\text{Vol}_J = \int_X J \wedge J \wedge J \quad \longleftarrow \text{metric dependent:} \quad J = \frac{i}{2} g_{a\bar{b}} dz^a \wedge d\bar{z}^{\bar{b}}$$

- Other conditions ensure closedness of Kähler form, compatibility between coordinate patches, that we remain in the same Kähler class

$$dJ = 0, \quad J \wedge \Omega = 0$$

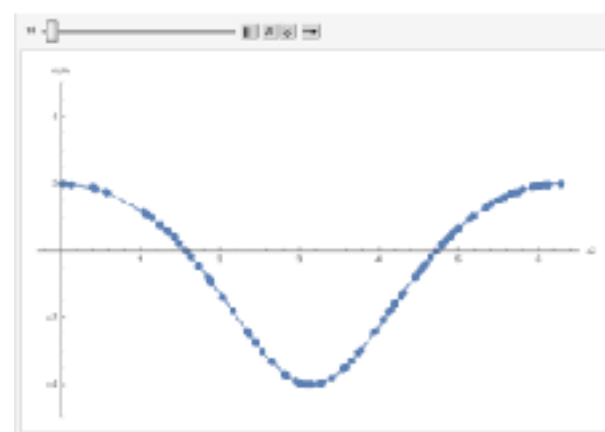
- Last can be achieved with Kähler–Ricci flow or Phi model $J^{\text{CY}} = J^{\text{FS}} + i \partial \bar{\partial} \phi$

$$g^{\text{CY}} = g^{\text{FS}} + \partial \bar{\partial} \phi$$

ϕ globally defined

} implemented in cymetric

T^2



VJ, Mayorga Peña, Mishra (2020)
cf. Halverson, Ruehle (2023)

Larfors, Lukas, Ruehle, Schneider (2021, 2022)

Neural Network Approximation

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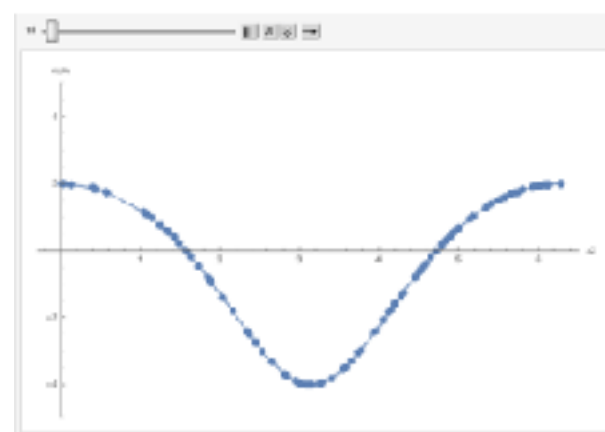
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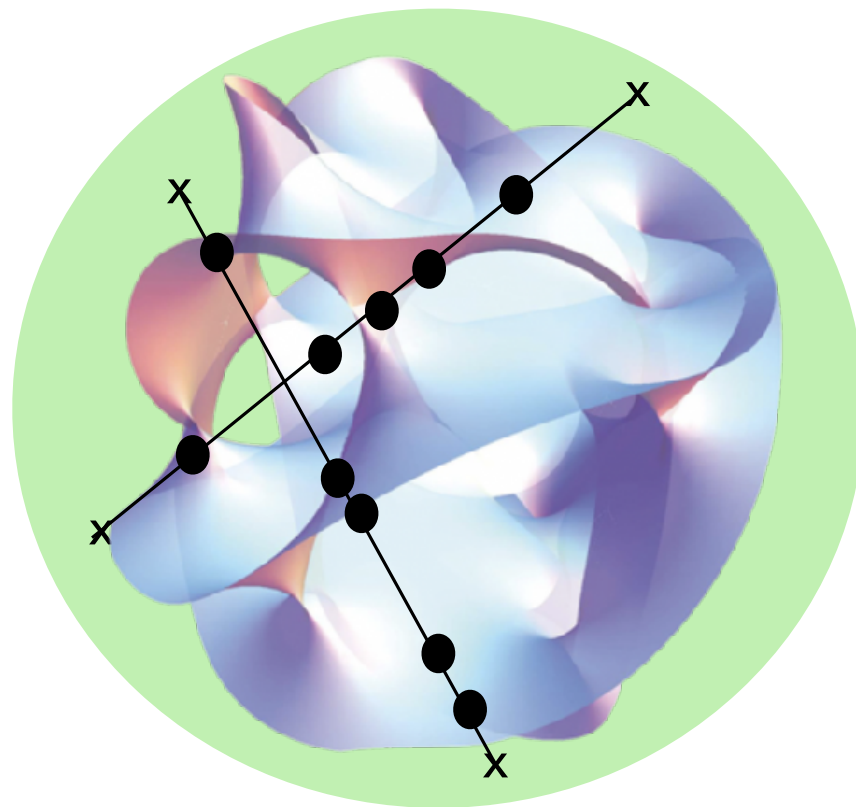


VJ, Mayorga Peña, Mishra (2020)
cf. Halverson, Ruehle (2023)

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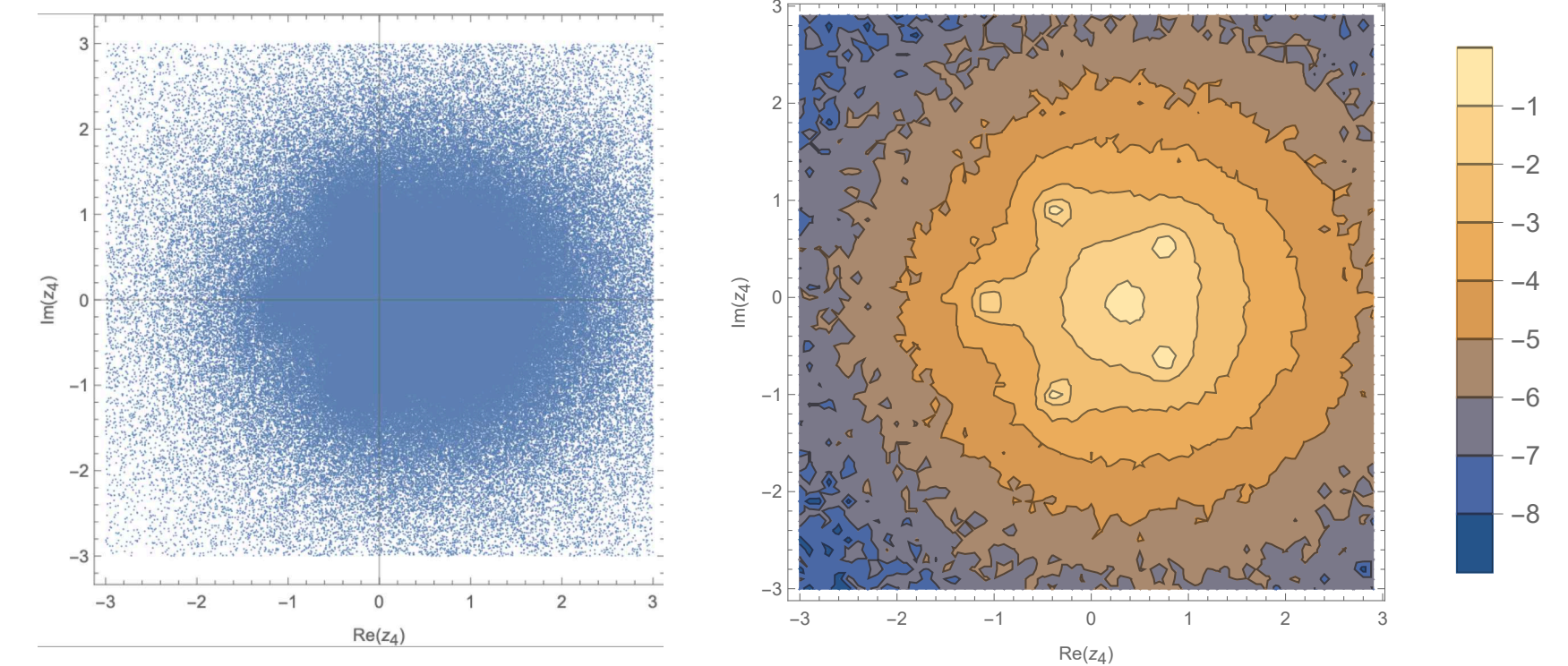
Point Selection

- Lines in $\mathbb{P}^n$ uniformly distributed with respect to Fubini–Study metric
- For CICY hypersurfaces, choose points in ambient space and see where it intersects manifold

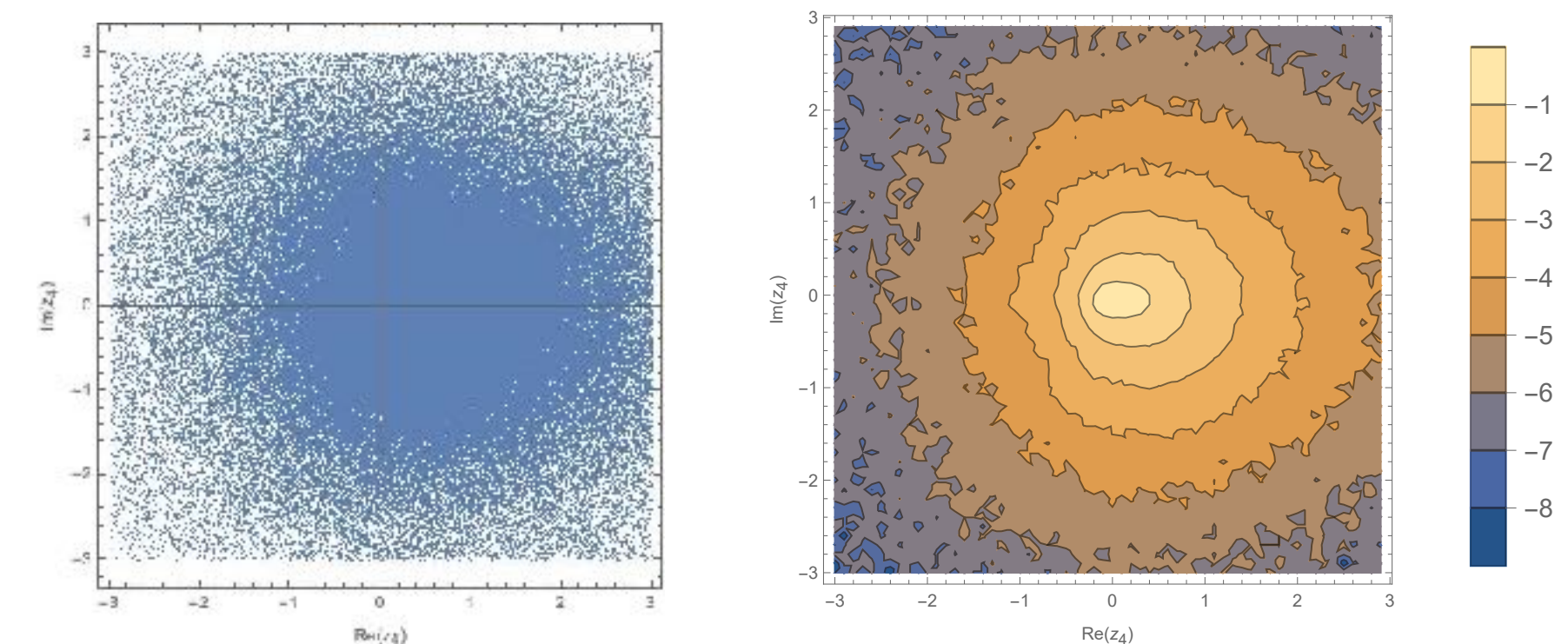


Shiffman, Zelditch (1998)
 Braun, Brelidze, Douglas, Ovrut (2007)
 Anderson, Braun, Karp, Ovrut (2010)
 Ashmore, He, Ovrut (2019)

$$\sum_{i=1}^5 z_i^5 = 0 \subset \mathbb{P}^4$$



$$\sum_{i=1}^5 z_i^5 + \prod_{i=1}^5 z_i = 0 \subset \mathbb{P}^4$$



VJ, Mayorga Peña, Mishra (2020)

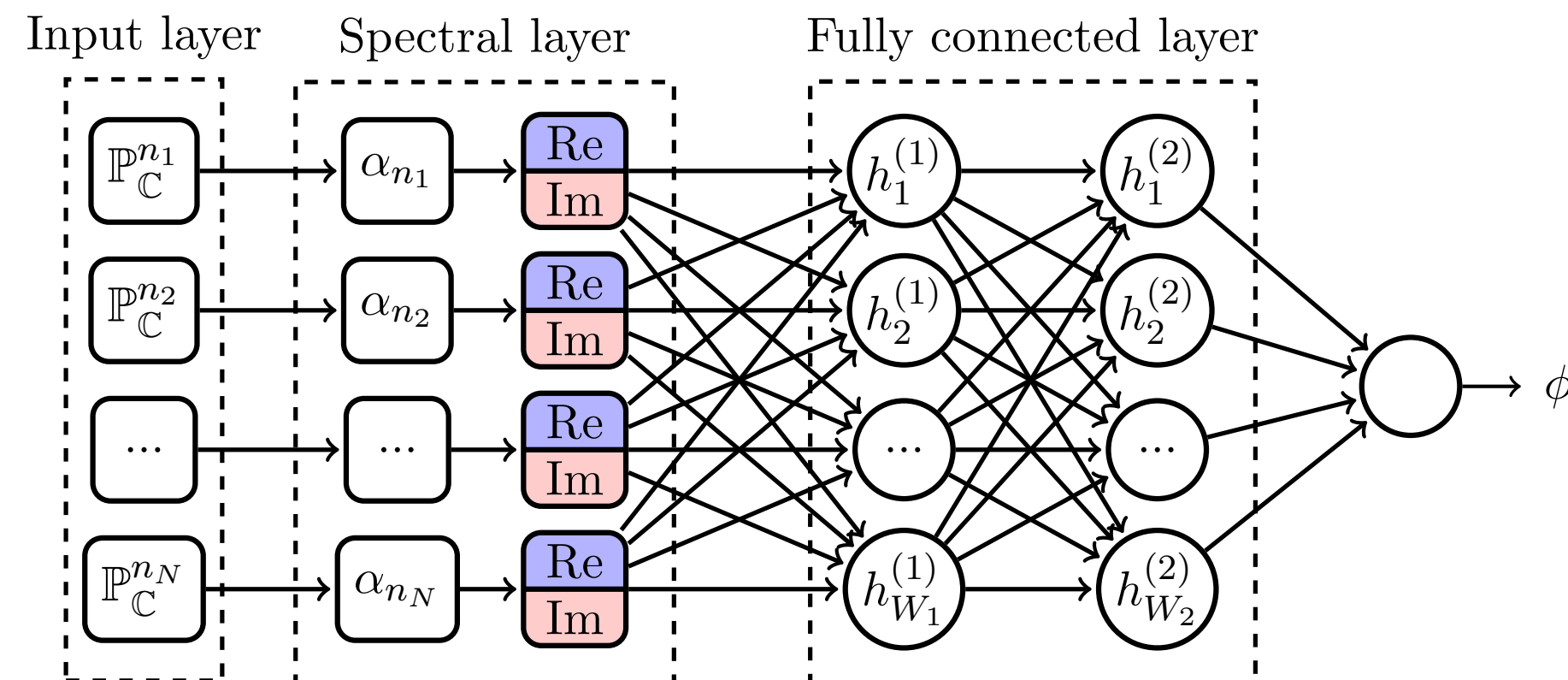
Spectral Network

- Define map $\alpha_n(p) : \mathbb{P}^n \rightarrow \mathbb{C}^{n+1, n+1}$

$$Z \rightarrow \lambda Z, \quad \lambda \in \mathbb{C}^*$$

invariant

$$\alpha_n(p) = \begin{bmatrix} \frac{Z_0 \overline{Z_0}}{|Z|^2} & \frac{Z_0 \overline{Z_1}}{|Z|^2} & \cdots & \frac{Z_0 \overline{Z_n}}{|Z|^2} \\ \frac{Z_1 \overline{Z_0}}{|Z|^2} & \frac{Z_1 \overline{Z_1}}{|Z|^2} & \cdots & \frac{Z_1 \overline{Z_n}}{|Z|^2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{Z_n \overline{Z_0}}{|Z|^2} & \frac{Z_n \overline{Z_1}}{|Z|^2} & \cdots & \frac{Z_n \overline{Z_n}}{|Z|^2} \end{bmatrix}$$



Berglund, Butbaia, Hübsch, VJ, Mayorga Peña, Mishra, Tan (2022)

