TREE-LEVEL COMPLETIONS OF GRAVITATIONAL AMPLITUDES

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The dispersion relations only utilizes imaginary part of the UV amplitude:

This does not distinguish between trees (particle spectrum) and loops

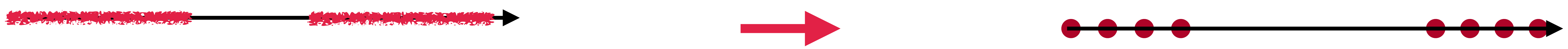
$$\frac{8\pi G_N s^2}{-t} + \cdots = s^2 \int_0^\infty \frac{ds'}{\pi} \frac{2T_s(s',t)}{(s')^3}, \quad t<0$$

Graviton pole comes from the UV.

Example: stringy graviton $\qquad T_s(s',t) \sim (s')^{2+\frac{\alpha' t}{2}}$

Example: eikonal $\qquad T_s(s,t=-\vec{q}^2) \sim s \int d^{D-2}\vec{b} e^{i\vec{b}\vec{q}} \sin^2\delta(s,b)$ $\qquad$ Sasha a few mins ago

Can we say more when we assume



?

Are the EFT bounds modified ?

Provide new handles to constrain ?

- The ``primal" approach: explicit amplitude construction
- Implementation in dispersion relations

- The ``primal" approach: closed (Gravity) stringy deformations

$$\frac{\Gamma(-\alpha' s)\Gamma(-\alpha' t)\Gamma(-\alpha' u)}{\Gamma(1+\alpha' s)\Gamma(1+\alpha' t)\Gamma(1+\alpha' u)} \to$$

$$\frac{\Gamma(-\alpha' s)\Gamma(-\alpha' t)\Gamma(-\alpha' u)}{\Gamma(1+\alpha' s)\Gamma(1+\alpha' t)\Gamma(1+\alpha' u)} + \epsilon\frac{\Gamma(1-\alpha' s)\Gamma(1-\alpha' t)\Gamma(1-\alpha' u)}{\Gamma(2+\alpha' s)\Gamma(2+\alpha' t)\Gamma(2+\alpha' u)}$$

Arkani-Hamed, Huang, Huang (20)

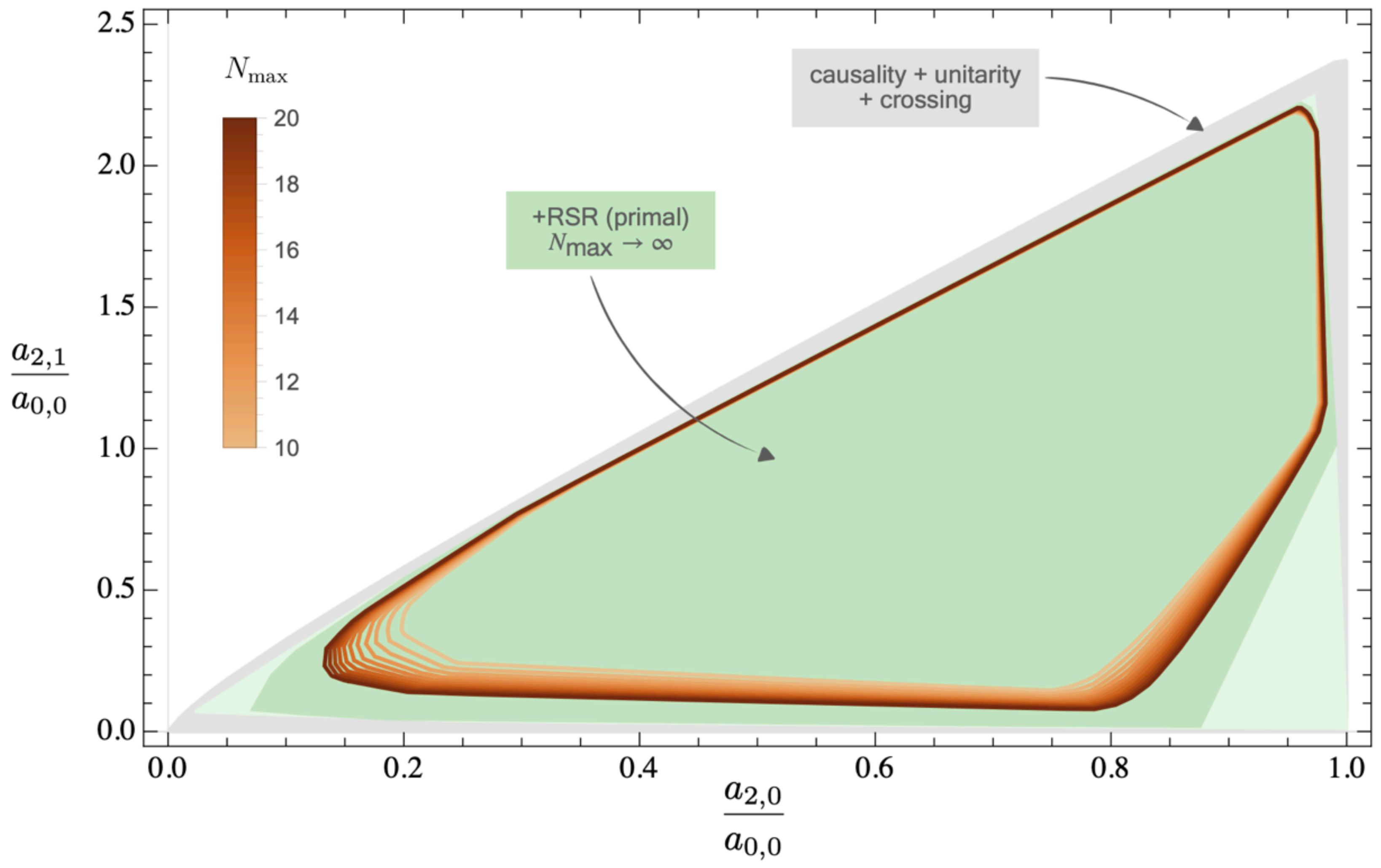
Residue is positive $\frac{1+n(1-\epsilon)}{n+1}\left(\frac{n^{n-1}}{2^{n-1}n!}\right)^2\left(P_n(x)^2 + \frac{4\epsilon(n-1)}{n(1+(1-\epsilon)n)}P_n(x)P^B_{n-4}(x)\right)$ unitary for $0 < \epsilon < 1$ in 10 D

