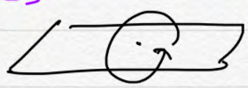
prepared: 2023/5/17

Q: Do we know all branes in string theories?

A: No!

[Heckman et al.] R-brane in IIB / F
2022, 2023

 T^2 fiber $\curvearrowleft \begin{pmatrix}1&0\\0&-1\end{pmatrix}$

[Kaidi - Ohmori - YT - Yonekura]

- Heterotic 7-, 6-, 4-, 1- branes

Bergshoeff-Gibbons-Townsend 0607193

Polchinski 0510033

- 7- and 6- branes are new. AFIAK.

- uniform worldsheet desc. of extremal near-horizon region

- mysterious relation to
  classif. of non-susy 10d het strings
  $\underset{\text{equiv.}}{\Longleftrightarrow}$ classif. of 2d spin CFTs with $c \leq 16$

- unexpected relation to TMF

- 'gravitational' discrete theta angles...

§1. 6-brane

§2. generalization

§3. TMF.

§1 the 6-brane

QG Folklore: $\exists$ obj carrying any charge

<u>proof (?)</u>

$\exists$ black semi-classical sol'n
with the said charge

Let it Hawking-evaporate!

(cf. [McNamara-Vafa])

The charge: $\mathbb{Z}_2$ magnetic charge of
Type I = heterotic so(32).

dynamical particles in adj, spinor
but not in vector, conj. spinor.

gauge gp is actually $Spin(32)/\mathbb{Z}_2$
which is <u>not</u> $SO(32)$

(already pointed out in the original paper
on the heterotic strings!)

$\rightsquigarrow$ $\mathbb{Z}_2$ charge measuring the impossibility of vector rep.

[Witten] 'Without vector structure'

Explicitly,

$$U(1)^{16} \subset U(16) \subset SO(32)$$

S^2 ← magn. flux $\left(\frac{1}{2}, \frac{1}{2}, \ldots, \frac{1}{2}\right)$

<u>not</u> compatible with electric charge

$(1, 0, \ldots 0)$ vector

but OK with

$(1, 1, \ldots 0)$ adj

or with $\left(\frac{1}{2}, \frac{1}{2}, \ldots \frac{1}{2}\right)$ spinor

inner product $= 4 \in \mathbb{Z}$

Clearly, $\exists$ black 6-brane with this charge

$$\mathbb{R}^6 \times \underbrace{\mathbb{R}^1 \times \mathbb{R}_{>0} \times S^2}$$

it's just an R-N. BH.

Surprisingly (?) the explicit supergravity solutions are known:

In heterotic frame,

$$\begin{cases} ds^2 = -\dfrac{1-r_+/r}{1-r_-/r}\,dt^2 + d\vec{x}^2 + \dfrac{dr^2}{\left(1-\frac{r_+}{r}\right)\left(1-\frac{r_-}{r}\right)} + r^2 d\Omega_{S^2}^2 \\ e^{-2\phi} = g_s^{-2}\left(1-\dfrac{r_-}{r}\right) \end{cases}$$

← dilaton diverges at the inner horizon.

$$M \propto r_+ + r_-$$
$$Q^2 \propto r_+ r_-$$

[Horowitz-Strominger] 1991

Extremal limit is given by $r_+ \to r_-$

$$\begin{cases} ds^2 = -dt^2 + d\vec{x}^2 + dy^2 + r_0^2\, d\Omega_{S^2}^2 \\ e^{-2\phi} = g_s^{-2}\, e^{y/r_0} \end{cases}$$

S^2 is constant stringy size.

dilaton diverges at the core

we have

$$r_0 \propto Q\, l_s$$
$$M \propto Q\, g_s^{-2}$$

Before proceeding.

Q. Shouldn't Type I be better?

As the S-dual, the coupling at the core should be very weak...