$$f_i : \mathbb{R}^{n_{i-1}} \rightarrow \mathbb{R}^{n_i}$$

$$\mathbf{v}_{i-1} \mapsto \mathbf{v}_i = \sigma(\mathbf{v}'_i)$$

$$\mathbf{v}'_i = \mathbf{W}^i \cdot \mathbf{v}_{i-1} + \mathbf{b}_i$$

number of neurons in i -th layer

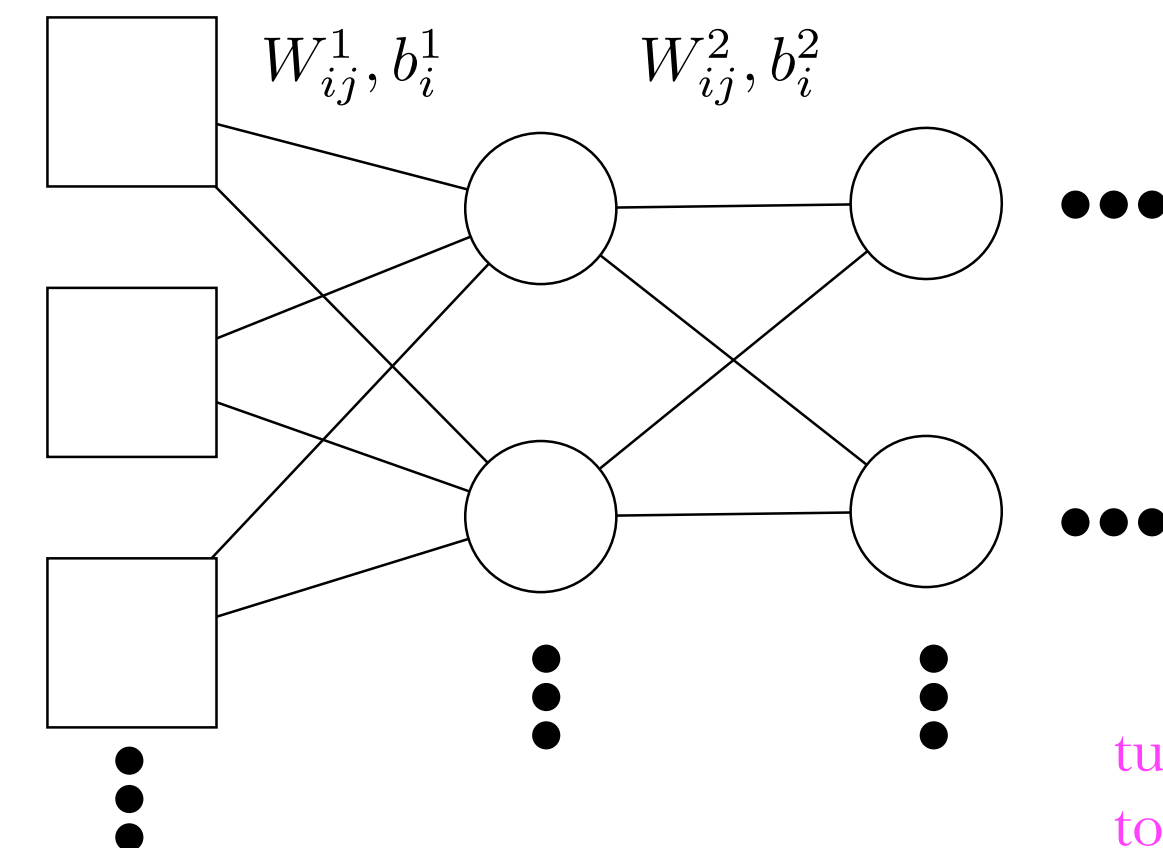
weight matrix

bias vector

non-linearity
e.g., ReLU

$$\mathbf{v}_\ell = f_\ell(f_{\ell-1}(\cdots(f_1(\mathbf{v}_0))\cdots))$$

$$a_{\text{out}} = \sum_{m=1}^{n_\ell} v_\ell^m$$



tune weights and biases
to minimize **loss function**
during **training**

Calabi–Yau Twofold

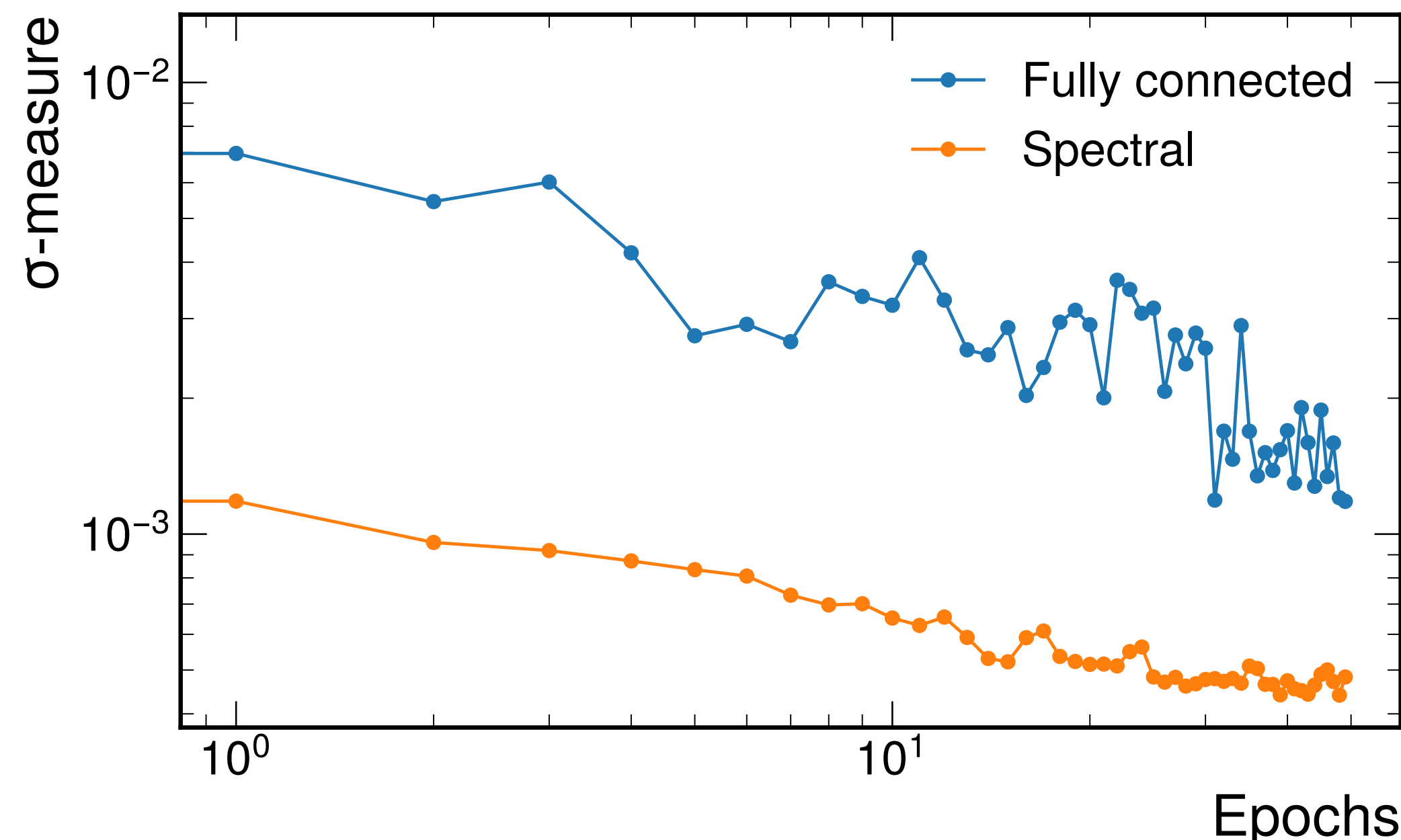
- Consider Cefalú quartic: $p_\lambda(z) = \sum_{i=1}^4 z_i^4 - \frac{\lambda}{3} \left(\sum_{i=1}^4 z_i^2 \right)^2 \subset \mathbb{P}^3$

singular when $\lambda \rightarrow \lambda^\# \in \left\{ \frac{3}{4}, 1, \frac{3}{2}, 3 \right\}$ $\leftarrow X_{\frac{3}{4}} \simeq X_0/\mathbb{Z}_2$, $X_1 \simeq T^4/\mathbb{Z}_2$

polynomial fails to be transverse
 $\frac{\partial p_\lambda}{\partial z_i} = 0 \quad \forall i$

respectively, $\{8, 16, 12, 4\}$ singularities of A_1 -type
 at $\lambda^\# + \varepsilon$: singular point becomes $\mathbb{P}^1$

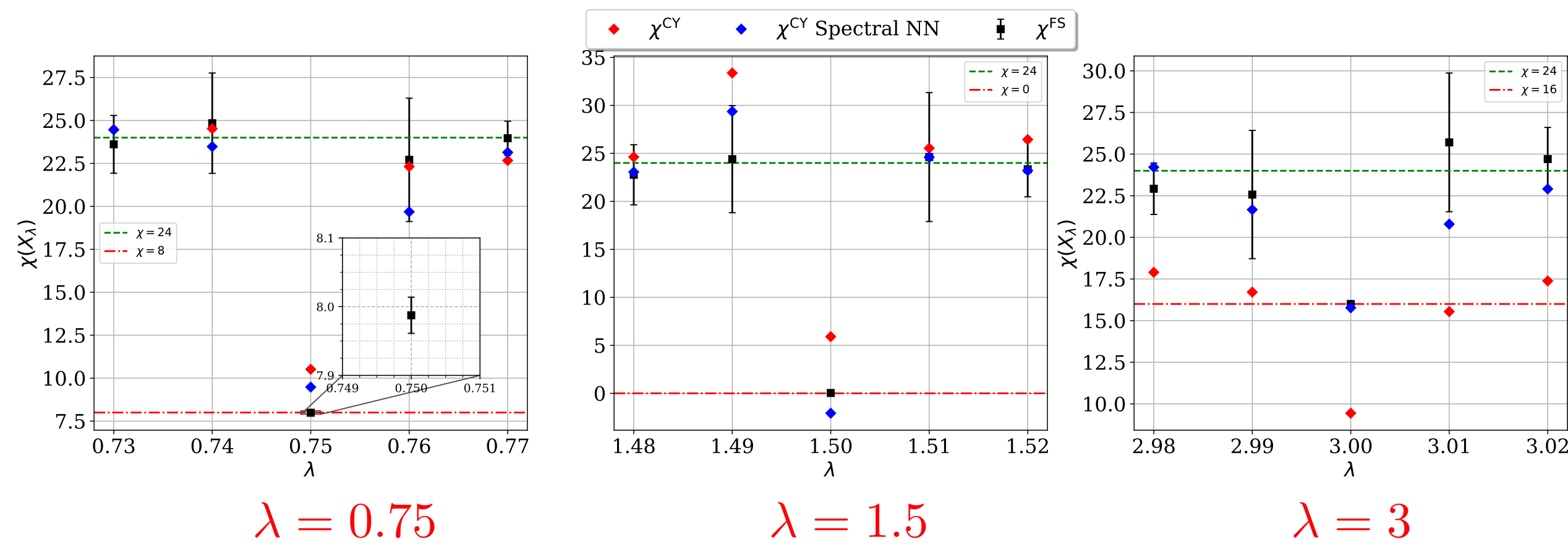
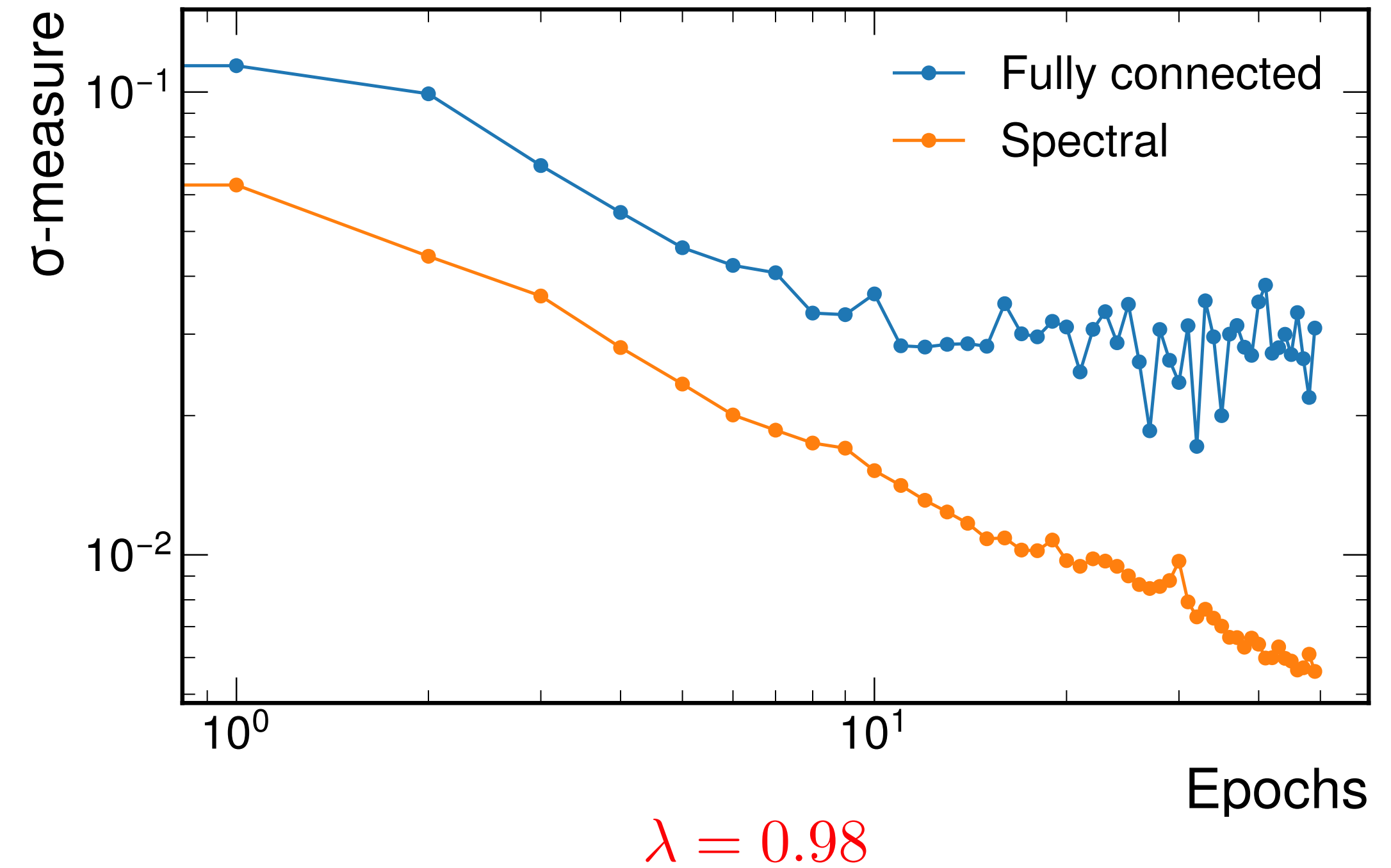
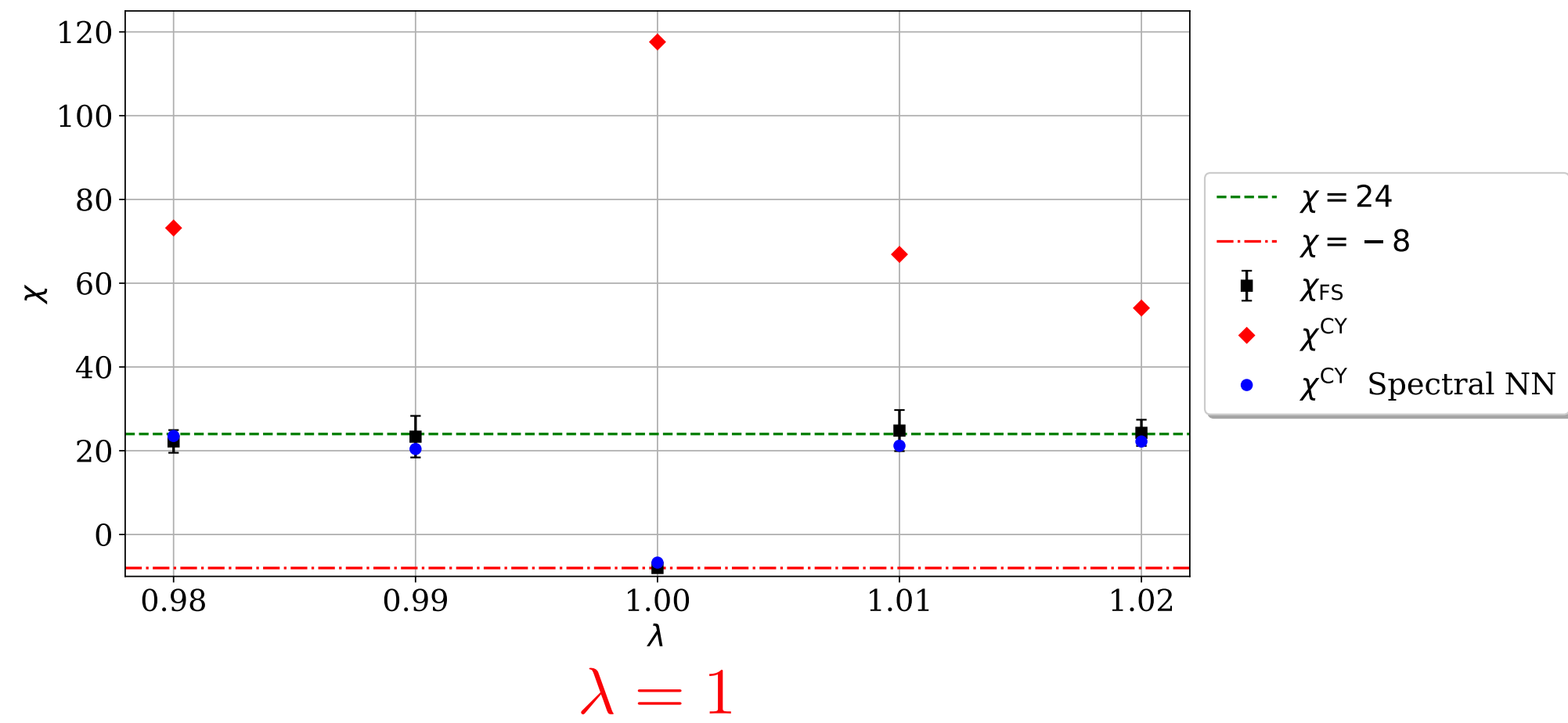
- $\lambda = 0$:



10^5 points for training
 performance equivalent to $k = 8$
 in Donaldson's algorithm

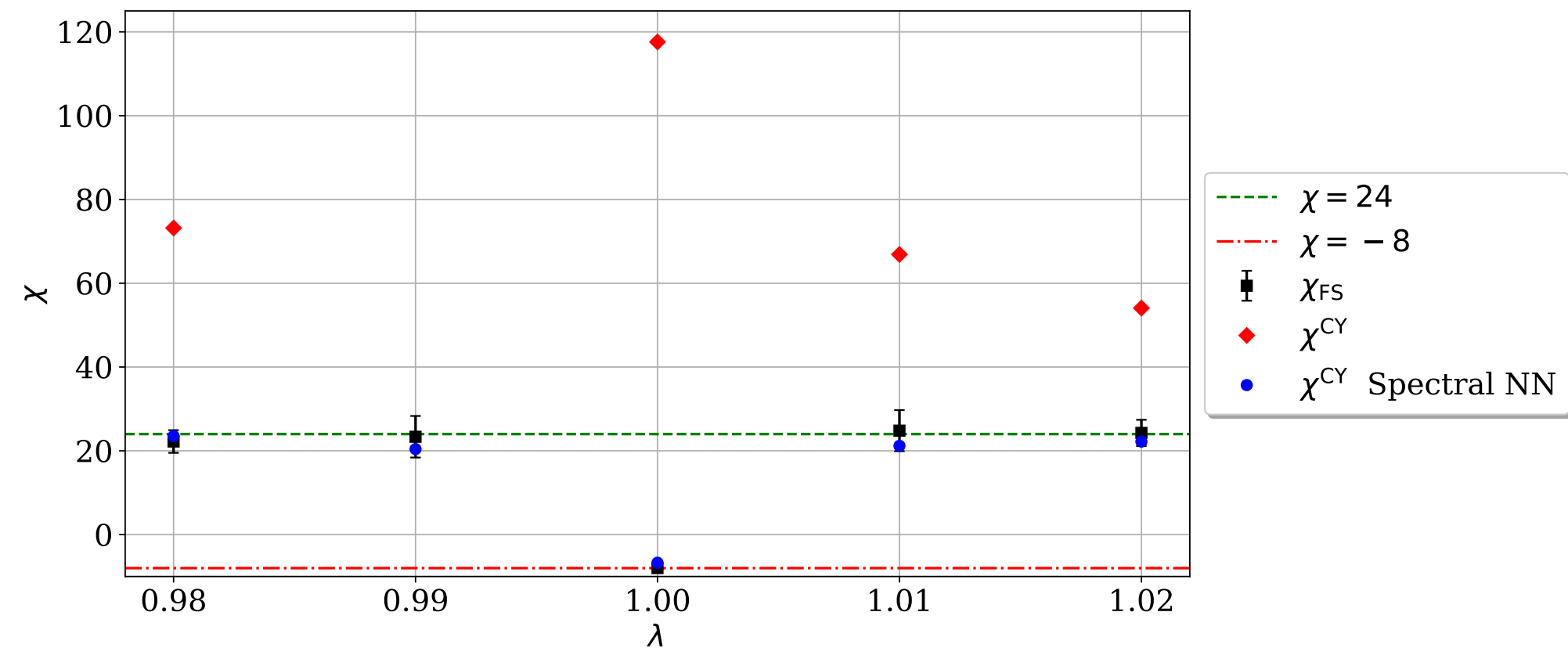
cf. Headrick, Nasser (2009)