Haring and Zhiboedov (23)

$$f_{N_{\max}}(s|t,u) = -8\pi G_N \frac{\Gamma(-s)\Gamma(-t)\Gamma(u)}{\Gamma(1+s)\Gamma(1+t)\Gamma(1+u)}$$

$$+ \sum_{c_s,c_{tu}=0}^{N_{\max}} \sum_{d_s,d_{tu}=1}^{2N_{\max}} \theta_{\alpha_{\text{ind}}}(\alpha)\, \alpha_{c_s,c_{tu},d_s,d_{tu}} \frac{\Gamma(c_s-s)\Gamma(c_{tu}-t)\Gamma(c_{tu}-u)}{\Gamma(d_s+s)\Gamma(d_{tu}+t)\Gamma(d_{tu}+u)},$$

Maximal spin, Regge sum rules



- The ``primal" approach: closed (Gravity) stringy deformations

Assuming dual resonance and level truncation (M(s,t) is rational when t take certain value) Cheung, Hillman, Remmen (24)

$$M(s,t)=\sum_{n=0}^{\infty}\left(\frac{1}{\mu(n)-s}+\frac{1}{\mu(n)-u}\right)R(n,t),$$

$$R(n,t)=c(n)\prod_{k=1}^{n-1}(t-\xi(k))^2$$

Crossing symmetry →

$$\mu(n)=\frac{n+\lambda-1}{\lambda}$$

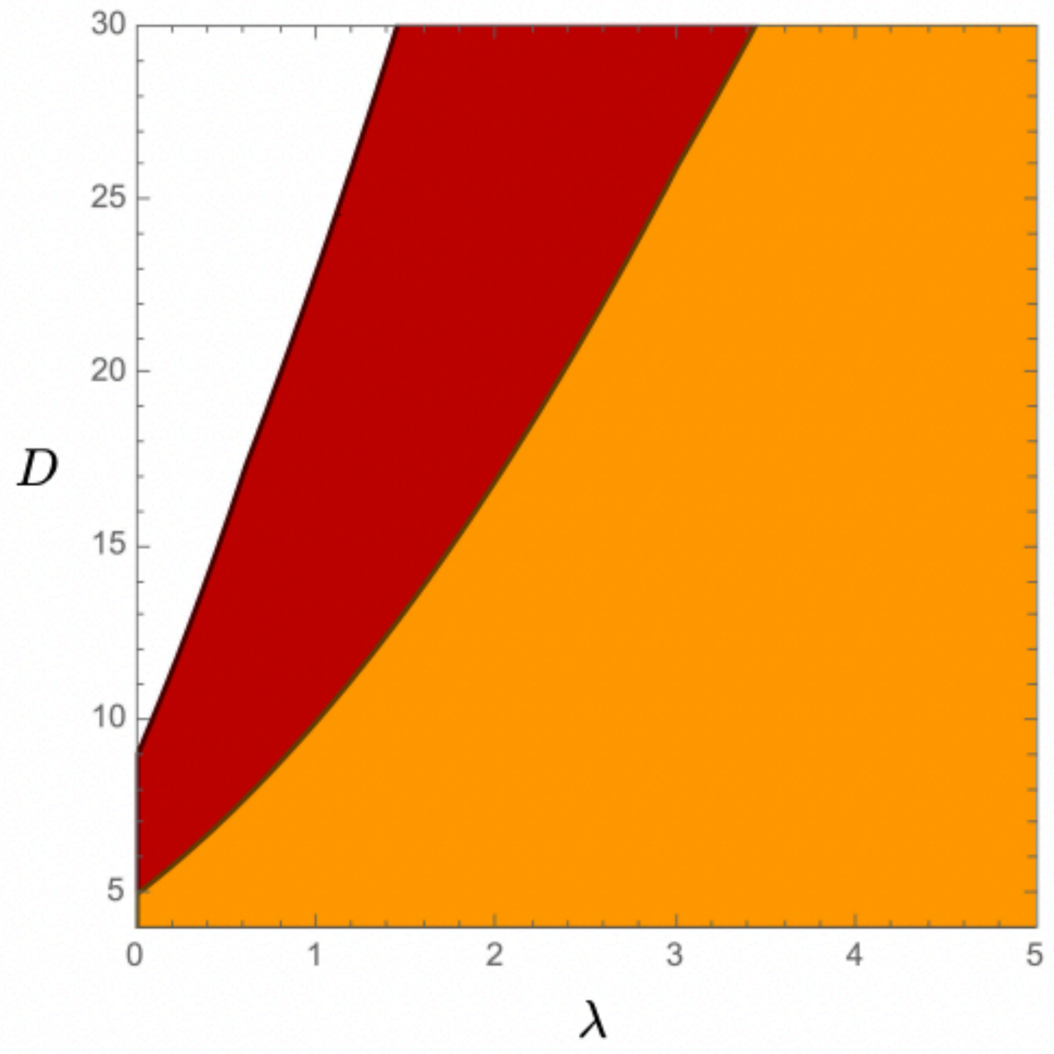
$$\xi(n)=-\frac{2n+\lambda-1}{2\lambda}$$