A. well, in type I frame, we have

$$\begin{cases} ds^2 = \underline{\underline{\rho^2}}\left(dt^2 + d\vec{x}^2\right) + r_0^2\left(\underline{\underline{16}}\, d\rho^2 + \rho^2 d\Omega_S^2\right) \\ e^{+\phi_{\text{type I}}} \sim \rho^2 \end{cases}$$

strange prefactor

$\exists$ conical singularity

Furthermore,

$$M \propto g_{\text{type I}}^{-3/2} \leftarrow$$

in between

$g_{\text{type I}}^{-1}$: D-brane

$g_{\text{type I}}^{-2}$: NS5-brane

so, not very helpful.

In fact we started thinking in Type I frame and got stuck for more than a year until May 2023 when we decided to switch to heterotic frame.

Coming back to heterotic,

$$\begin{cases} ds^2 = \boxed{\boxed{-dt^2 + d\vec{x}^2} + dy^2} + \boxed{r_0^2\, d\Omega^2_{S^2}} \\ e^{-2\phi} = g_s^{-2}\, e^{y/r_0} \end{cases}$$

flat

linear dilaton.

???

(strong coupling region is problematic, though)

Recall

$$U(1)^{16} \subset U(16) \subset SO(32)$$

$\uparrow$ magnetic charge $(\frac{1}{2}, \frac{1}{2}, \cdots \frac{1}{2})$.

$Spin(32)/\mathbb{Z}_2$ current algebra is

$$\Big(\underbrace{\psi^{1,\cdots,16}}_{\text{charge } +\frac{1}{2}}\ \underbrace{\bar{\psi}^{\bar{1},\cdots\bar{16}}}_{\text{charge } -\frac{1}{2}}\Big)/(-1)^F$$

$\updownarrow$ equivalent [Yunqin's talk]

$$\Big([SU(16)/\mathbb{Z}_4]_1 \times \underbrace{\Psi_L}_{\text{charge } 2},\ \underbrace{\bar{\Psi}_L}_{-2} \Big)/(-1)^F$$

We also have het σ-model on S^2, i.e.

$\mathcal{N}=(2,2)$ σ-model on S^2 $\longrightarrow$ $S^2 \times \underbrace{\Psi_R}_{\text{charge } 2},\ \underbrace{\bar{\Psi}_R}_{\text{charge } -2}$.

$\Downarrow$ IR

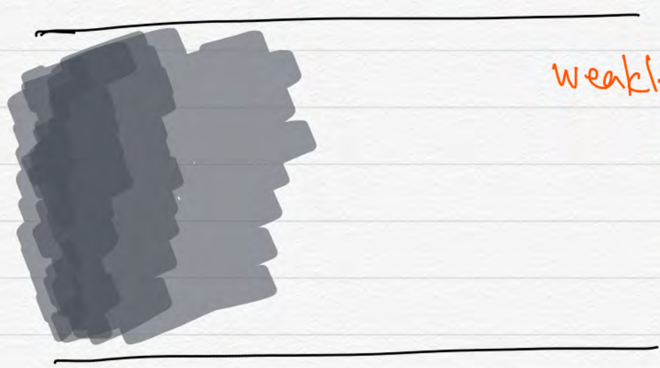
two gapped vacua exchanged by $(-1)^F$.

We end up with

$$\underset{\text{flat}}{\mathbb{R}^{1,6}} \times \underset{\substack{\text{linear}\\ \text{dilaton}}}{\mathbb{R}} \times \left[SU(16)/\mathbb{Z}_4\right]_1$$

NOTE : previously found as an endpoint of the closed tachyon condensation from $SU(16)/\mathbb{Z}_4 \times \Phi\,\Phi$ heterotic string [Kaidi 2020]

ALSO, not perfectly satisfying, because of strong-coupling region of linear dilaton.



weakly coupled spectrum :

- $SU(16)$ vector
- spin-1/2 fermion in $\Lambda^4 \mathbf{16}$

$\longleftarrow$ strongly coupled.