Calabi–Yau Twofold

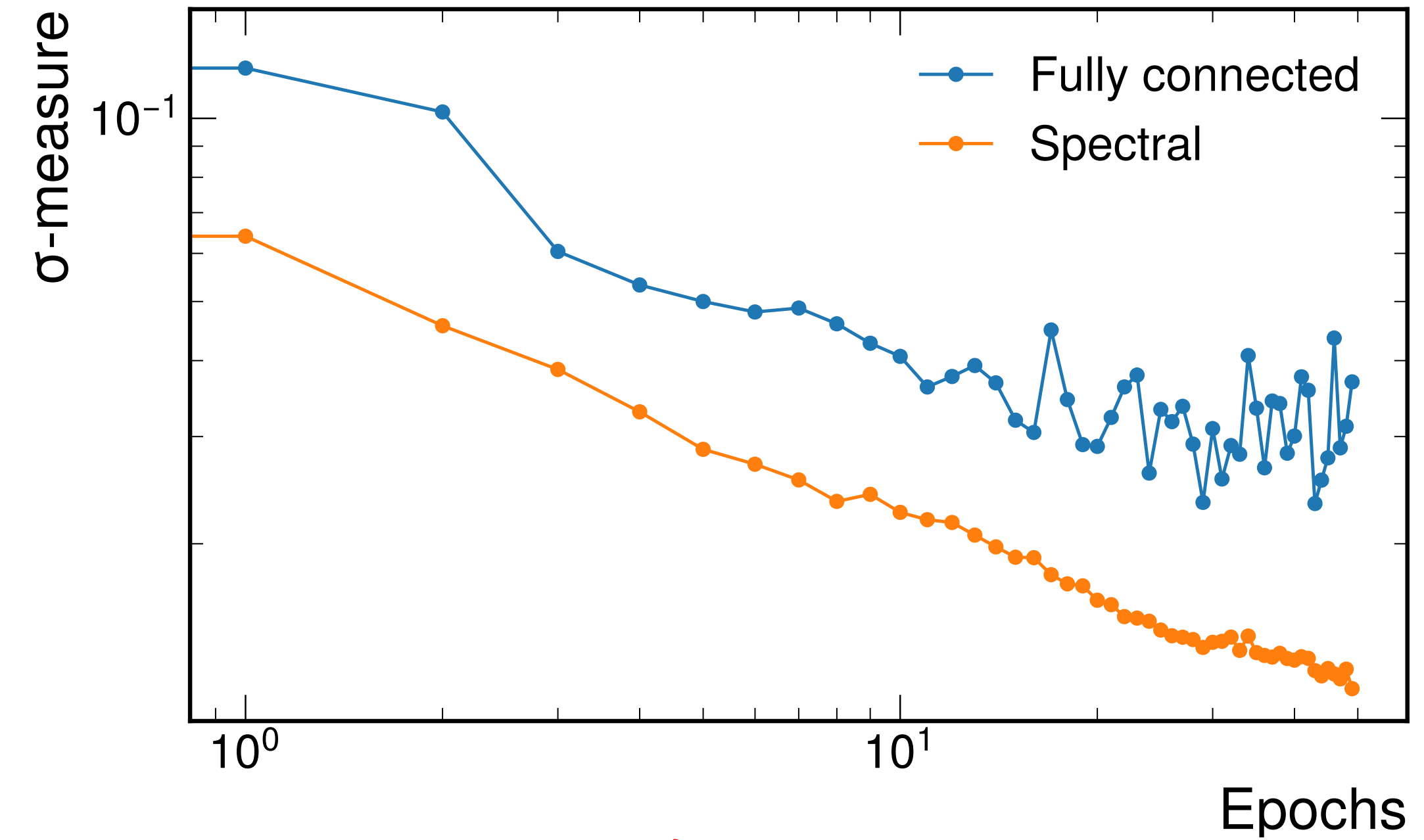


$$\chi = \int_{X_s} e(J) + 2 |\text{Sing}(X)| = 24 .$$

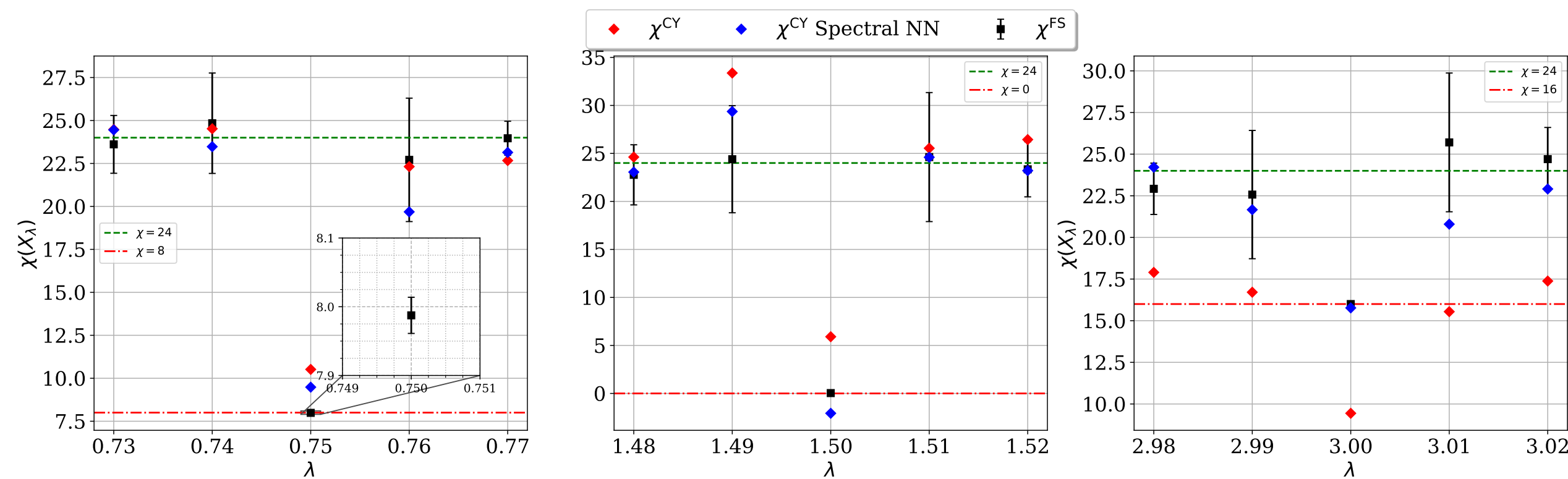
Calabi–Yau Twofold



$\lambda = 1$



$\lambda = 0.99$



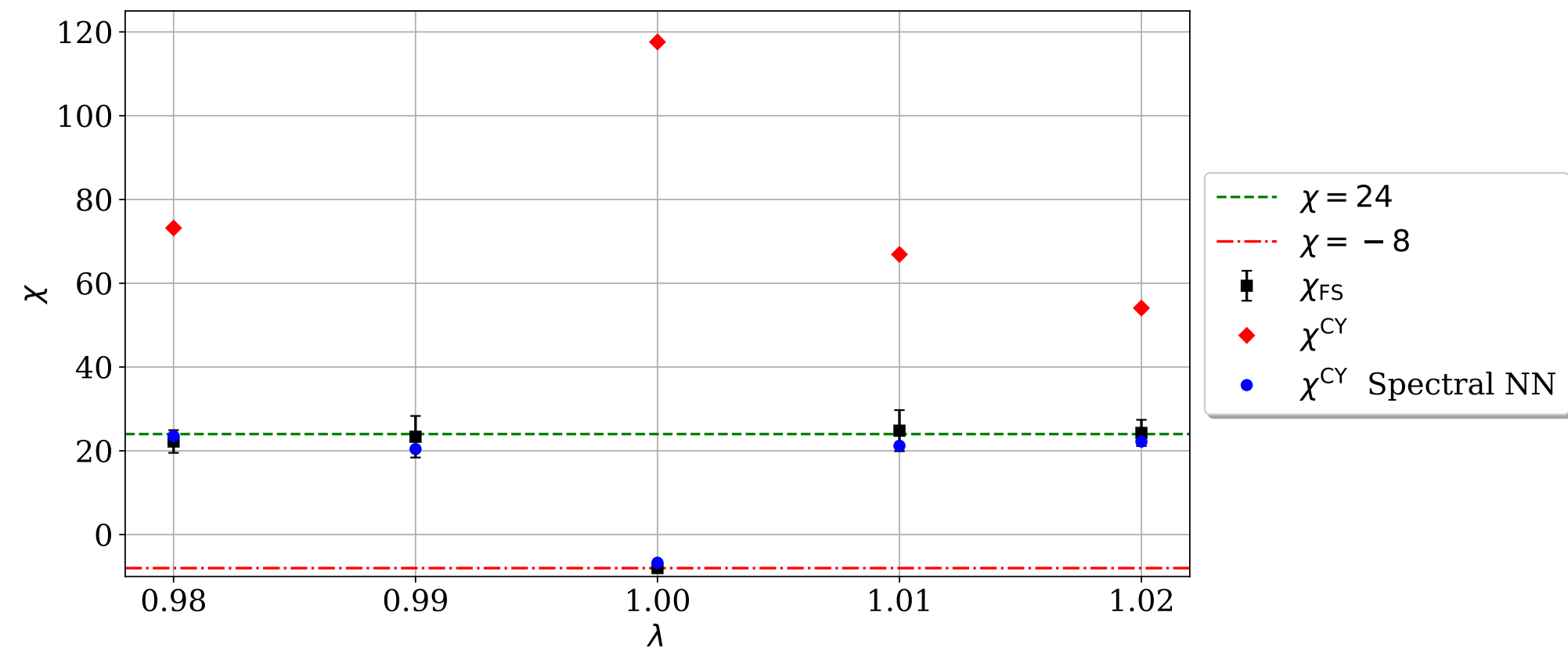
$\lambda = 0.75$

$\lambda = 1.5$

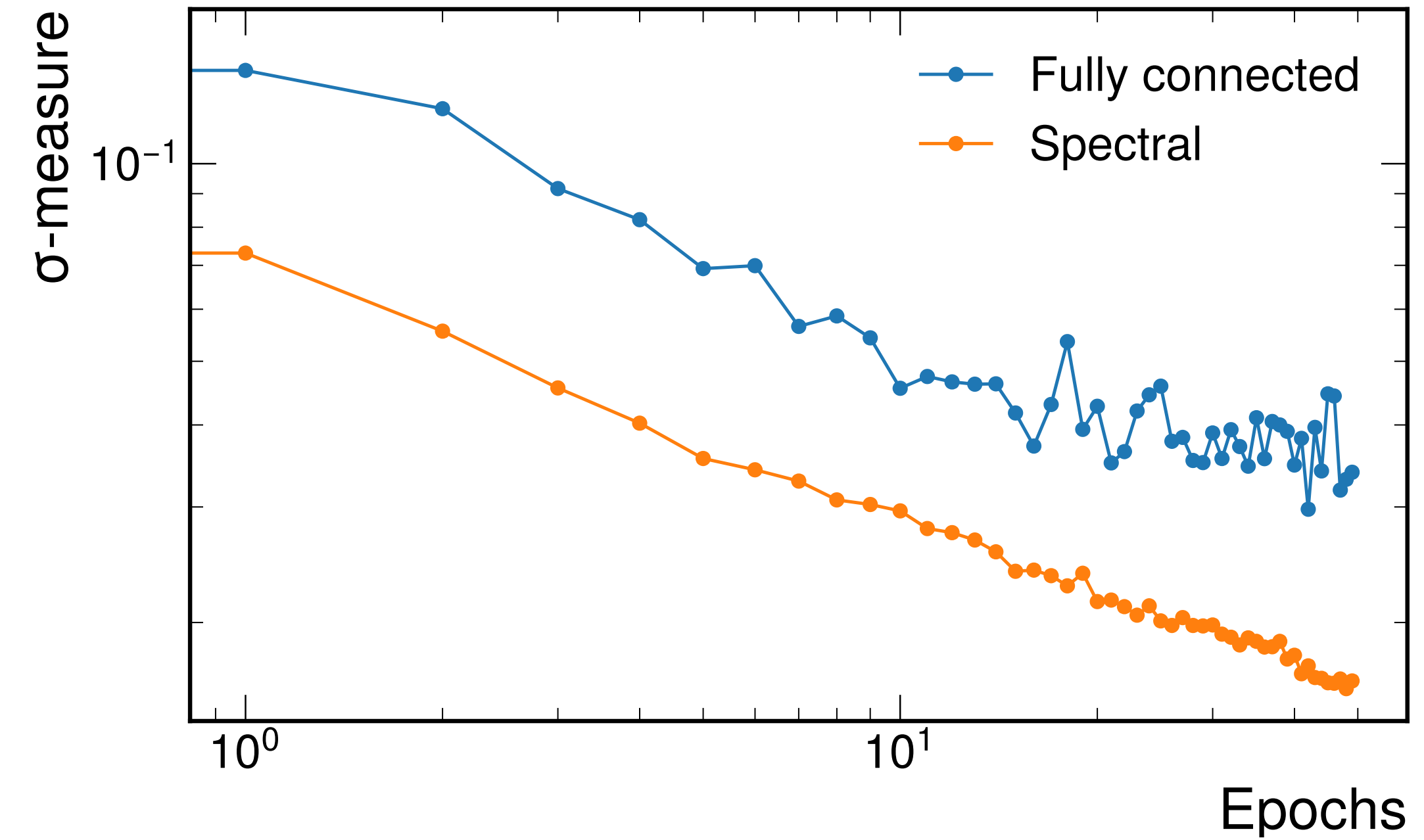
$\lambda = 3$

$$\chi = \int_{X_s} e(J) + 2 |\text{Sing}(X)| = 24 .$$

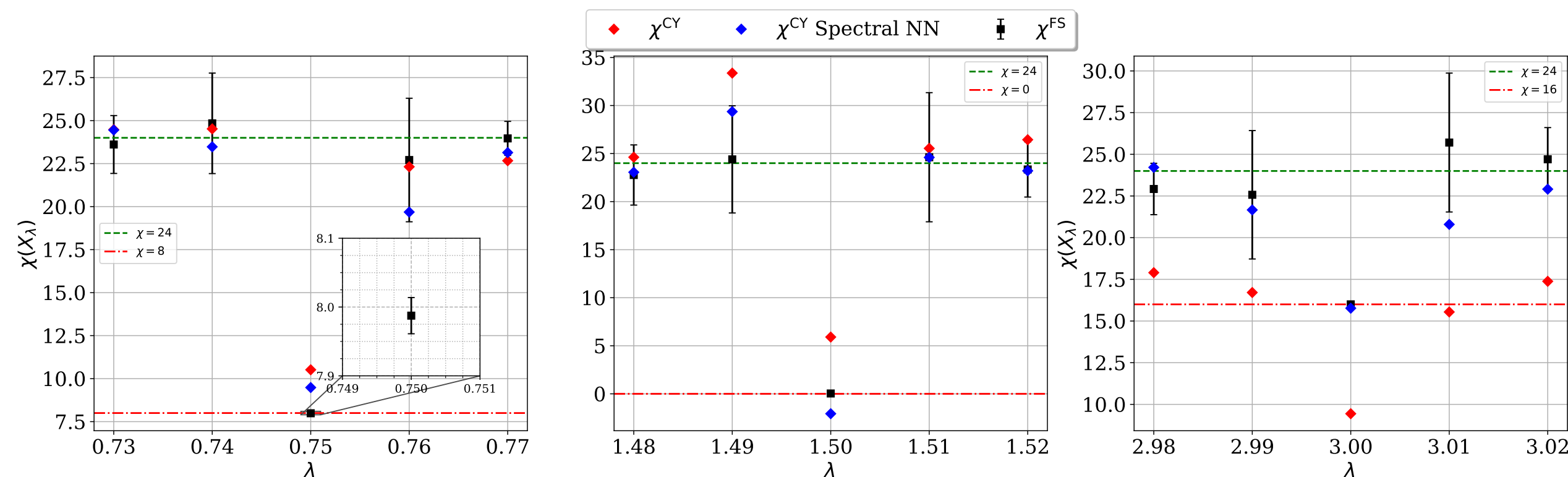
Calabi–Yau Twofold



$\lambda = 1$



$\lambda = 1.00$



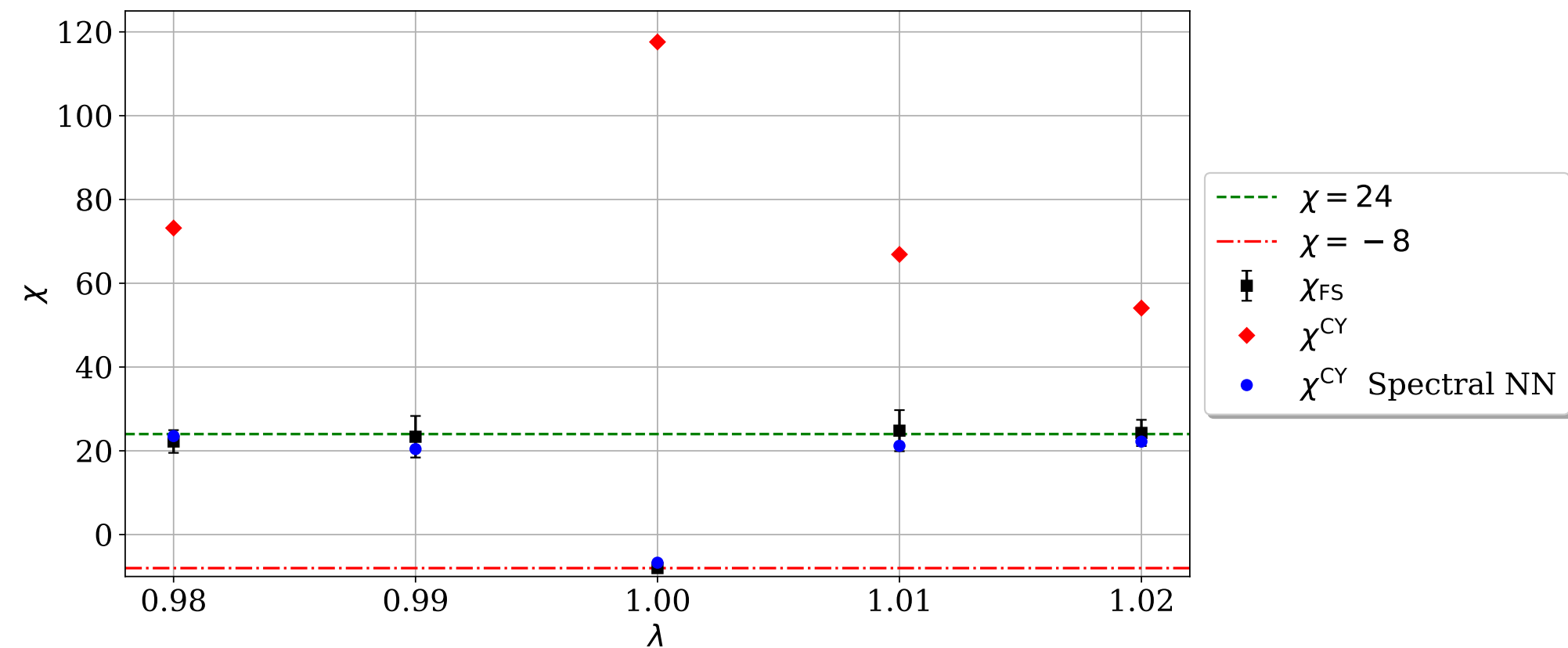
$\lambda = 0.75$

$\lambda = 1.5$

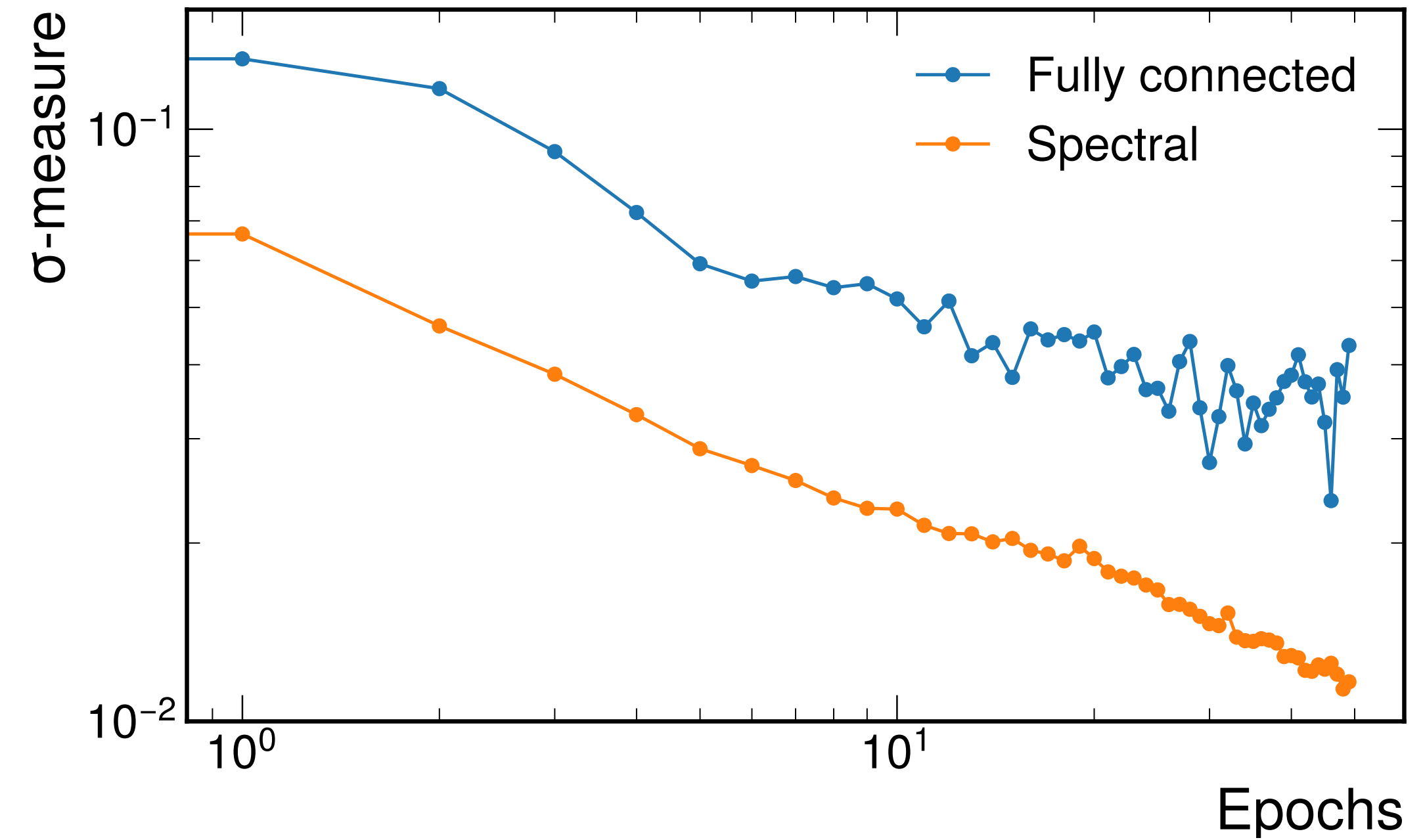
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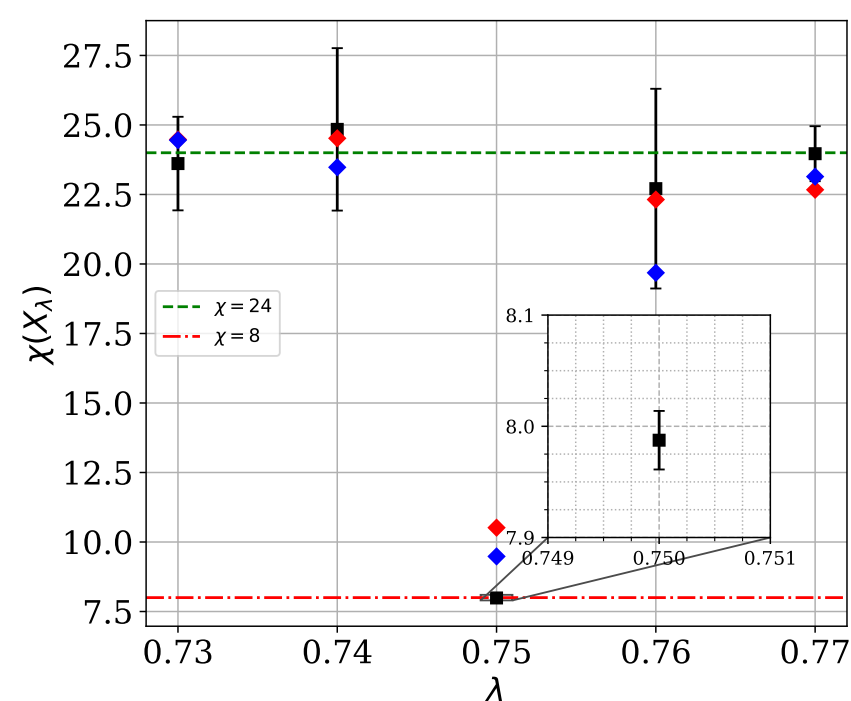
Calabi–Yau Twofold



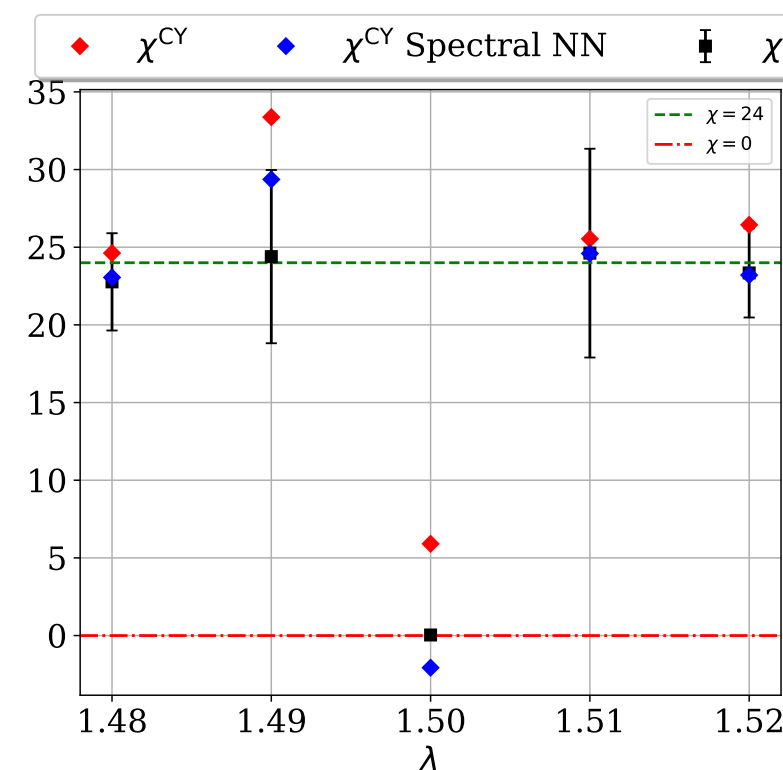
$\lambda = 1$



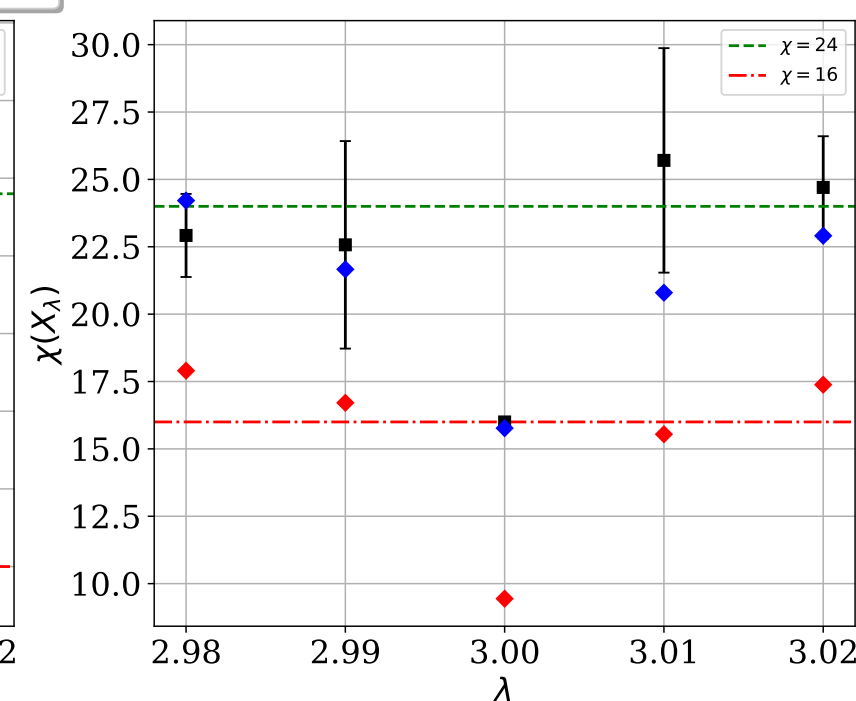
$\lambda = 1.01$



$\lambda = 0.75$



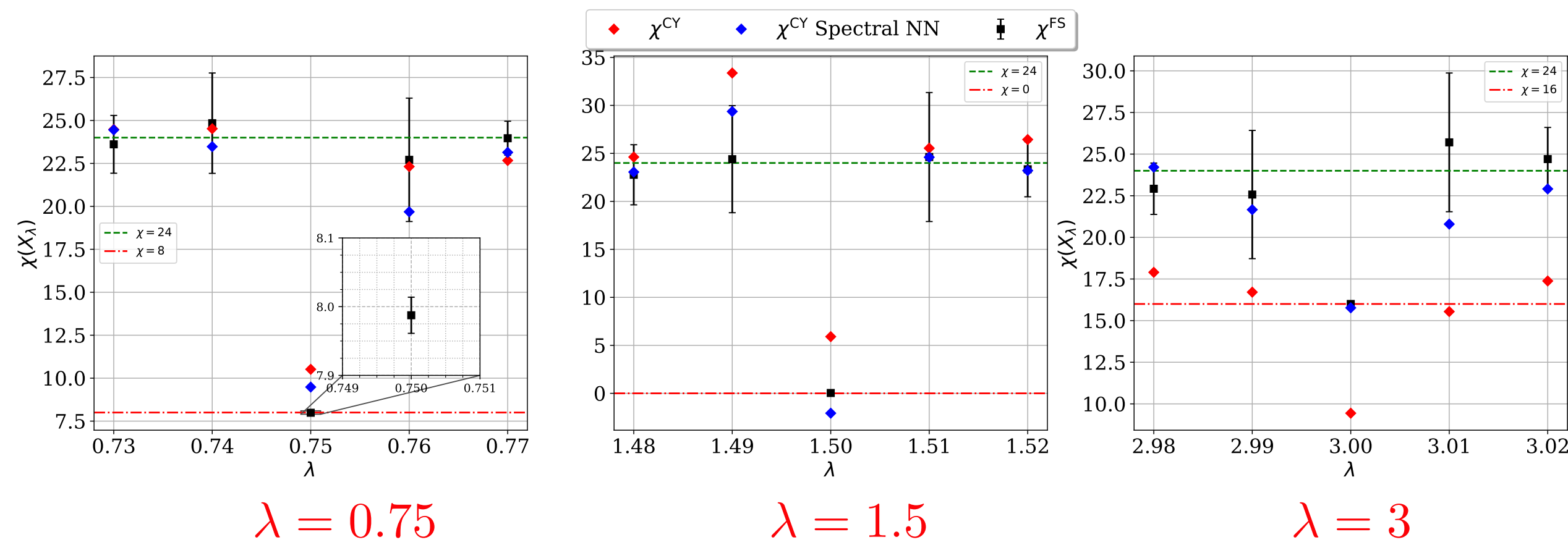
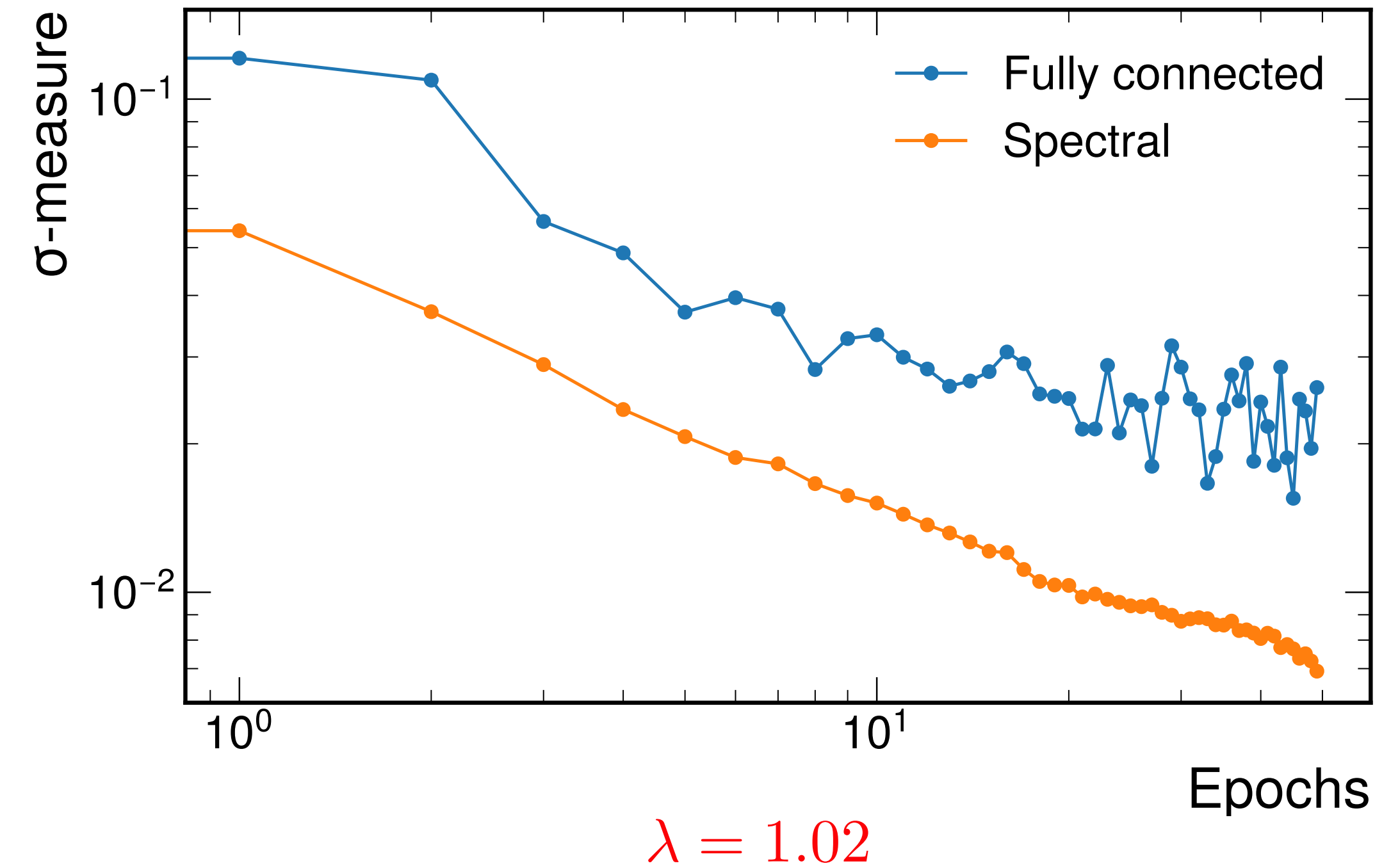
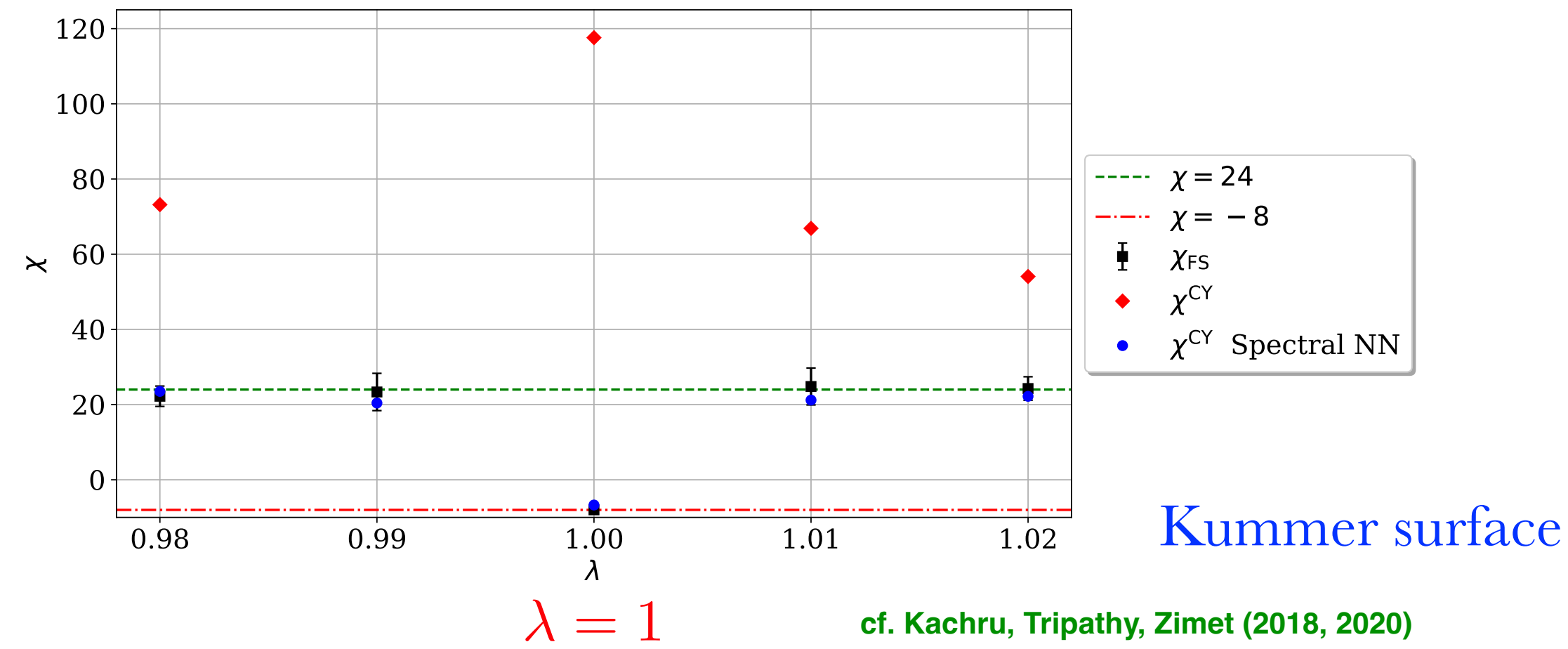
$\lambda = 1.5$



$\lambda = 3$

$$\chi = \int_{X_s} e(J) + 2 |\text{Sing}(X)| = 24 .$$

Calabi–Yau Twofold



$$\chi = \int_{X_s} e(J) + 2 |\text{Sing}(X)| = 24 .$$

Low-energy Matter Spectrum

- Charged matter multiplets are counted by $h^1(V)$, $h^1(V^*) = h^2(V)$
