On a certain non-supersymmetric heterotic 6-brane

Yuji Tachikawa (Kavli IPMU)

based on an ongoing work mainly with **Kazuya Yonekura**,
with occasional helps from Y. Lee, K. Ohmori, and V. Saxena,
and a lot of mathematical helps from **Mayuko Yamashita**

Hirosifest @ IPMU, 2022

(Words typeset in purple are hyperlinked if you download the slides.)

Ooguri san has been very supportive
from the very early days of my career.

In 2005, the penultimate year of my PhD study,
(and when Ooguri san was at my age now),
he was instrumental in arranging my five-month visit
to the KITP program Mathematical Structures in String Theory
as a graduate fellow.
(For a while I hesitated. Ookouchi san encouraged me to go.)

That kickstarted my career, allowing me to interact with
many notable string theorists, to find collaborators ...

He also invited me to Caltech during my stay.
He even brought me to the Norton Simon Museum.
(Shigemori san was there at the time and took care of my visit.)

Unfortunately I didn't find my photo with Ooguri san from those days.
Let me instead show me having fun in California,
which was enabled by Ooguri san:

Ryu and Takayanagi were in KITP at that time too as postdocs,
working on their famous paper!

In May 2011, there was a conference **Three String Generations at IHÉS.**

In May 2011, there was a conference **Three String Generations at IHÉS.**

It was a joint 50th-ish anniversary meeting for ten string theorists, where some of their mentors and students were also invited.

Since I gave a talk there, **ten years have already passed**, and we're now celebrating Ooguri-san's 60th anniversary.

Time flies.

Concerning these 0 mod 10-years old anniversaries,
there is the following saying of Confucius/孔子:

> 吾十有五而志于學，三十而立，
> 四十而不惑，五十而知天命，
> 六十而耳順，七十而從心所欲，不踰矩

(Analects 2:4)

Let me give a tentative English translation,
based on the Japanese translation by Shigeki Kaizuka,
a famous Chinese classicist and a brother of Hideki Yukawa.

吾十有五而志于學

When I was fifteen, I set my mind on learning.

三十而立

When I was thirty, I stood on my own.

四十而不惑

When I was forty, I stopped vacillating.

五十而知天命

When I was fifty, I understood what heaven wants from me.

六十而耳順,

When I was sixty, my ears became obedient.

(That is, he learned to calmly accept inconvenient truths spoken to him.)

七十而從心所欲，不踰矩

When I was seventy, even when I acted as I wished,
I did not transgress any norm.

五十而知天命

When I was fifty, I understood what heaven wants from me.

Soon after the conference **Three String Generations at IHÉS** was over,
I realized I should have used this quote and
asked Hirosi and other 50-ish participants
if they understood what heaven wanted from them.

I have regretted this missed opportunity ever since.

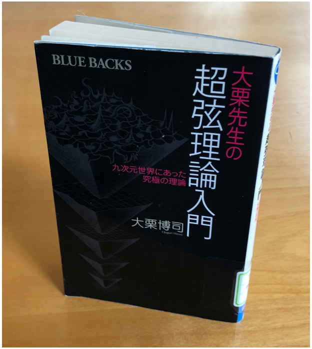
So I won't miss today's chance to ask Hirosi
if he has attained the level of Confucius at sixty:

六十而耳順

> **When I was sixty, my ears became obedient**
> **= I became able to accept inconvenient truths told.**

Let us see how it goes.

He has a popular introduction to String Theory in Japanese



which sold $\mathbf{10^4 \sim 10^5}$ copies.
This book is perfect.

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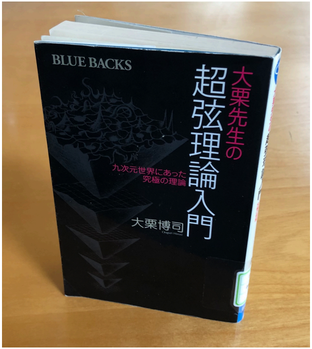
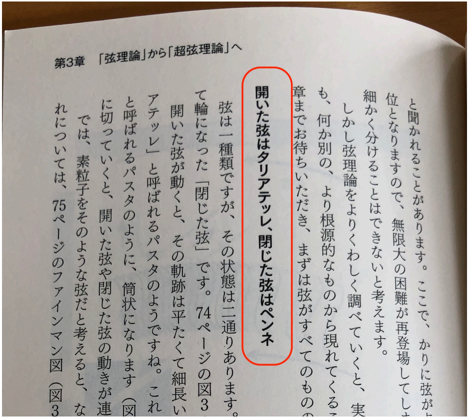


which sold $\mathbf{10^4 \sim 10^5}$ copies.
This book is perfect.