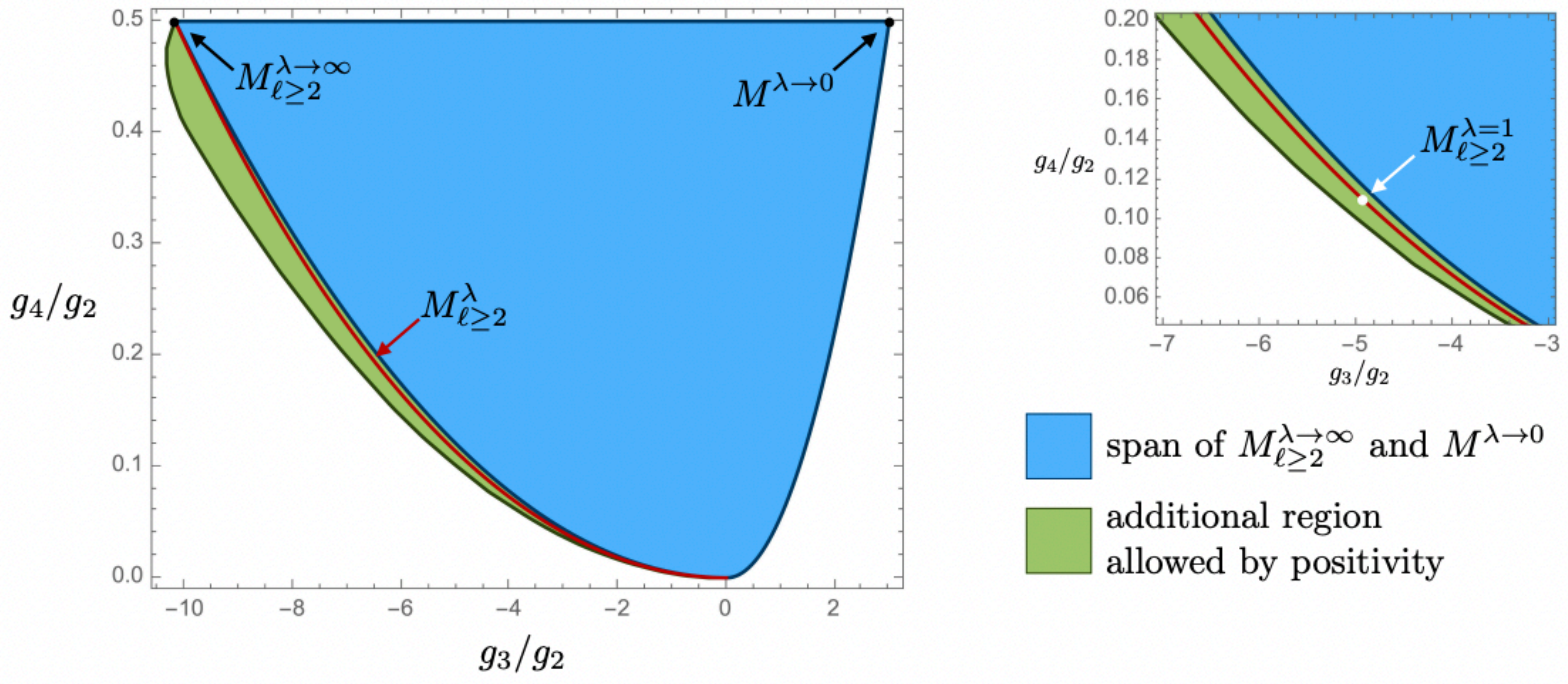
$$c(n)=\frac{\lambda^{2n-2}}{\left(\frac{1+3\lambda}{2}\right)^2_{n-1}}$$

$$M=\frac{\rho^2}{stu}+\sum_{k=0}^{\infty}\frac{\lambda(3\lambda-1)^2\Gamma(3\lambda-1+k)}{(3\lambda-1+2k)k!}M_k$$

$$M_k=\frac{\Gamma(k+\lambda-\lambda s)\Gamma(k+\lambda-\lambda t)\Gamma(k+\lambda-\lambda u)}{\Gamma(k+2\lambda+\lambda s)\Gamma(k+2\lambda+\lambda t)\Gamma(k+2\lambda+\lambda u)}.$$

Unitarity up to n=30





Can these amplitude correspond to a complete theory ?