- We specialize to the case of E_6 , the commutant of $SU(3)$ holonomy in E_8
- $(1,1)$ -forms and $(2,1)$ -forms correspond to number of generations, anti-generations
- Yukawa couplings of $(1,1)$ -forms are topological
- There is a nowhere vanishing top holomorphic form $\Omega = \frac{1}{3!} \Omega_{ijk} dz^i \wedge dz^j \wedge dz^k$
- Given a $(2,1)$ -form, there is an equivalent $(0,1)$ -form: $\omega = \frac{1}{2} \omega_{ij\bar{k}} dz^i \wedge dz^j \wedge dz^{\bar{k}} \longleftrightarrow \omega^i = \frac{1}{2} \bar{\Omega}^{ijk} \omega_{j\bar{k}\bar{m}} dz^{\bar{m}}$
- Unnormalized Yukawa couplings computed as $\kappa = \int_X \Omega \wedge \omega^i \wedge \omega^j \wedge \omega^k \Omega_{ijk}$ (27)³ couplings

quasi-topological: depends on cohomology class of ω but not actual representative

Normalized Yukawa Couplings

- We have unnormalized trilinear couplings $\kappa_{ijk} = \int_X \Omega \wedge \tilde{\Omega}(a_i, a_j, a_k)$
- Generically, normalizations are computed using inner product that depends on Ricci-flat metric

$$(a, b)_V := \int_X a \wedge \overline{\star_V} \mathcal{H}b \quad \text{with} \quad \mathcal{H} : H^1(V) \rightarrow \mathcal{H}^1(V) \quad \text{denoting harmonic projection}$$