6d spacetime

has anomaly polynomial

$$\left[\frac{p_1(R)}{2} + c_2(F)\right] \frac{c_3(F)}{2}.$$

canceled by GS. coupling

$$\begin{cases} dH = \dfrac{p_1(R)}{2} + c_2(F) \\ \displaystyle\int_{X_8} B_2 \wedge \frac{c_3(F)}{2} \end{cases}$$

$\rightsquigarrow$ strong. coupling regim is effectively a boundary for chiral fermion (with GS cancellation.) How????

§2 Generalization

Consider $E_8 \times E_8$ het. string instead.

$$\mathbb{R}^{1,4} \times \mathbb{R}_{>0} \times S^4$$

$$\Uparrow$$

$$dH = \underbrace{\frac{1}{2} p_1(R)}_{0} - n(F_{E_8}) - n(F_{E_8'})$$

exact

should cancel each other

e.g. $\underbrace{+1 \qquad -1}$

TS^4 as $SU(2) \times SU(2)$ bundle exactly has this.

couple

$$(E_8 \times E_8)_1 \underset{\text{equivalent}}{=} \left[\left(\frac{E_7 \times E_7}{\mathbb{Z}_2} \right)_1 \times \psi^1 \psi^2 \psi^3 \psi^4 \right] / (-1)^F$$

we also have het σ-model on S^4, e.g.

$$S^4 + \psi^1_R \psi^2_R \psi^3_R \psi^4_R$$

$\mathcal{N}=(1,1)$ σ-model on S^4

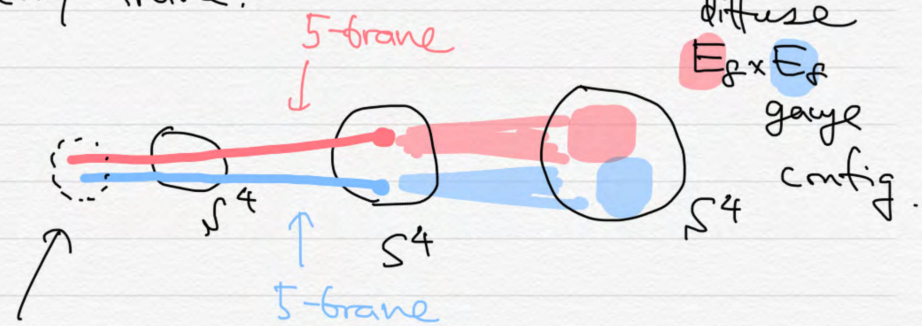
$\Downarrow$ IR

two gapped vac exchanged by $(-1)^F$

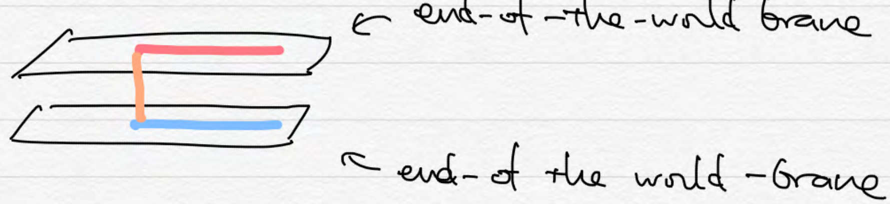
$$\rightsquigarrow \underset{\text{flat}}{\mathbb{R}^{1,4}} \times \underset{\text{linear dilaton}}{\mathbb{R}} \times \left(\frac{E_7 \times E_7}{\mathbb{Z}_2} \right)_1$$

again, the same as an endpoint of 10d tachyonic het. string of [Kaidi 2020]

- In the M-theory frame.



at the core,



surely an allowed configuration in M-theory.

This gives me confidence for the existence of 'our' 6-brane too.