Similar deformations of open-string amplitudes often meet their demise at higher points

Lovelace-Shaprio

$$A(s,t) = C_{4\text{-pt}} \frac{\Gamma(1-\alpha_\rho(s))\Gamma(1-\alpha_\rho(t))}{\Gamma(1-\alpha_\rho(s)-\alpha_\rho(t))}$$

$$\alpha_\rho(s) = \frac{1}{2} + \alpha' s \;\; ; \;\; \alpha' = \frac{1}{2m_\rho^2}$$

n-point generalization has negative norm state at n $\geq$ 8

Bianchi, Consoli, Di Vecchia (20)

Bespoke amplitudes (dual resonance + spectrum)

$$A_{\text{bespoke}}(s,t) = \sum_{\alpha,\beta} A_V(\nu_\alpha(s), \nu_\beta(t))$$

$$f(\mu,\nu) = P(\nu) - \mu Q(\nu) = 0$$

Cheung, Remmen (23)

fails consistent factorization at 5, 6-point

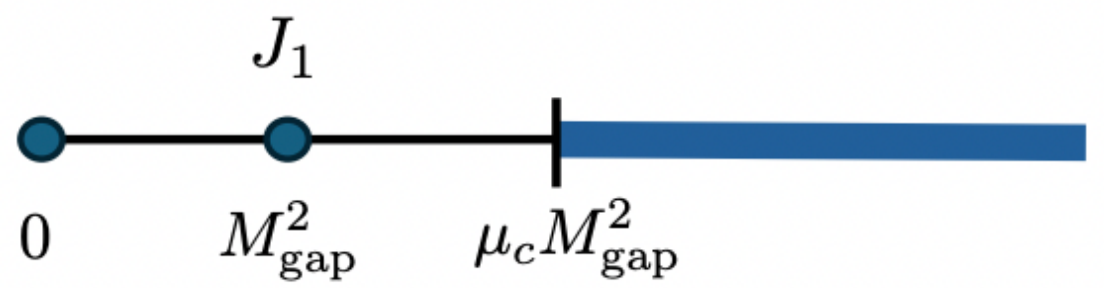
Arkani-Hamed, Cheung, Figueiredo, Remmen (23)

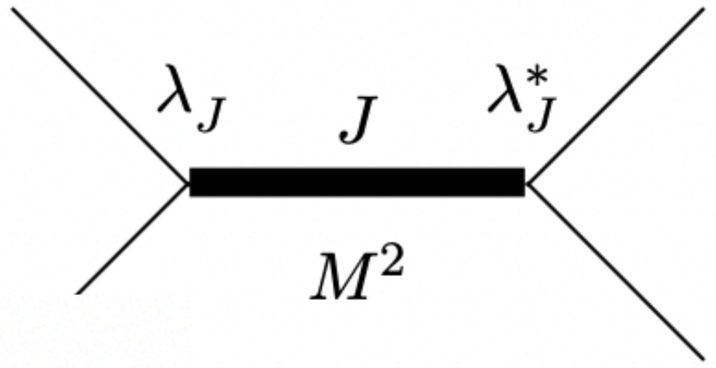
But also at higher levels for 4-point

Bhardwaj, Spradlin, Volovich, Weng (24)

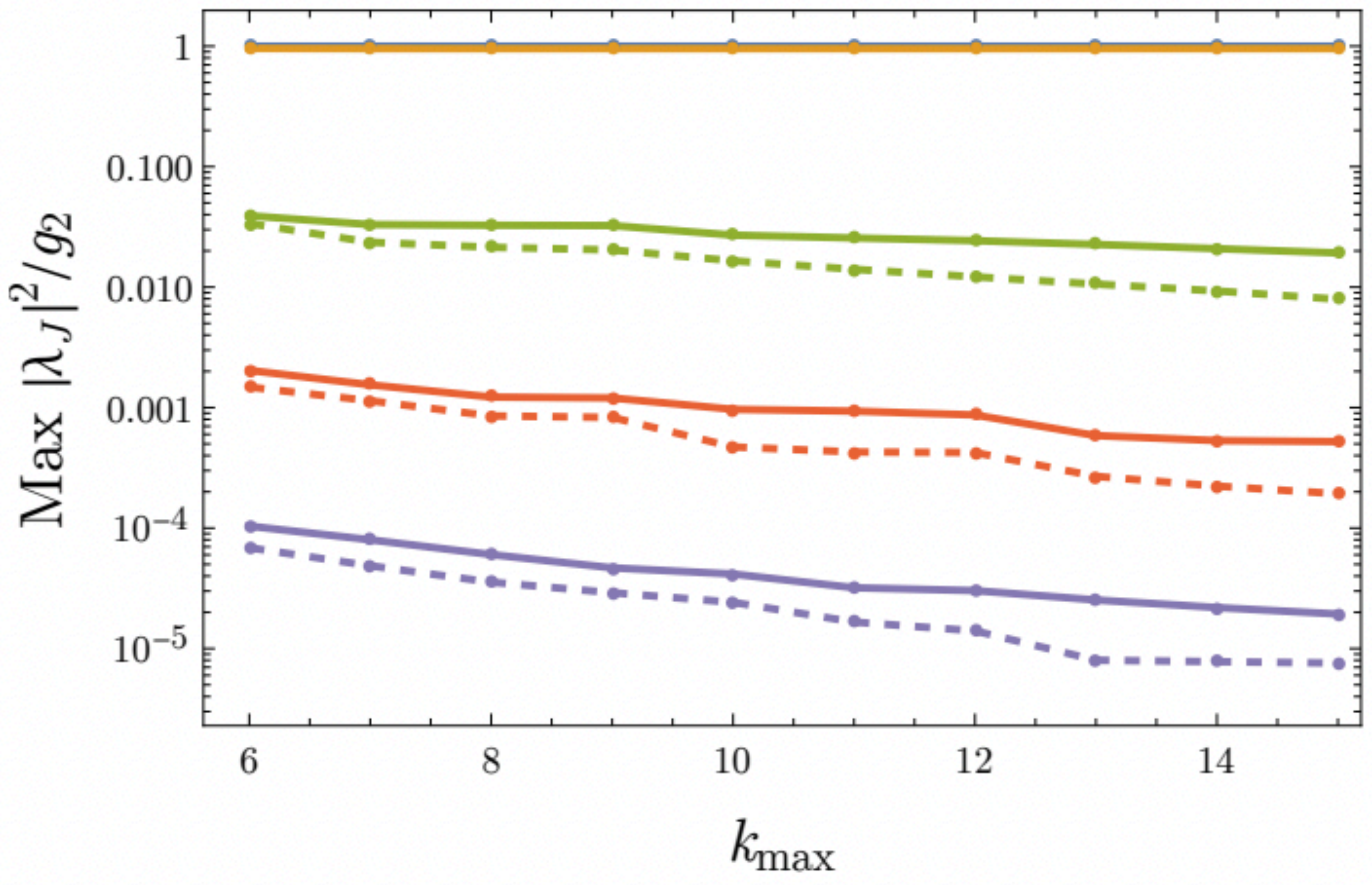
- Implementation in dispersion relations