- The physical Yukawa couplings are $\lambda_{ijk} = \frac{\kappa_{ijk}}{\|a_i\|_V \|a_j\|_V \|a_k\|_V} \left(\int_X \Omega \wedge \overline{\Omega} \right)^{-1}$
- Here, $\{a_i\}_{i=1}^{h^{2,1}}$ is orthogonal basis for $H^1(V)$: $K_{i\bar{j}} = \int_X J \wedge J \wedge a_i \wedge \bar{a}_{\bar{j}} = \delta_{i\bar{j}}$
- In standard embedding, because normalization can be computed independent of Ricci-flat metric and only requires knowledge of metric on complex structure moduli space, the Yukawa couplings are same in every Kähler class

Standard Embedding

- Let holomorphic vector bundle that solves Hermitian Yang–Mills equations be tangent bundle so that metric on X does double duty as the fiber metric
- When $V \cong T_X$, the number of generations $N_{\text{gen}} = \text{ind } \mathcal{D} = \frac{1}{2}|\chi| = |h^{1,1} - h^{2,1}|$, where $\mathcal{D}^2 = \square$
- Deformations of V correspond to complex structure moduli of X
- Metric on complex structure moduli space is Weil–Petersson metric
- Using Weil–Petersson geometry, can compute Yukawa couplings without knowing Ricci-flat metric
- In standard embedding, will compute Yukawa couplings in three different ways
 - period integrals
 - Kodaira–Spencer map
 - explicit harmonic forms using Ricci-flat metric

} Weil–Petersson geometry

Period Integrals

- Symplectic basis of 3-cycles $A^a \cap B_b = \delta_b^a$, with $a, b = 1, \dots, h^{2,1}$

$$A^a \cap A^b = B_a \cap B_b = 0$$

- Dual 3-forms $\int_{A^a} \alpha_b = \delta_b^a$, $\int_{B_a} \beta^b = \delta_a^b$

$$\int_X \alpha_a \wedge \beta^b = \delta_a^b, \quad \int_X \alpha_a \wedge \alpha_b = \int_X \beta^a \wedge \beta^b = 0$$

- Holomorphic (3,0)-form is $\Omega = z^a \alpha_a - \mathcal{G}_b \beta^b$ so that $\mathcal{V} = \int_X \Omega \wedge \bar{\Omega} = \bar{z}^a \mathcal{G}_a - z^a \bar{\mathcal{G}}_a$

$$z^a = \int_{A^a} \Omega, \quad \mathcal{G}_b = \int_{B_b} \Omega \quad \text{satisfy Picard–Fuchs equation}$$

- Weil–Petersson metric is $g_{\text{WP}} = -i\partial\bar{\partial} \log \mathcal{V}$

Candelas, de la Ossa, Green, Parkes (1991)
Morrison (1991)

- Analytic result from period integrals — however, it is only practical for $h^{2,1} \sim \text{few}$

Kodaira–Spencer Geometry

- Kodaira–Spencer map is $\rho : T_{t_0}B \rightarrow H^1(X; T_X)$
- On overlaps $U_i \cap U_j$, we have transition functions $f_{ij} : z_j \mapsto z_i$
- To account for complex structure deformations, take $f_{ij}(z_j; t)$, where $t = (t^1, \dots, t^m)$, $m = h^{2,1}(X)$
- Locally, this is fibration of X over a base B with reference point t_0
- Explicitly, consider diffeomorphism $X := X_{t_0} \xrightarrow{\zeta_t} X_t$; ξ denotes infinitesimal diffeomorphism ζ_t at $t = t_0$
 Then, $\rho(\frac{\partial}{\partial t}) = [\bar{\partial}\xi]$ under identification $H^1(T_X) \simeq H_{\bar{\partial}}^{2,1}(X)$
- Weil–Petersson metric is $\langle a, b \rangle_{\text{WP}} = \int_{X_t} \rho(a) \wedge \overline{\star_{g_t} \mathcal{H} \rho(b)} \propto \left(\frac{d\Omega_t}{dt^a}, \frac{d\Omega_t}{dt^b} \right) + \frac{\left| \left(\Omega, \frac{d\Omega_t}{dt} \right) \right|^2}{(\Omega, \Omega)} \Bigg|_{t_0}$ ← elides knowledge of Ricci-flat metric
- In standard embedding, $N_{a\bar{b}} \sim (a, b)_V = \langle a, b \rangle_{\text{WP}}$

Machine Learning Sections

- Start from harmonic projection $\mathcal{H} : H^1(T_X) \rightarrow \mathcal{H}^1(T_X)$
 $\psi \in \rho(\partial_t) \mapsto \mathcal{H}\psi$
- Find smooth section s of T_X such that $\Delta(\psi + \bar{\partial}s) = 0$ — *i.e.*, $\mathcal{H}\psi - \psi = \bar{\partial}s$
- Strategy is to model s with a neural network using the machine learned Ricci-flat metric
 - start with an overcomplete basis of sections
 - parametrize coefficients to get globally defined object
- In principle, can do this for any holomorphic vector bundle on a Kähler manifold

Douglas (2023)
cf. Constantin, Fraser-Taliente, Harvey, Lukas, Ovrut (2024)

Machine Learning Sections

- Given a reference form $\psi \in H^1(X; V)$, approximate cohomologous harmonic projection

$\mathcal{H}\psi \approx \eta := \psi + \bar{\partial}_V s_{\text{NN}}$, where s_{NN} is given by a spectral network

- In particular, $s_{\text{NN}} = \sum_{ijkl} \psi_{ijkl}^{(\text{NN})} \frac{\overline{\alpha_\nu^{ij}} z^k z^l}{|z|^4} g^{\mu\bar{\nu}} \cdot \frac{\partial}{\partial z^\mu} \in \Gamma(T_X)$

output of spectral network

$$\alpha_\nu^{ij} dz^\nu = \iota^*(z^i dz^j - z^j dz^i), \text{ where } \iota : X \hookrightarrow A$$

Greek Latin

Ashmore (2023)

- Harmonic forms minimize the inner product $(\eta, \Delta_V \eta)$

- $\bar{\partial}_V$ closed by construction

- Minimize loss function $\ell_H = (\bar{\partial}_V^\dagger \eta, \bar{\partial}_V^\dagger \eta)$

- Methods apply to any $h^{2,1}$

Quintic

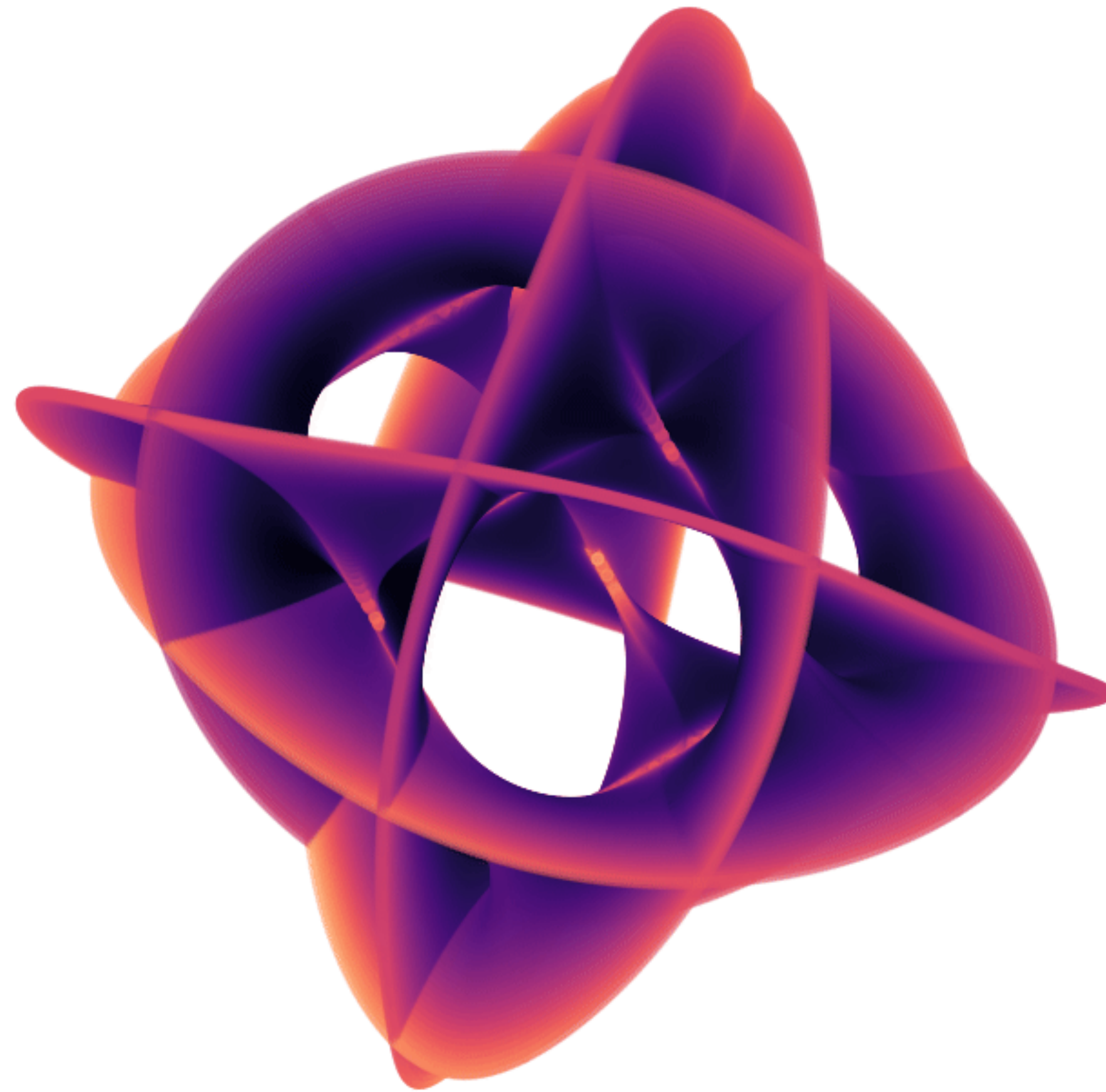
- Fermat quintic, $\psi = 0$

$$\sum_{i=1}^5 z_i^5 - 5\psi z_1 z_2 z_3 z_4 z_5 = 0 \subset \mathbb{P}^4$$

singular when $\psi^5 = 1$

$$h^{1,1} = 1, \quad h^{2,1} = 101$$

ϕ



Quintic

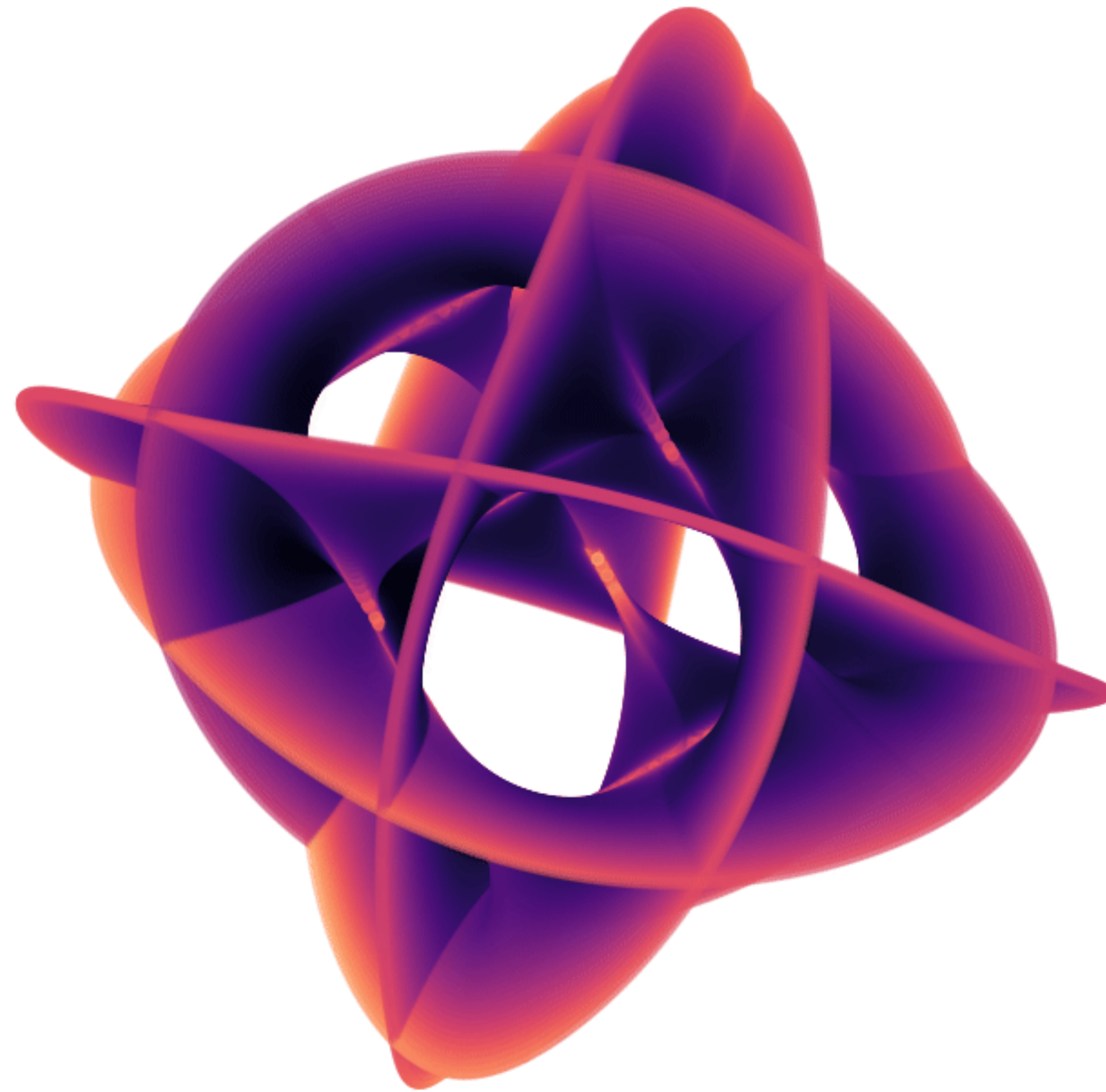
- Fermat quintic, $\psi = 0$

$$\sum_{i=1}^5 z_i^5 - 5\psi z_1 z_2 z_3 z_4 z_5 = 0 \subset \mathbb{P}^4$$

singular when $\psi^5 = 1$

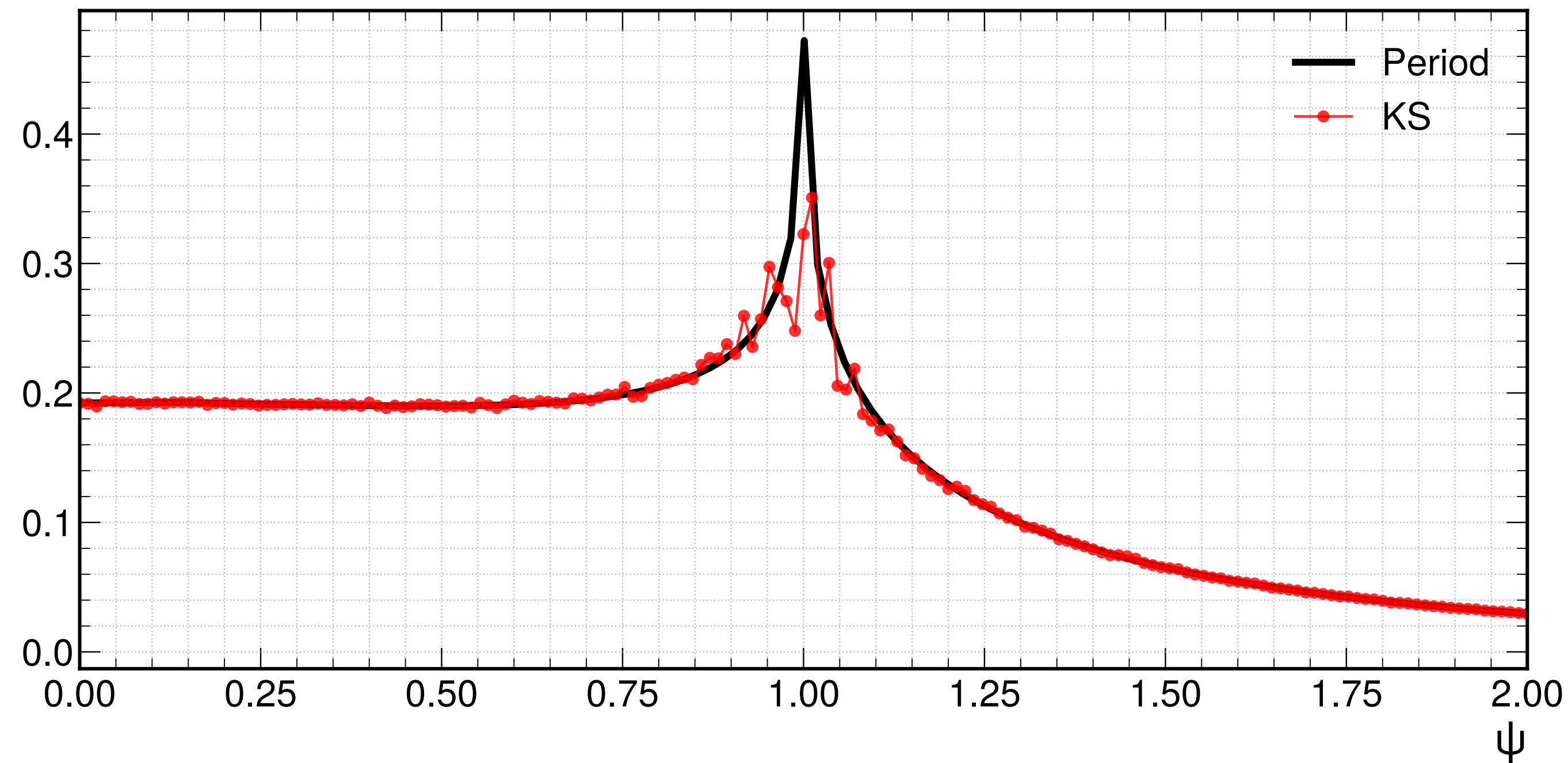
$$h^{1,1} = 1, \quad h^{2,1} = 101$$

ϕ



Quintic

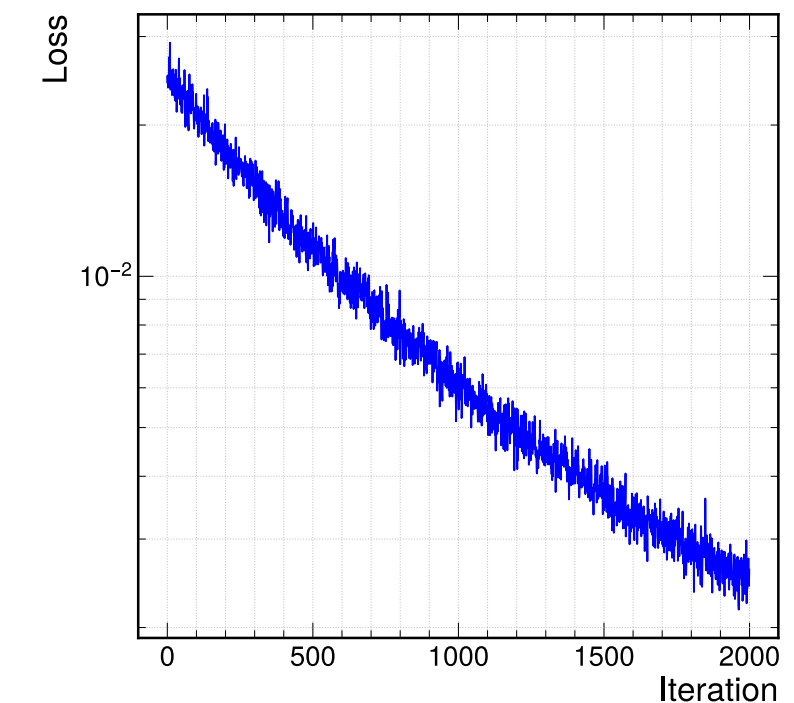
- Mirror of Dwork quintic



$$\sum_{i=1}^5 z_i^5 - 5\psi z_1 z_2 z_3 z_4 z_5 = 0 \subset \mathbb{P}^4$$

singular when $\psi^5 = 1$

$$h^{1,1} = 1, \quad h^{2,1} = 101$$



loss for finding harmonic representative

$$g_{\psi\bar{\psi}}(\psi=0) : 0.1914 \quad \text{vs.} \quad 25 \left(\frac{\Gamma(\frac{4}{5})\Gamma(\frac{2}{5})}{\Gamma(\frac{1}{5})\Gamma(\frac{3}{5})} \right)^5 \approx 0.1923$$