- $$\underset{\text{flat}}{\mathbb{R}^{1,4}} \times \underset{\substack{\text{linear}\\ \text{dilaton}}}{\mathbb{R}} \times \left[\frac{E_7 \times E_7}{\mathbb{Z}_2}\right]_1$$

spectrum:

$E_7 \times E_7'$ gauge field

left-chiral $\mathbf{56}$

right-chiral $\mathbf{56}'$

$\rightsquigarrow$ non-zero anomaly

$$\left[\frac{p_1(R)}{2} - n(F_{E_7}) - n(F_{E_7'})\right]$$

$$\times \frac{1}{2}\left[n(F_{E_7}) - n(F_{E_7'})\right].$$

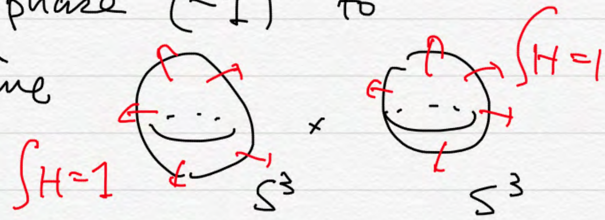
? cancelled by GS ... the same Q with 6-brane case.

HOW ????

· the resulting GS coupling $2\pi i \cdot \int \frac{1}{2} B \wedge \left(n(F_{E_7}) - n(F_{E_7'}) \right)$

assigns the phase (-1) to

6d spacetime



Two more cases:

$\mathbb{R}^{1,7} \times \mathbb{R}_{>0} \times S^1 \circlearrowleft E_8 \times E_8$ exchanged

$\Downarrow$ IR

$(E_8)_2$, using $(E_8 \times E_8)_1 = (E_8)_2 \times \psi \,/\, (-1)^F$.

$\mathbb{R}^{1,0} \times \mathbb{R}_{>0} \times S^8$: tangent bundle $\subset$ triality $\frac{Spin(32)}{\mathbb{Z}_2}$

$\Downarrow$ IR

$\left(\frac{Spin(24)}{\mathbb{Z}_2}\right)_1$, using $\left(\frac{Spin(32)}{\mathbb{Z}_2}\right)_1 = \frac{Spin(24)}{\mathbb{Z}_2} \times \psi^{1 \cdots 8} \,/\, (-1)^F$.

§3 Relation to TMF.

Stolz-Teichner conj says:

$$TMF_d(pt) \simeq \frac{\{ \text{2d } \mathcal{N}=(0,1) \text{ SQFT with } d = 2(c_R - c_L) \}}{\text{continuous deformation}}.$$

heterotic worldsheet theories !

$S^1 \circlearrowright E_8 \times E_8$ exchange

$\Downarrow$

$(E_8)_2$

$\chi_{-31} \in TMF_{-31}(pt)$

S^2 : without vector structure $Spin(32)/\mathbb{Z}_2$

$\Downarrow$

$[SU(16)/\mathbb{Z}_4]_1$

$\chi_{-30} \in TMF_{-30}(pt)$

S^4 : $n(E_8) = 1$, $n(E_8') = -1$

$\Downarrow$

$\left[\frac{E_7 \times E_7}{\mathbb{Z}_2}\right]_1$

$\chi_{-28} \in TMF_{-28}(pt)$

S^8 : tangent bundle $\xrightarrow{\text{triality}}$ $\frac{Spin(32)}{\mathbb{Z}_2}$

$\Downarrow$

$[Spin(24)/\mathbb{Z}_2]_1$

$\chi_{-24} \in TMF_{-24}(pt)$

They turn out to be all very interesting elements.

A function on

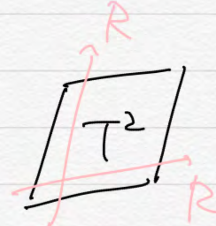
$$TMF_d(pt) \simeq \frac{\{ \text{2d } \mathcal{N}=(0,1) \text{ SQFT with } d = 2(c_R - c_L) \}}{\text{continuous deformation}}$$

is a function on

$\{ \text{2d } \mathcal{N}=(0,1) \text{ SQFT with } d = 2(c_R - c_L) \}$

invariant under continuous def.

A typical such function is
the <u>elliptic genus</u>

i.e. the part. func. on 

It's a modular function.

When $2(c_R - c_L) = 24$,

$$Z_{ell}(\tau) = a\,\eta(q)^{12} + b\,E_4(q)\,\eta(q)^{-12}$$

from the theory of modular func ; $a, b \in \mathbb{Z}$.