We can successively introduce factorization poles in numeric bootstrap





Berman, Elvanga, Geiser, Lin (24)



Isolated gap state must have J=2, 0

Can one derive $\frac{\lambda_J^2}{g_2}(\mu)$ and $\frac{\lambda_J^2}{G}(\mu)$?

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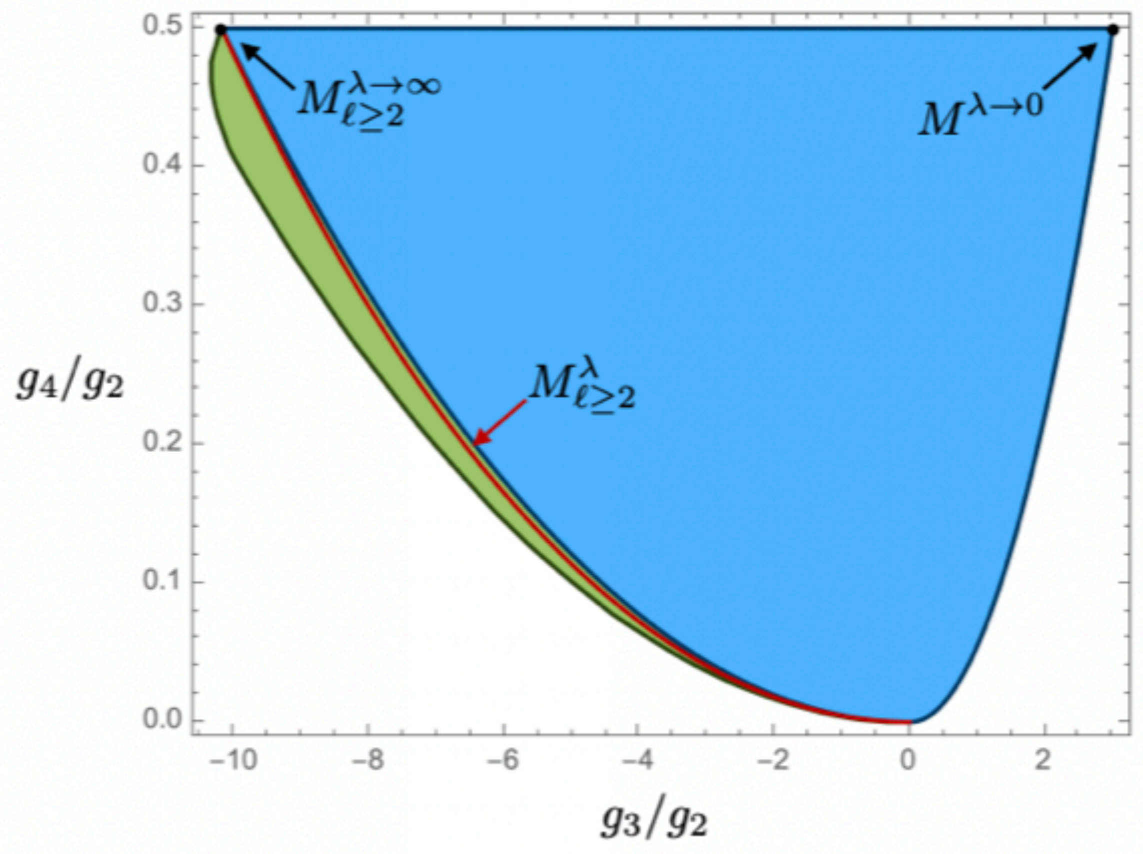
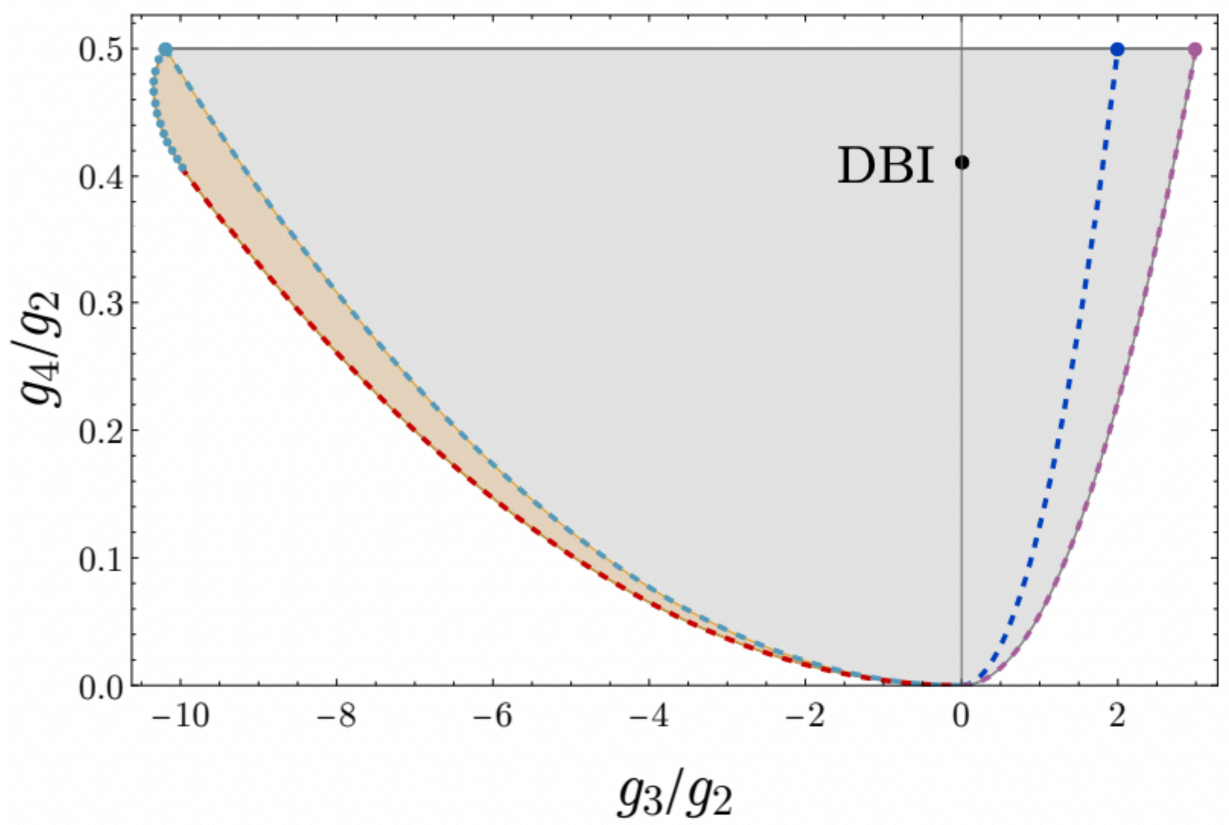
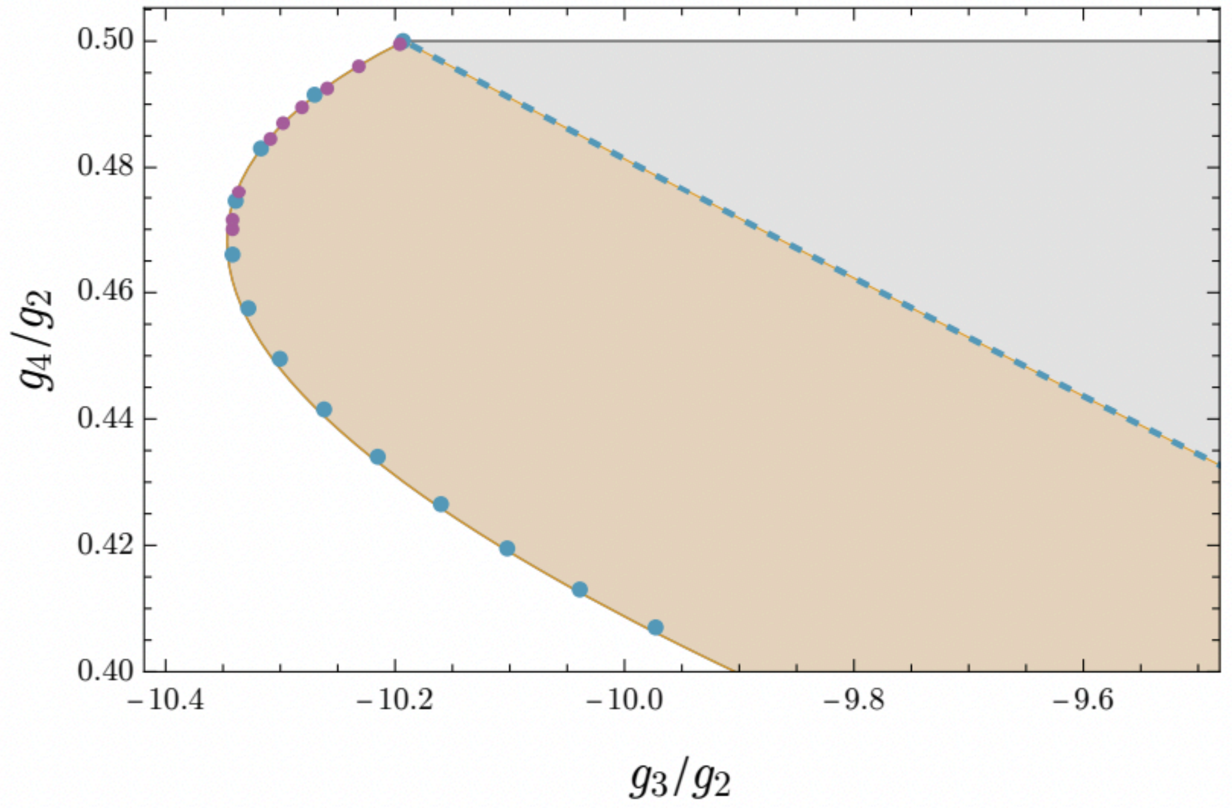
Consider maximized spin-2 coupling at the gap with no scalars