ML

periods

Quintic

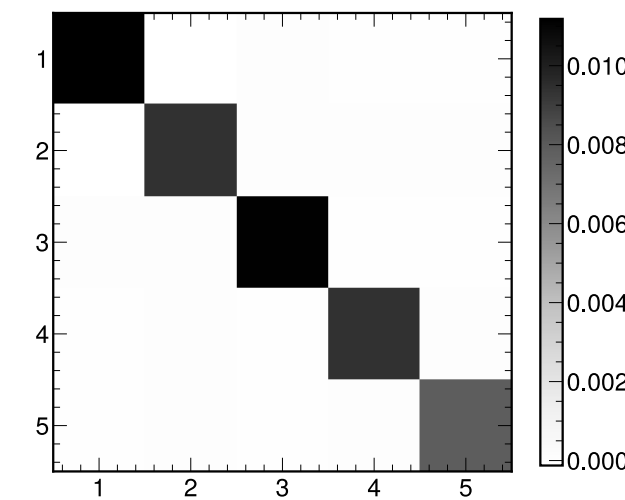
$$\sum_{i=1}^5 z_i^5 - 5\psi z_1 z_2 z_3 z_4 z_5 = 0 \subset \mathbb{P}^4$$

- Gepner model $Y_{4;5}$ is specific point in moduli space
mod Fermat quintic by $\mathbb{Z}_5 \times \mathbb{Z}_5$ to get manifold with $h^{1,1} = 1$, $h^{2,1} = 5$

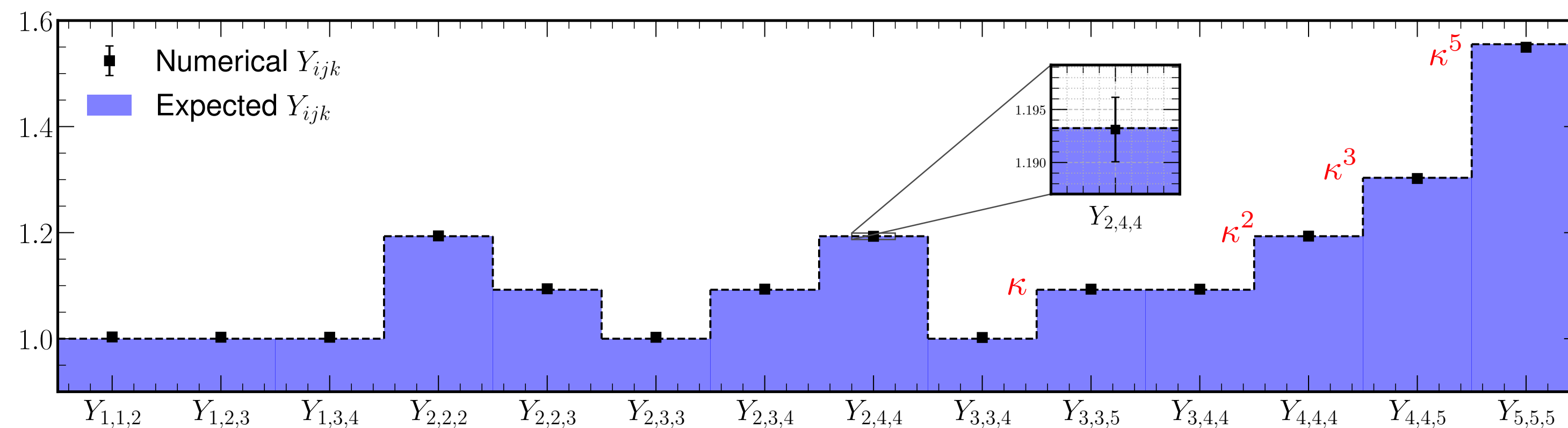
- Yukawa couplings are powers of $\kappa = \sqrt{\frac{\Gamma(3/5)^3 \Gamma(1/5)}{\Gamma(2/5)^3 \Gamma(4/5)}} \approx 1.092$

Distler, Greene (1988)

- Normalization with respect to Weil–Petersson metric



- Normalized Yukawa couplings



Butbaia, Mayorga Peña, Tan, Berglund, Hübsch, VJ, Mishra (2024)

Quarti-Quadric

- Two Kähler classes descending from projective factors

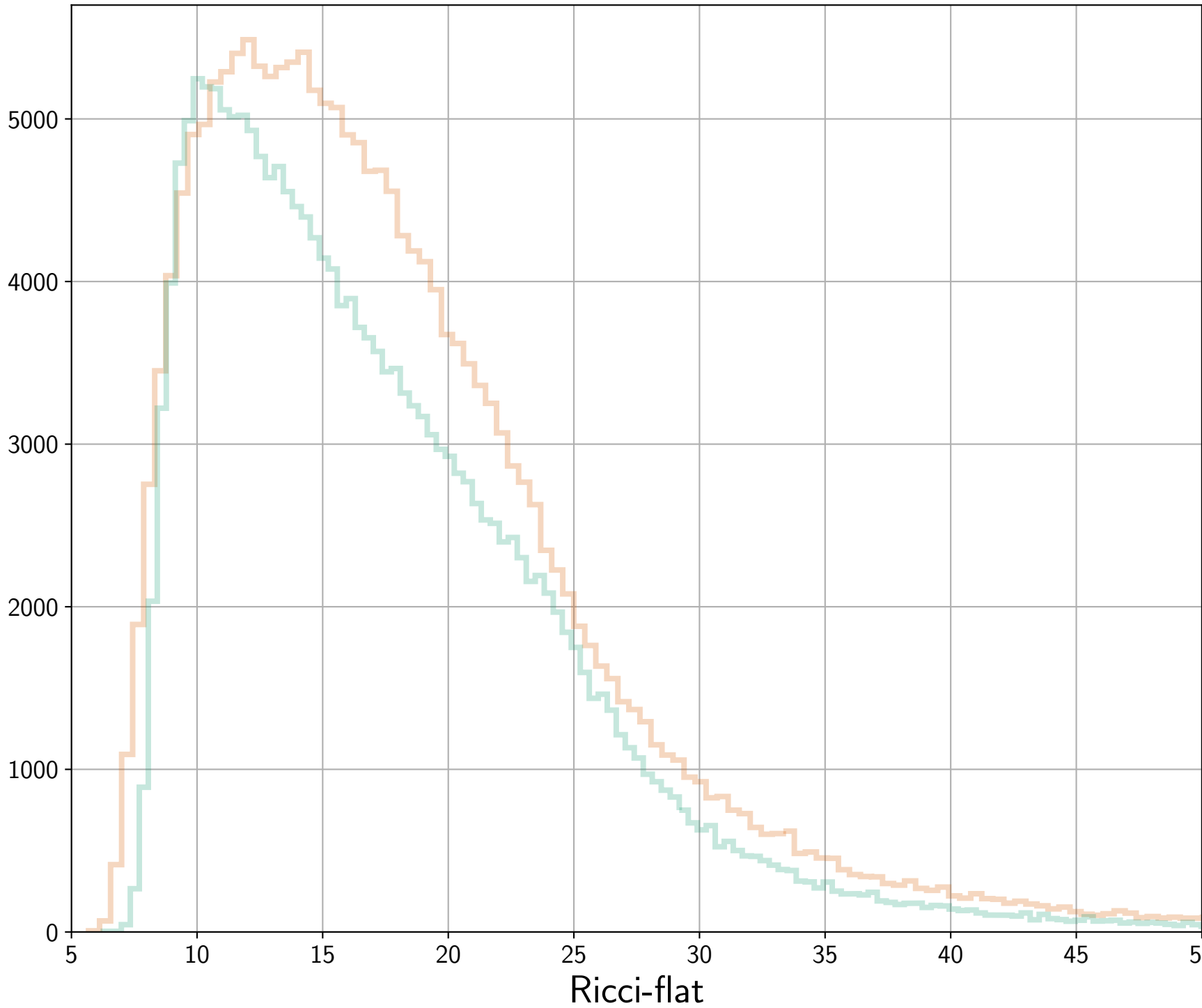
$$J = t_1 J_1 + t_2 J_2$$

- Kretschmann scalar $(R_{a\bar{b}c\bar{d}}R^{a\bar{b}c\bar{d}})^{\frac{1}{2}}$

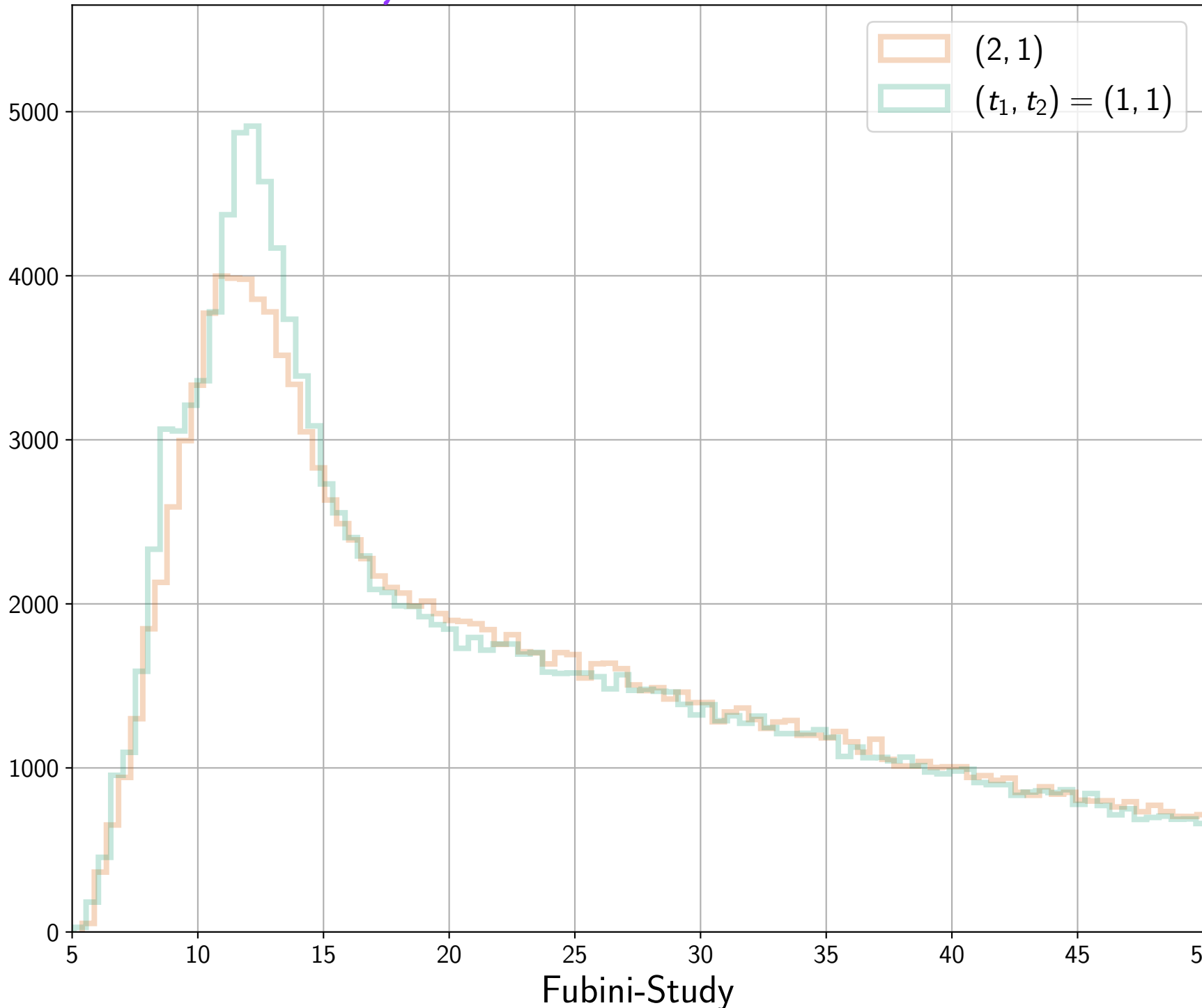
$$\left[\begin{array}{c|c} \mathbb{P}^3 & 4 \\ \mathbb{P}^1 & 2 \end{array}\right]_{\chi=-168}^{2,86}$$

$$x_1^4(y_1^2+y_2^2) + x_2^4(y_1^2+2y_2^2) + x_3^4(y_1^2-y_2^2) + x_4^4(y_1^2-2y_2^2) - 4\psi x_1x_2x_3x_4y_1y_2 = 0$$

Ricci-flat



Fubini-Study



singular when $\psi^4 = -1, -9$

$\psi = 0.2$

Quarti-Quadric

- Two Kähler classes descending from projective factors

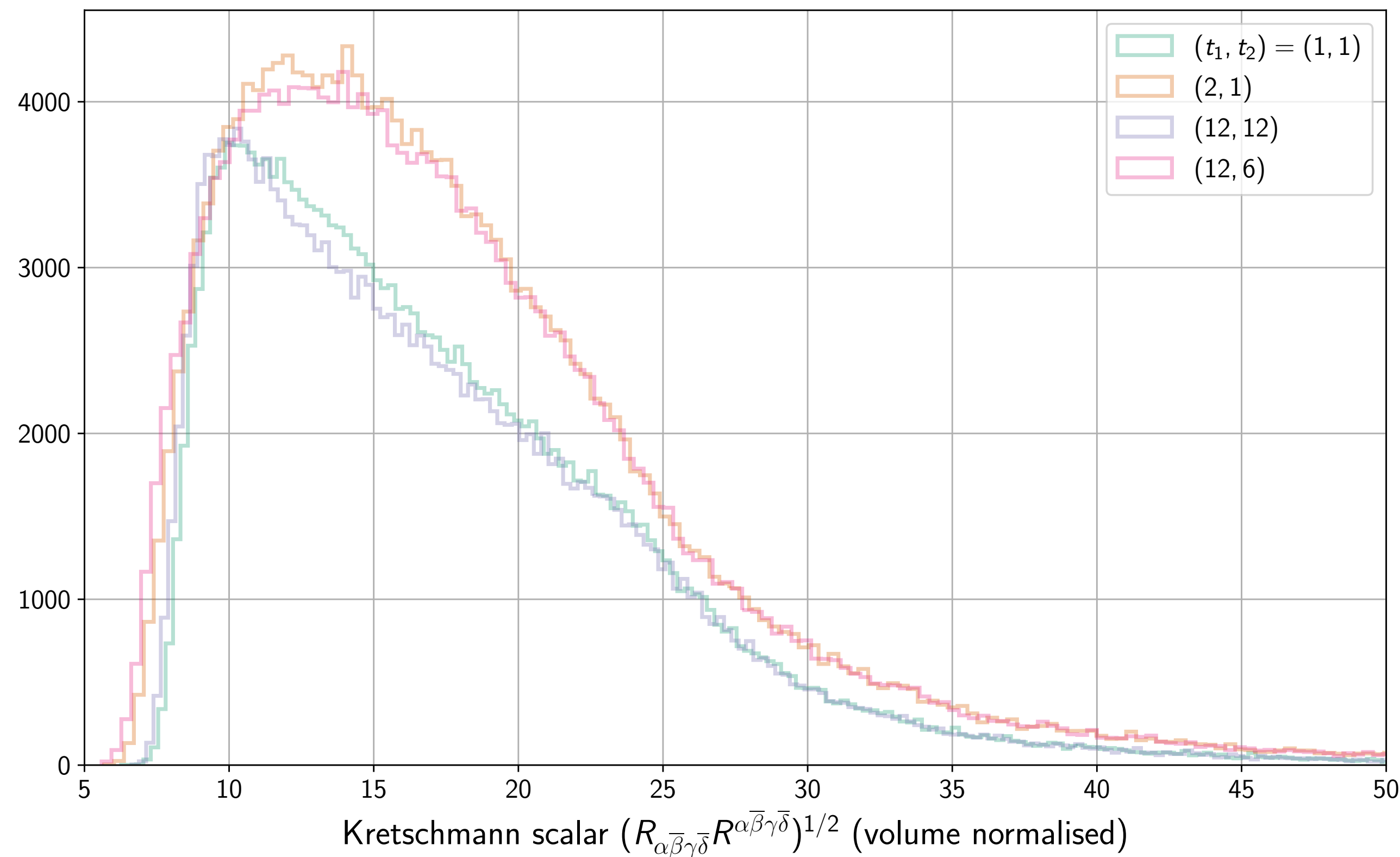
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singular when $\psi^4 = -1, -9$



$\psi = 0.2$

different distributions in two Kähler classes

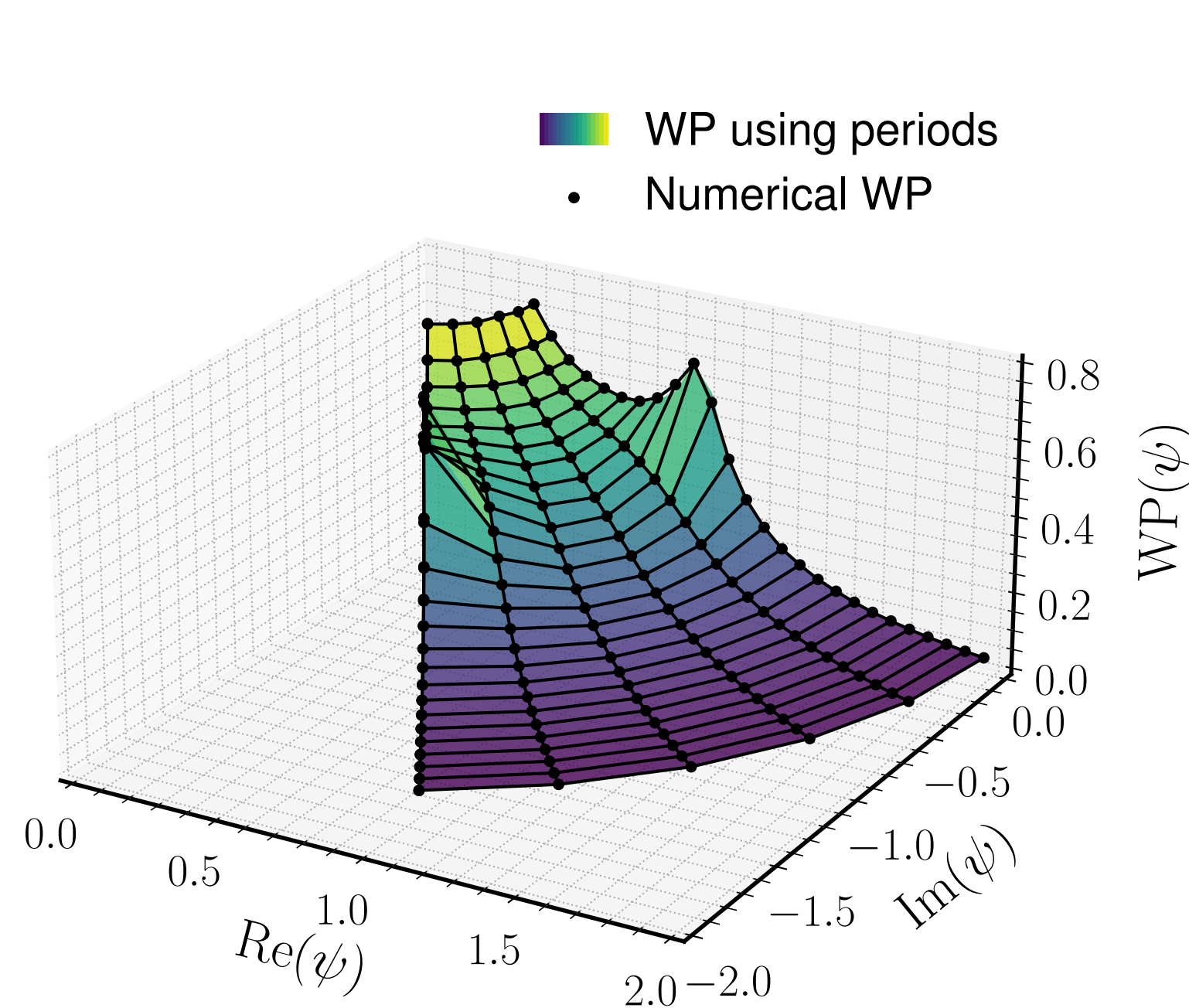
same in different descriptions of same Kähler class