TMF + Stolz–Teichner predicts
a is divisible by 24.

And our

S^8 : tangent bundle $\xrightarrow{\text{triality}}$ $\frac{Spin(32)}{\mathbb{Z}_2}$

$\Downarrow$

$[Spin(24)/\mathbb{Z}_2]$,

$x_{-24} \in TMF_{-24}(pt)$

saturates it : $\underset{\sim\sim\sim}{a = 24,\ b = 0}$.

This divisibility is related to the GS coupling on the
2d spacetime side:

$$2\pi i \cdot \int B_2 \cdot \frac{a}{24}$$

anomalous unless
a is divisible by 24.

[YT 2021]

· X_{-31}, X_{-30}, X_{-28} are more subtle.

Their elliptic genus (both $\mathbb{Z}$- and $\mathbb{Z}_2$-versions) vanish.

· SQFTs with zero elliptic genus form a subgroup of $TMF_d(pt)$ we call A_d.

According to math literature, non-trivial A_d are in the range $-31 \le d \le 9$ are

$$A_3 = \mathbb{Z}_{24} \qquad \boxed{A_6 = \mathbb{Z}_2} \qquad \boxed{A_8 = \mathbb{Z}_2} \qquad \boxed{A_9 = \mathbb{Z}_2}$$

$$\qquad\qquad\qquad\quad \updownarrow \qquad\qquad\quad \updownarrow \qquad\qquad\quad \updownarrow$$

$$\qquad\qquad\qquad A_{-28} = \mathbb{Z}_2 \qquad A_{-30} = \mathbb{Z}_2 \qquad A_{-31} = \mathbb{Z}_2$$

and they say $A_d \longleftrightarrow A_{-22-d}$ are Pontryagin dual

unless $d \not\equiv 3 \mod 24$.

as part of the Anderson self-duality of TMF.

Now, note that $A_{-28,-30,-31}$ appear exactly for the angular part of our branes ∇

and the paired $A_{6,8,9}$ appear exactly for the non-angular part of our branes.

Furthermore, it's known that

$\mathcal{N}=(0,1)$ σ-models on

A_3	:	$S^3 \simeq SU(2)$	with H-flux
A_6	:	$S^3 \times S^3$	with H
A_8	:	$SU(3)$	with H
A_9	:	$S^3 \times S^3 \times S^3$	with H

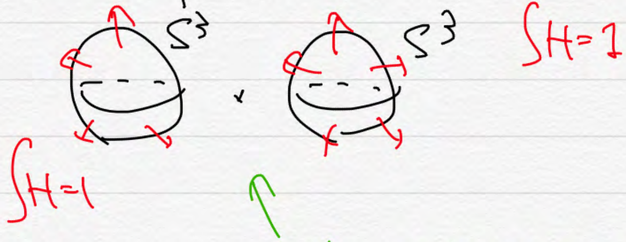
$A_{-28} \subset TMF_{-28}(pt)$

We already saw that

$\boxed{\text{4-brane}}$'s Green-Schwarz coupling

assigns the phase -1

to



$A_6 \subset TMF_6(pt)$

The same turns out to be true:

$A_3 = \mathbb{Z}_{24}$ $\quad \boxed{A_6 = \mathbb{Z}_2} \quad \boxed{A_8 = \mathbb{Z}_2} \quad \boxed{A_9 = \mathbb{Z}_2}$

$\qquad\qquad\qquad\quad \updownarrow \qquad\qquad\quad \updownarrow \qquad\qquad\quad \updownarrow$

$x_{-24} \in TMF_{24}(pt)$ $\quad \underbrace{A_{-28} = \mathbb{Z}_2 \quad A_{-30} = \mathbb{Z}_2 \quad A_{-31} = \mathbb{Z}_2}_{\text{GS coupling ?}}$

$\uparrow$

consistency of
GS coupling.

In general,

Anderson self-duality of TMF

= consistent Green-Schwarz coupling of
heterotic string theory ?

[YT- Yamashita 2305.06196]