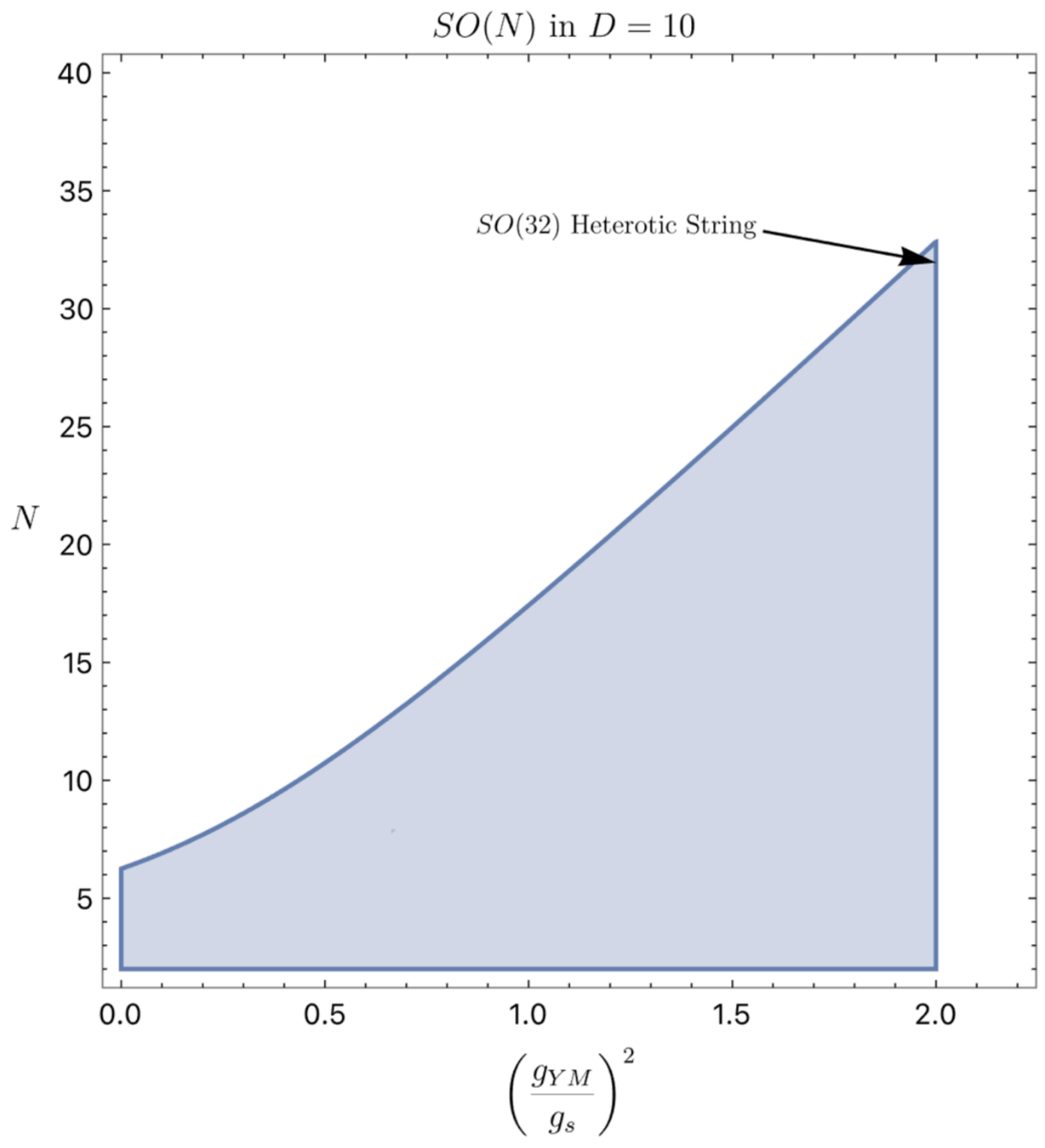
Tree-level completions and constraint on color: ``primal'' approach

$$\mathcal{A}^{\mathrm{UV}}(1^{-a},2^{-b},3^{+c},4^{+d}) = \Gamma^{\mathrm{str}}\mathcal{A}(1^{-a},2^{-b},3^{+c},4^{+d})$$