Intersection of Two Cubics

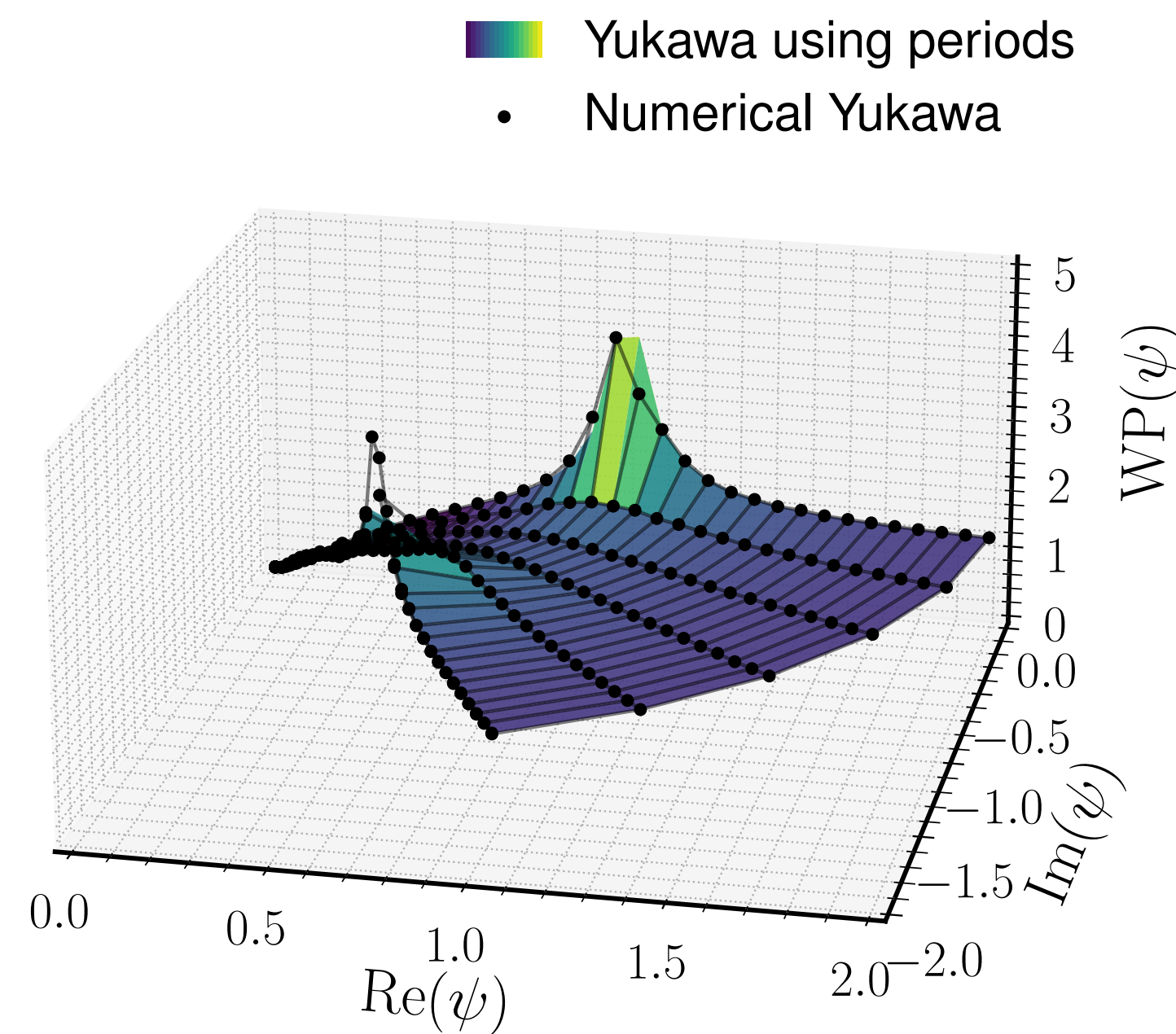
- Mirror of $\mathbb{P}^5[3, 3]$

$$\sum_{i=1}^3 x_i^3 - 3\psi x_4 x_5 x_6 = \sum_{i=4}^6 x_i^3 - 3\psi x_1 x_2 x_3 = 0$$

$$h^{1,1} = 1, \quad h^{2,1} = 73$$



(a) Weil–Petersson metric



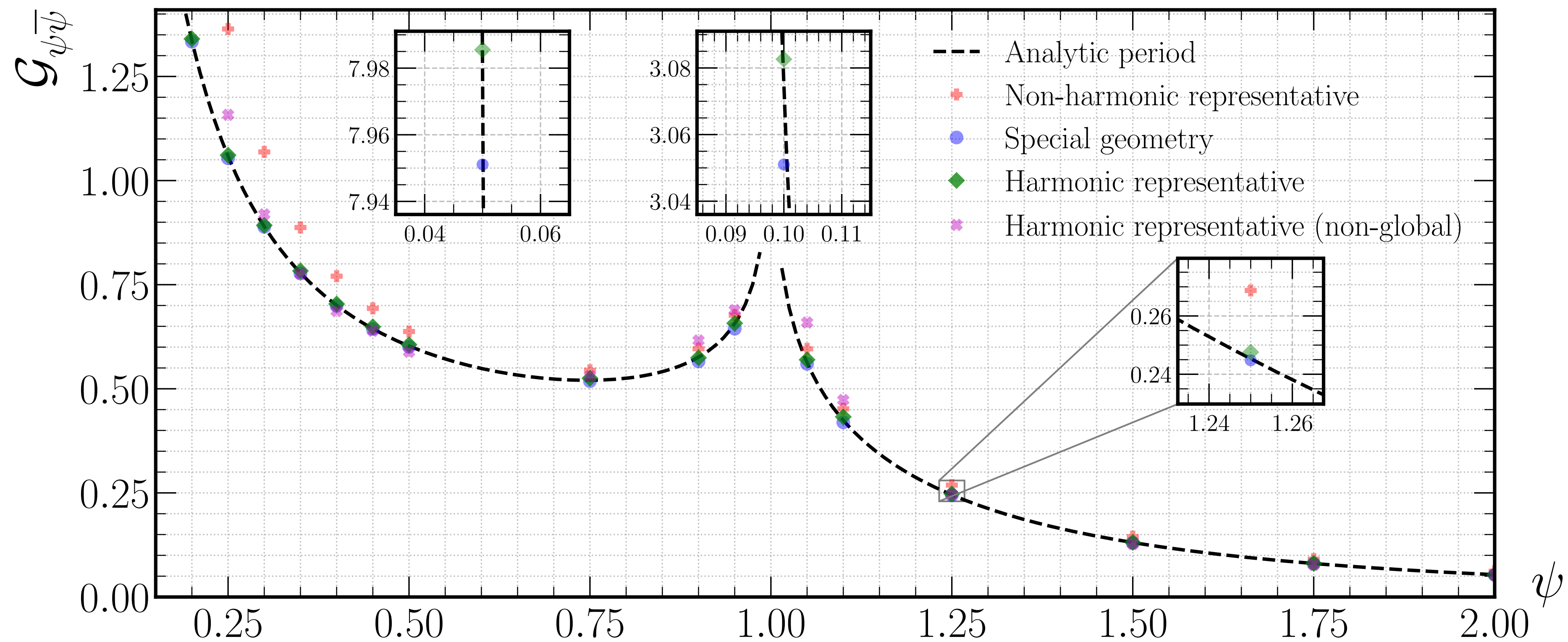
(b) Normalized Yukawa coupling

Intersection of Two Cubics

- Mirror of $\mathbb{P}^5[3, 3]$

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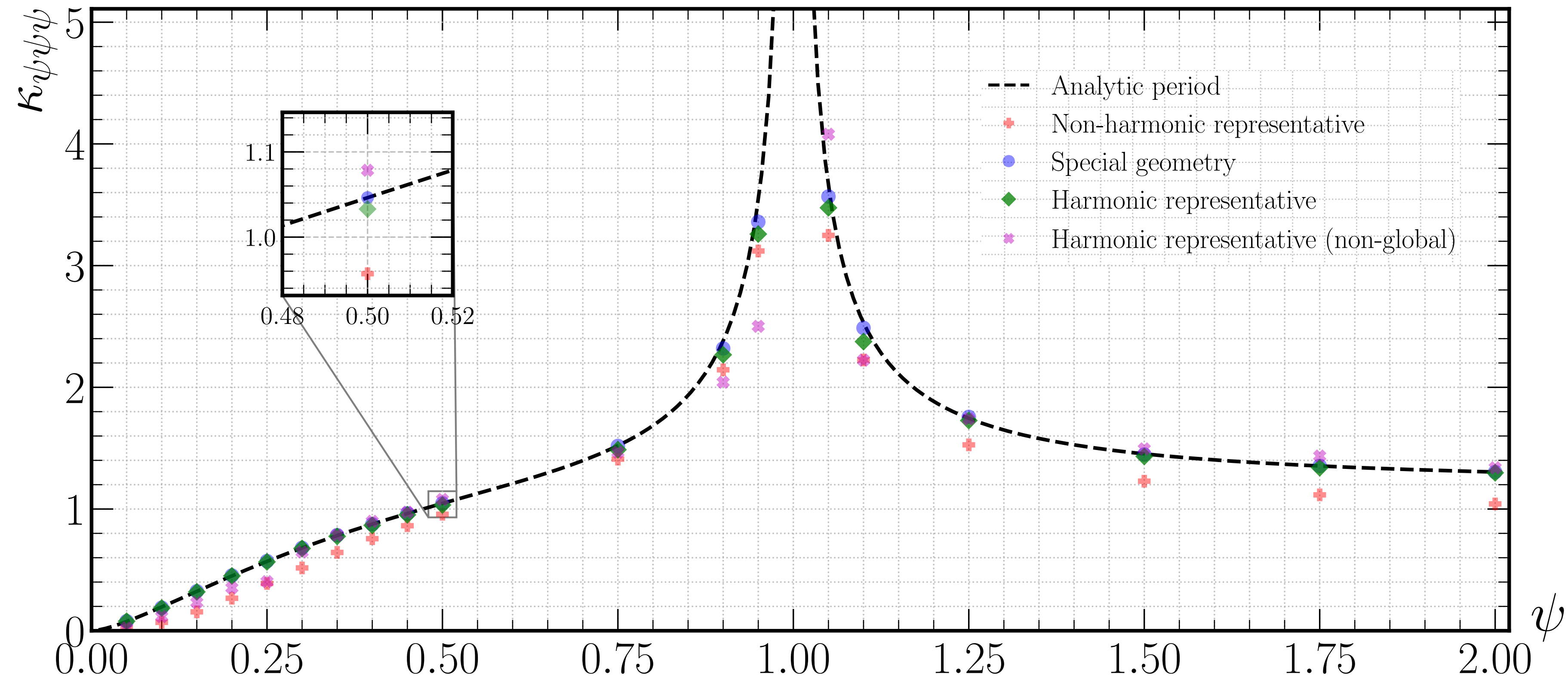


Intersection of Two Cubics

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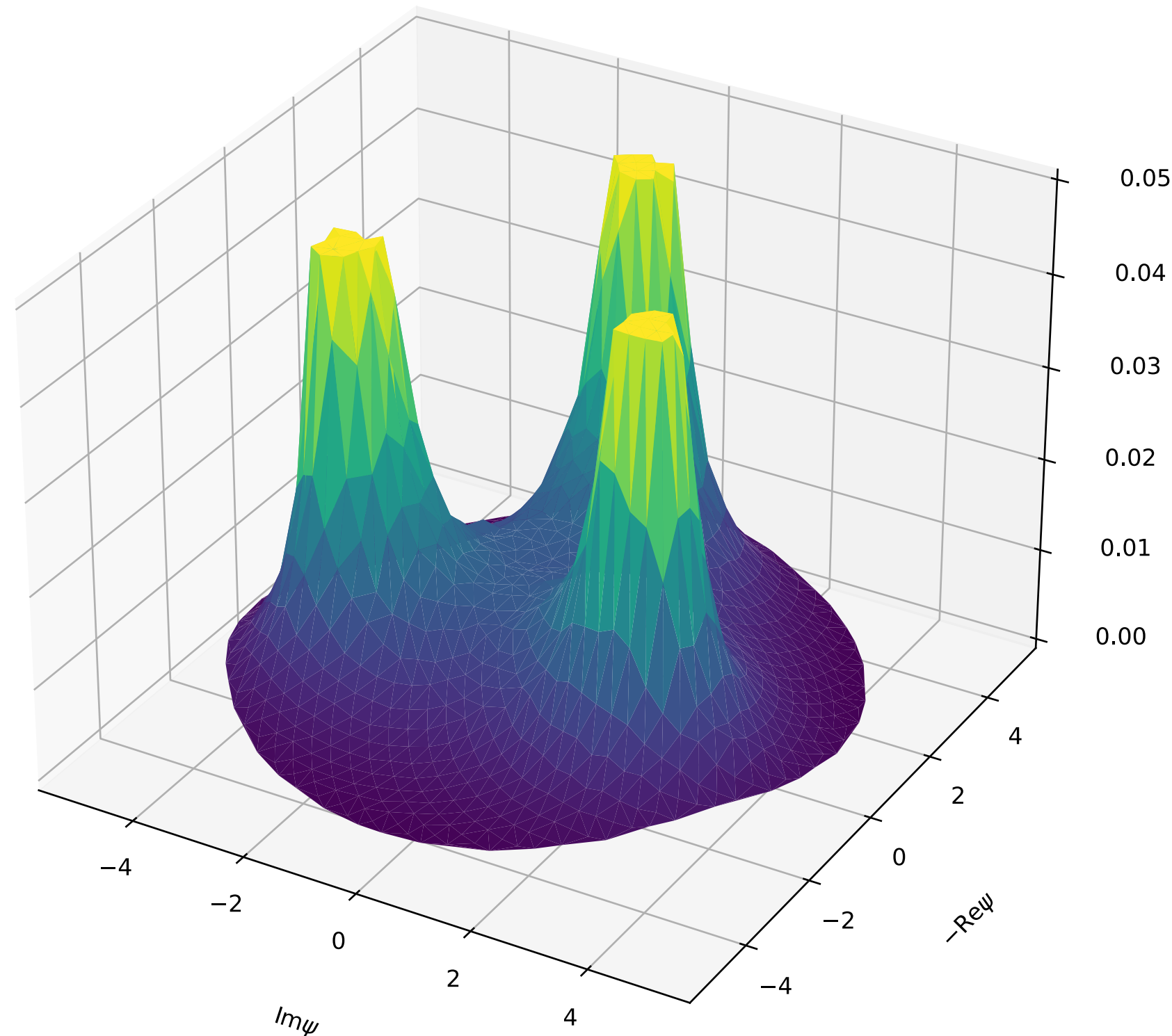
Tian–Yau

- Tian–Yau quotient

- Weil–Petersson metric along one direction of moduli

$$\frac{1}{3} \sum_{i=1}^4 x_i^3 + \psi x_1 x_2 x_3 = 0, \quad \epsilon = 0$$

$$g_{\lambda_1 \bar{\lambda}_1} (p_1 \rightarrow p_1 + \psi x_0 x_1 x_2)$$



$$\left[\begin{array}{c} \mathbb{P}^3 \\ \mathbb{P}^3 \end{array} \middle| \begin{array}{ccc} 3 & 0 & 1 \\ 0 & 3 & 1 \end{array} \right]_{\chi=-18}^{14, 23}$$

$$\frac{1}{3} \sum_{i=1}^4 x_i^3 = \frac{1}{3} \sum_{i=1}^4 y_i^3 = \sum_{i=1}^4 x_i y_i + \epsilon \sum_{i=3}^4 x_i y_i = 0$$

$$\begin{aligned} \mathbb{Z}_3 : \quad (x_1, x_2, x_3, x_4) &\mapsto (x_1, \omega_3^{-1} x_2, \omega_3 x_3, \omega_3 x_4) \\ (y_1, y_2, y_3, y_4) &\mapsto (y_0, \omega_3 y_2, \omega_3^{-1} y_3, \omega_3^{-1} y_4) \end{aligned}$$

$$h^{1,1} = 6, \quad h^{2,1} = 9, \quad \chi = -6$$

$$SU(3)^3 \text{ model, } 9\lambda_s, 7Q_s, 7Q^c_s$$

Tian, Yau (1986)
Kalara, Mohapatra (1987)

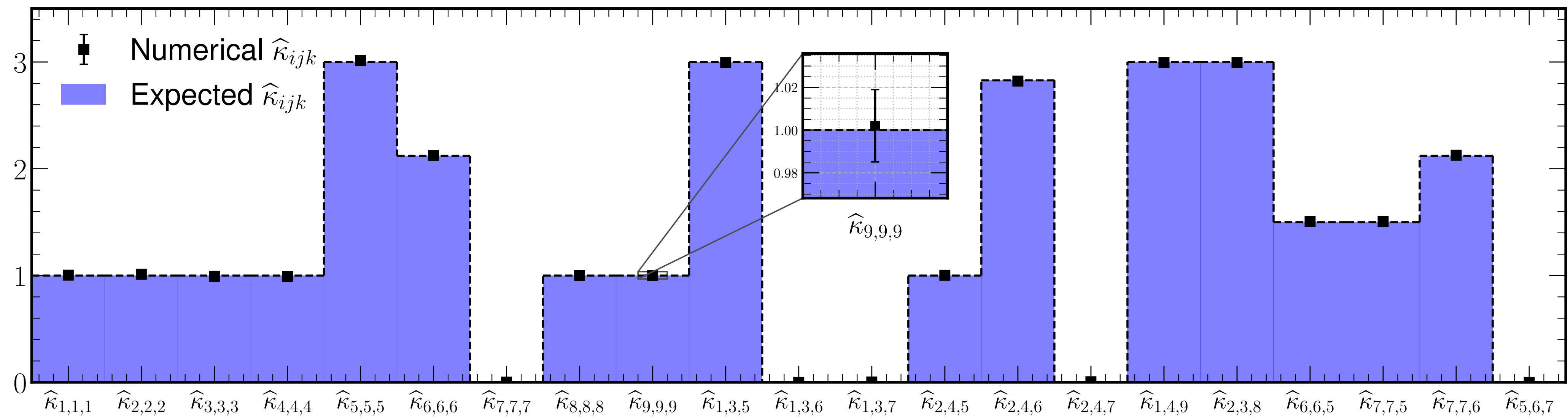
cf. Greene, Kirkland, Myron, Ross (1986)

Tian–Yau

- Tian–Yau quotient

$$\left[\begin{array}{c} \mathbb{P}^3 \\ \mathbb{P}^3 \end{array} \middle| \begin{array}{ccc} 3 & 0 & 1 \\ 0 & 3 & 1 \end{array} \right] / \mathbb{Z}_3$$

- Unnormalized Yukawa couplings



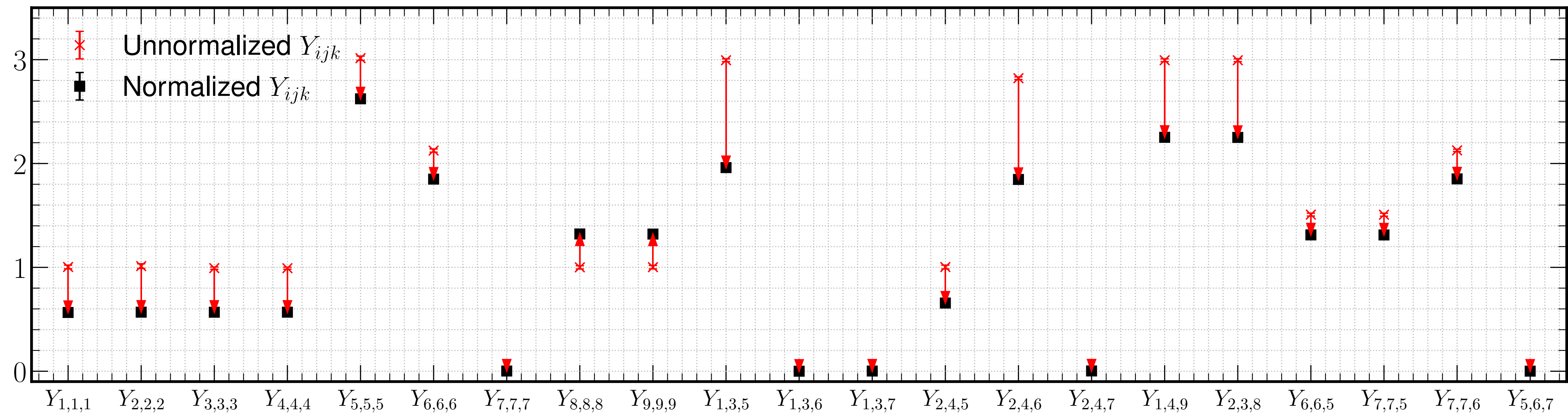
Candelas, Kalara (1988)

Tian–Yau

- Tian–Yau quotient

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- Physical Yukawa couplings



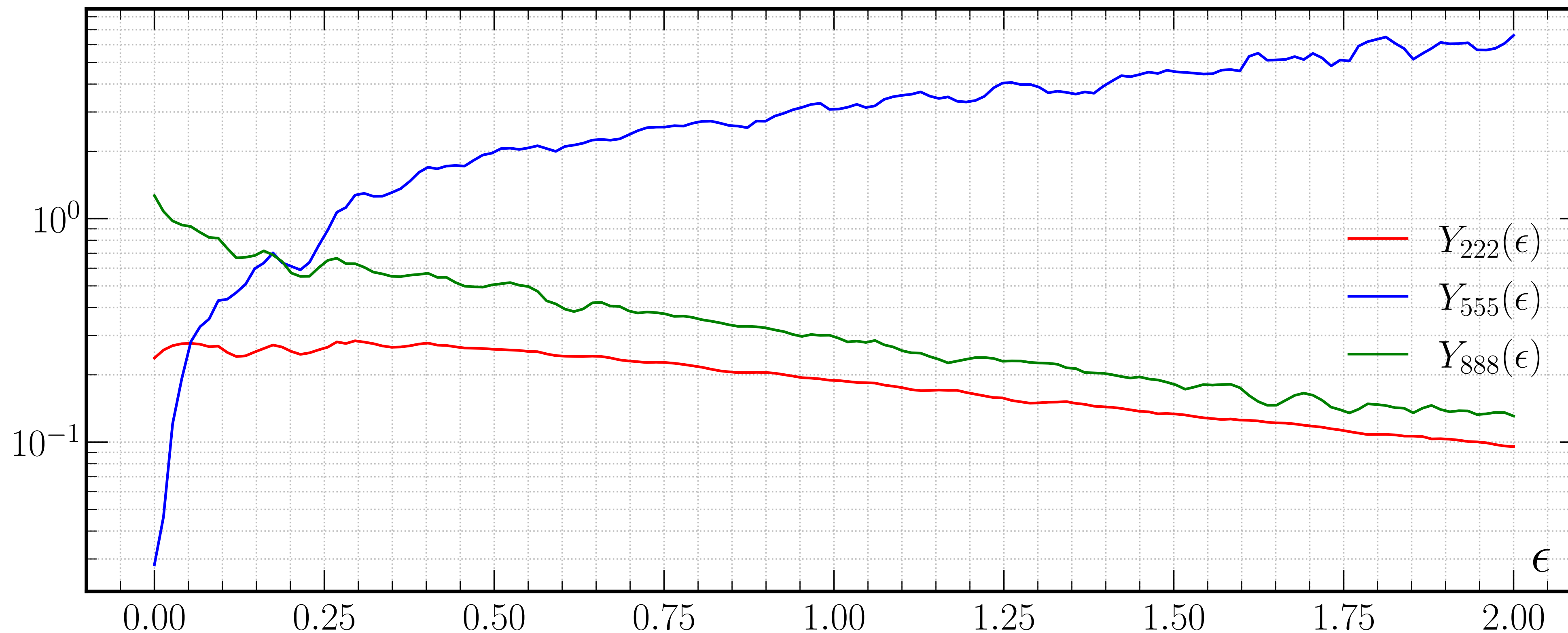
Tian–Yau

- Tian–Yau quotient

$$\left[\begin{array}{c} \mathbb{P}^3 \\ \mathbb{P}^3 \end{array} \middle| \begin{array}{ccc} 3 & 0 & 1 \\ 0 & 3 & 1 \end{array} \right] / \mathbb{Z}_3$$

- Physical Yukawa couplings as a function of moduli

$$x_0 y_0 + x_1 y_1 + (1 + \epsilon)(x_2 y_2 + x_3 y_3) = 0$$



λ^3 Yukawa couplings

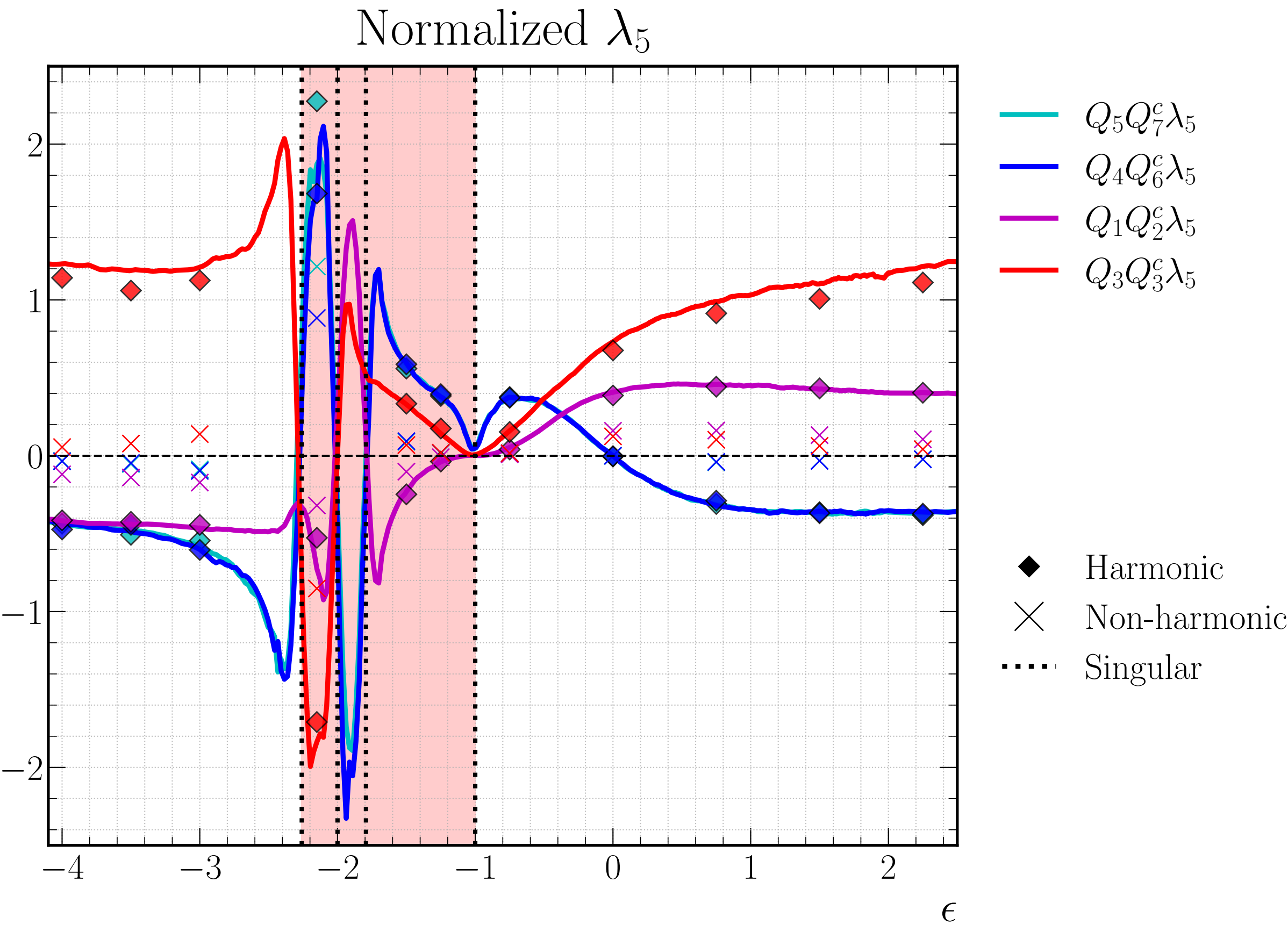
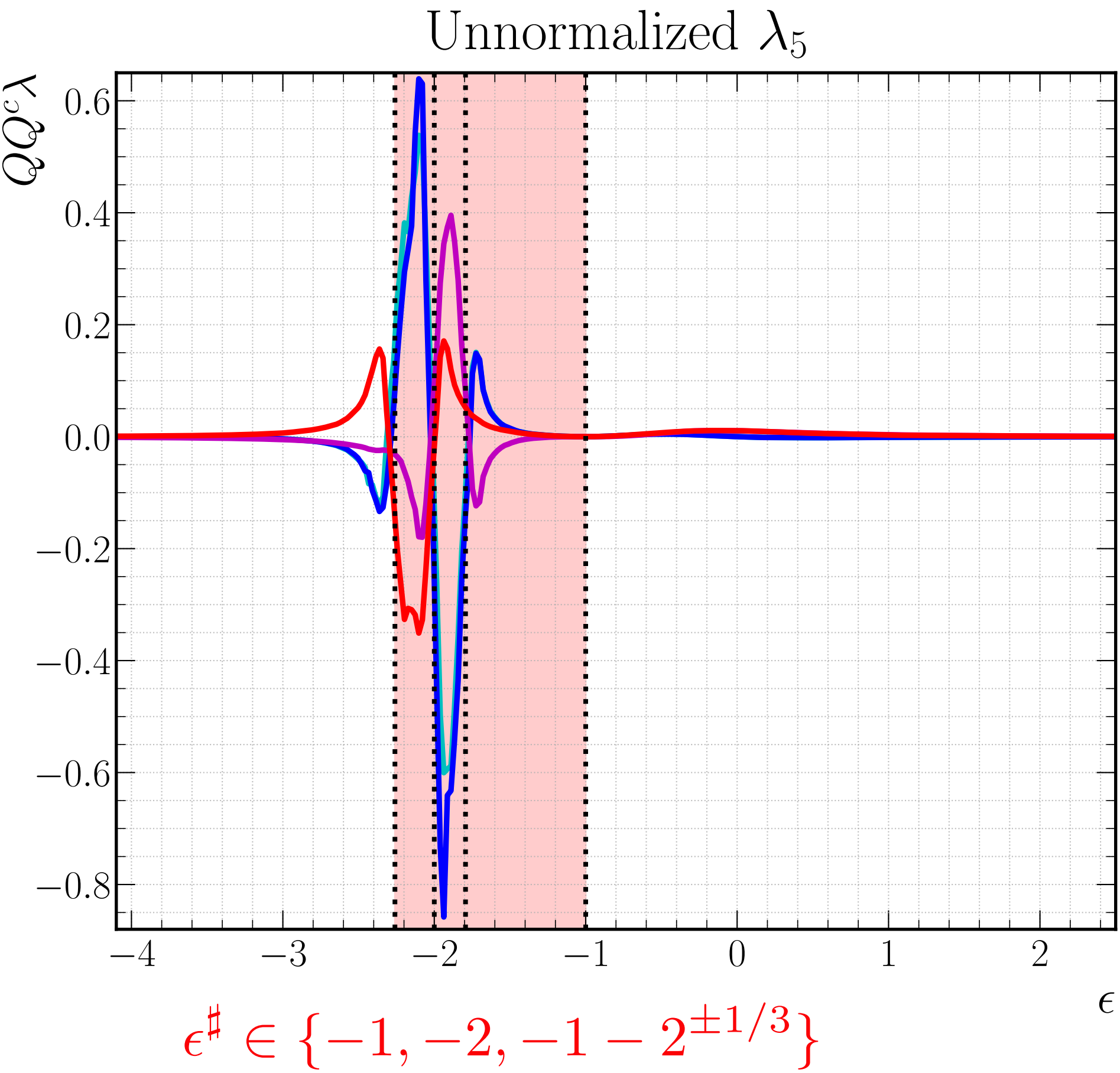
hierarchies arise upon moving away from symmetric points in complex structure moduli space

Tian–Yau

- Tian–Yau quotient

$$\left[\begin{array}{c} \mathbb{P}^3 \\ \mathbb{P}^3 \end{array} \middle| \begin{array}{ccc} 3 & 0 & 1 \\ 0 & 3 & 1 \end{array} \right] / \mathbb{Z}_3$$

$$x_0 y_0 + x_1 y_1 + (1 + \epsilon)(x_2 y_2 + x_3 y_3) = 0$$



Code

📖 README 📄 License

CYMYC

`cymyc` is a library for numerical differential geometry on Calabi-Yau manifolds written in JAX, enabling performant:

- Approximations of useful tensor fields;
- Computations of curvature-related quantities;
- Investigations of the complex structure moduli space;

in addition to many other features.

Installation

First, clone the project:

```
git clone git@github.com:Justin-Tan/cymyc.git
cd cymyc
```

Next, with a working Python installation, create a new virtual environment and run an editable install, which permits local development.

```
pip install --upgrade pip
python -m venv /path/to/venv
source /path/to/venv/bin/activate

python -m pip install -e .
```

[Calabi–Yau Metrics](#), [Yukawas](#), and [Curvature](#)

<https://github.com/Justin-Tan/cymyc>

arXiv:2410.19728



[MLgeometry](#)

Douglas, Lakshminarasimhan, Qi (2020)
Douglas (2021)
Douglas, Platt, Qi (2024)

[cymetric](#)

Larfors, Lukas, Ruehle, Schneider (2021, 2022)
Hendi, Larfors, Walden (2024)

[cyjax](#)

Gerdes, Krippendorf (2022)

Code

cymyc



Dwork quintic

```
import jax
from cymyc.calabi_yau import *

key = jax.random.key(0)
keys = jax.random.split(key, 3)

cy = DworkQuintic(0.0)
pts = cy.sample_points(
    key = keys[1],
    max_pts = int(1e5))

metric = RicciFlatMetric(
    key = keys[2],
    cy = cy,
    tx = optax.adamw(1e-4))

metric.fit(pts, epochs=10)

e_X = metric.chern3(
    pts.generator(
        batch_size = 100,
        include_pullbacks = True),
    pts.kappa)
print(cy.integrate(pts = pts, func_vals = e_X))
~
~
~
~
~
~
~
```

Define CY
Sample points

Solve for
Ricci-flat metric

Compute χ

```
(cymyc) gpu@gpu-P55A-UD3 ~ $ python3 metric.py
```

x 20

TERMINAL /bin/bash

in:1/1715

[2] 0:vim*

"gpu-P55A-UD3" 20:25 10-Dec-24

Code

cymyc



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cymyc



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TERMINAL /bin/bash

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[2] 0:vim*

"gpu-P55A-UD3" 20:25 10-Dec-24

Prospectus

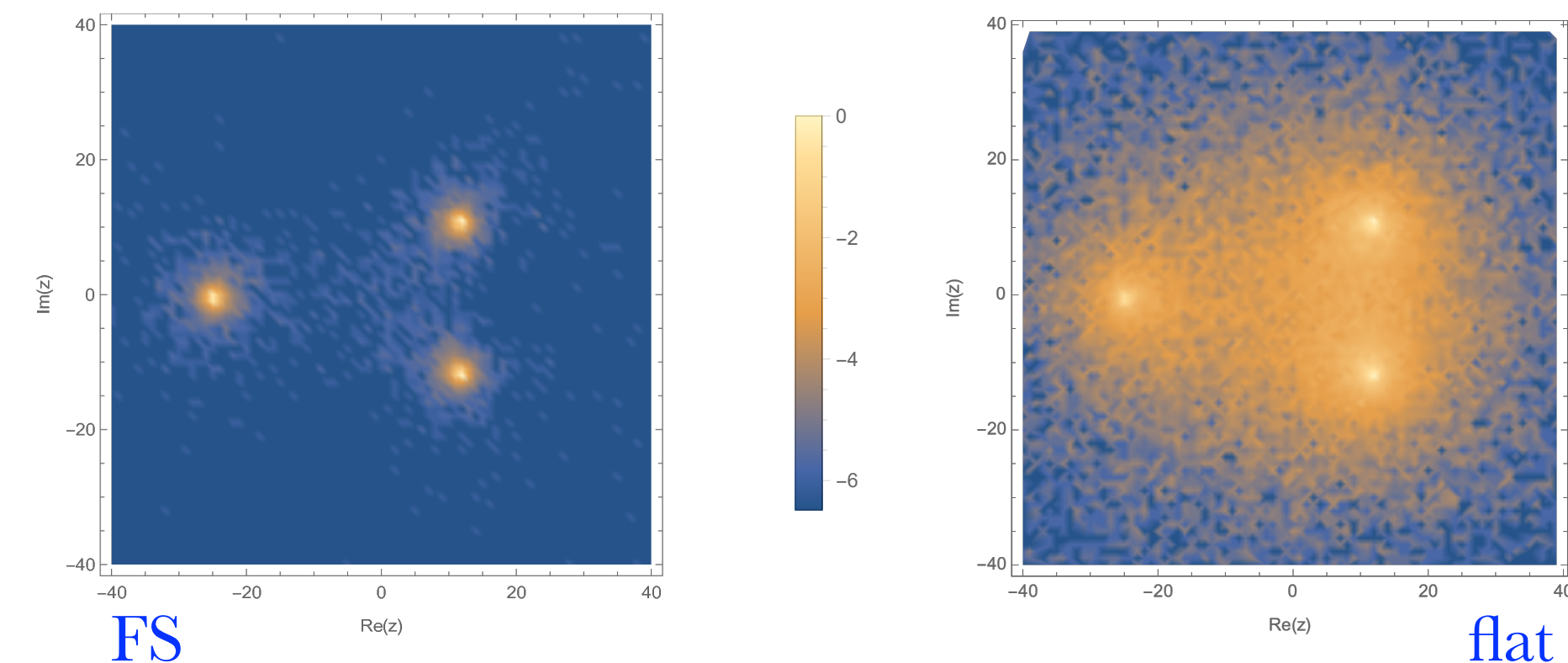
- Precision string phenomenology is possible
- Physical Yukawa couplings in non-standard embeddings
- α' corrections
- Toric Calabi–Yau manifolds, G_2 manifolds, $SU(3)$ structure
- Swampland distance conjectures
- de Sitter and cosmology in type II compactification
- Standard Model building
- Better point selection
- Toward analytics; SYZ limit

Constantin, Fraser-Taliente, Harvey, Lukas, Ovrut (2024)

Fraser-Taliente, Harvey, Kim (2024)

Anderson, Gerdes, Gray, Krippendorf, Raghuram, Ruehle (2020)
Larfors, Lukas, Ruehle, Schneider (2021, 2022)
Douglas, Platt, Qi (2024)

Ashmore (2020)
Ashmore, Ruehle (2021)
Ahmed, Ruehle (2023)



THANK YOU!



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Research
Foundation**