$$\mathcal{A}(1^{-a},2^{-b},3^{+c},4^{+d}) = \langle 12\rangle^2 [34]^2 \Bigg[\frac{1}{M_P^2}\left(\frac{\mathbb{P}_1^s}{s}+\frac{\mathbb{P}_1^t}{t}+\frac{\mathbb{P}_1^u}{u}\right)+ \frac{g_{\mathrm{YM}}^2}{3}\left(\frac{\mathbb{P}^s_{\mathrm{Adj}}-\mathbb{P}^t_{\mathrm{Adj}}}{st}+\frac{\mathbb{P}^t_{\mathrm{Adj}}-\mathbb{P}^u_{\mathrm{Adj}}}{tu}+\frac{\mathbb{P}^u_{\mathrm{Adj}}-\mathbb{P}^s_{\mathrm{Adj}}}{su}\right)\Bigg]$$

The residues are now expanded on color projectors

$$\lim_{s\to m^2} \mathcal{A}^{\mathrm{UV}}(1^{-a},2^{-b},3^{+c},4^{+d}) \sim \frac{1}{s-m^2}\sum_J \rho_{J,\alpha}\mathbb{P}^{abcd}_\alpha \mathbb{G}_j(\cos\theta)$$



Bachu, Hillman (23)

Tree-level completions and constraint on color: In dispersion relaitons

Hillman, Huang, Rodina, Rumbutis (23)

$$\frac{B^{i_1i_2i_3i_4}_{\text{poly}}(s,t)}{s(s+t)} - \frac{B^{i_1i_2i_3i_4}_{\text{poly}}(0,t)}{st} + \frac{B^{i_1i_2i_3i_4}_{\text{poly}}(-t,t)}{(s+t)t} - \frac{8\pi G}{t}\mathbb{P}_0^{i_2i_3;i_4i_1} =$$
$$n^{(D)}_{J,R}\sum_{JR}\mathbb{G}^{(D)}_J(cos(\theta))\int_{M^2}^{\infty}\frac{ds'}{\pi s'(s'+t)}\text{Im}[f_{J,R}(s')]\left(\frac{\mathbb{P}_R^{i_1i_2;i_3i_4}}{(s'-s)}+\frac{\mathbb{P}_R^{i_1i_3;i_2i_4}}{(s'+t+s)}\right)$$

Converting the projectors of different channels into a common basis

$$\mathbb{P}^s = M_{st}\mathbb{P}^t, \quad \mathbb{P}^u = M_{ut}\mathbb{P}^t$$

If we can find $v = (1,0,0,\cdots)$ such that

$$[M_{st}v]_i = [M_{ut}v]_i = 0$$

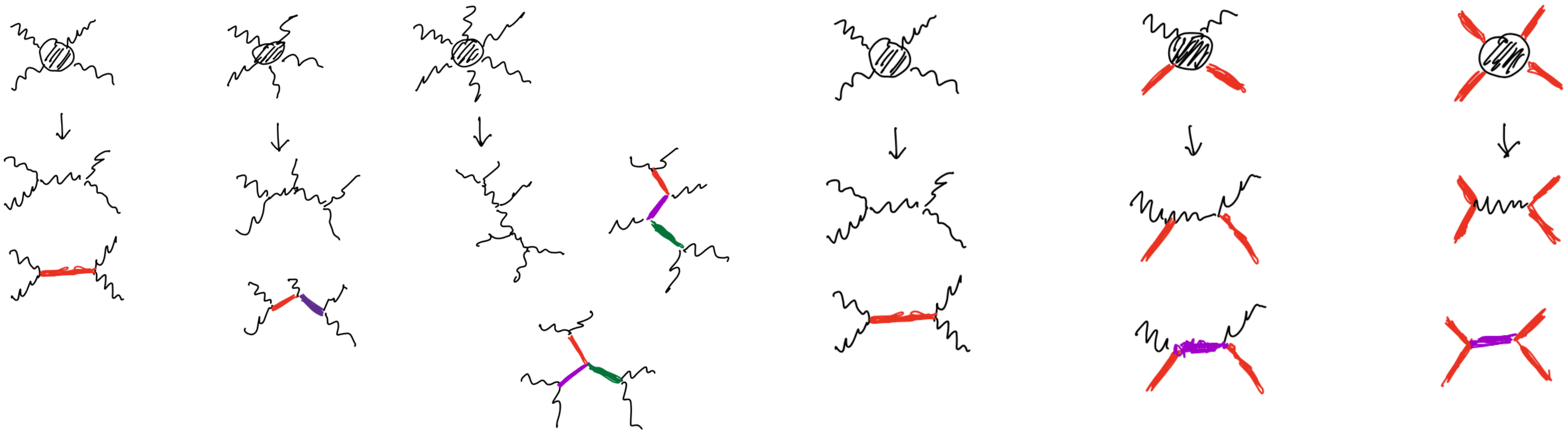
Then if all irreps $\neq i$ are absent, we arrive at

$$\frac{8\pi G_N}{-t} + Poly(t) = 0$$

such spectrum are inconsistent

Can mixed bootstrap at four-points replace higher-point bootstrap ?

Consistent higher point generalizations ⟶ Consistent mixed amplitudes



Karateev, Marucha, Penedones, Sahoo (22)