Well, *almost* perfect.

Here are a couple of comments I wanted to make for some time ...



第3章　「弦理論」から「超弦理論」へ

と聞かれることがあります。ここで、かりに弦が
位となりますので、無限大の困難が再登場してし
細かく分けることはできないと考えます。
しかし弦理論をよりくわしく調べていくと、実
も、何か別の、より根源的なものから現れてくる
章までお待ちいただき、まずは弦がすべてのもの

開いた弦はタリアテッレ、閉じた弦はペンネ

弦は一種類ですが、その状態は二通りあります。
て輪になった「閉じた弦」です。74ページの図3
開いた弦が動くと、その軌跡は平たくて細長い
アテッレ」と呼ばれるパスタのようですね。これ
と呼ばれるパスタのように、筒状になります（図
に切っていくと、開いた弦や閉じた弦の動きが連
では、素粒子をそのような弦だと考えると、な
れについては、75ページのファインマン図（図3

In p.73, he compared **open string worldsheets to tagliatelle**, and **closed string worldsheets to penne**.

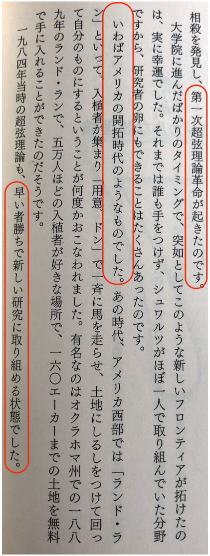
Tagliatelle are good

but not very common in Japan.

(photo taken from https://en.wikipedia.org/wiki/Tagliatelle)

He should have used きしめん (kishimen) instead!

(photo taken from https://www.flickr.com/photos/tamaiyuya/8314125524)


相殺を発見し、第一次超弦理論革命が起きたのです。

　大学院に進んだばかりのタイミングで、突如としてこのような新しいフロンティアが拓けたのは、実に幸運でした。それまでは誰も手をつけず、シュワルツがほぼ一人で取り組んでいた分野ですから、研究者の卵にもできることはたくさんあったのです。

　いわばアメリカの開拓時代のようなものでした。あの時代、アメリカ西部では「ランド・ラン」といって、入植者が集まり「用意、ドン」で一斉に馬を走らせ、土地にしるしをつけて回って自分のものにするということが何度かおこなわれました。有名なのはオクラホマ州での一八八九年のランド・ランで、五万人ほどの入植者が好きな場所で、一六〇エーカーまでの土地を無料で手に入れることができたのだそうです。

　一九八四年当時の超弦理論も、早い者勝ちで新しい研究に取り組める状態でした。

Another is in p.178, where he likened
the **First Superstring Revolution** (from 1984)
with the American Wild West,
in particular the Land Run (around 1890),
where the settlers could occupy the land
as much as they wanted ...

... which I do not think was a particularly good idea.

The free lands were there because the US government
took them from Native Americans.

The phenomenon of American Wild West was contemporary with,
and part of, the global colonization process,
including that of Korea and parts of China by Japan, about a decade later.

So I didn't think that American Wild West was a particularly apt analogy.

I thought a small dose of criticism is the
best possible birthday present to an adult (like a vaccine!)

That's why I chose this topic for this occasion.

Happy 60th anniversary, Ooguri san!

Many happy returns!

For the scientific part, let me talk about something
I've been thinking for a couple of years,
mainly with **K. Yonekura**,
with occasional helps from **Y. Lee, K. Ohmori, and V. Saxena.**

Recently we've been greatly helped by **M. Yamashita** on the math side.

On a certain non-supersymmetric heterotic 6-brane

Consistent quantum gravity should/would have dynamical objects with all possible charges.

There are refinements of this statement, such as the cobordism conjecture of [McNamara-Vafa, 1909.10355].

Ooguri san has played a crucial role in the development of this subject in general.

Today I'd like to consider the case of a certain $\mathbb{Z}_2$ charge.

Consider heterotic $\mathfrak{so}(32)$ theory = Type I theory.

It has **dynamical particles in the adjoint and in the spinor**,
but **not in vector and in the conjugate spinor**.

The gauge group is then $\mathbf{Spin(32)}/\mathbb{Z}_2$ which is *not* $\mathbf{SO(32)}$.

You can consider a background of $\mathbf{Spin(32)}/\mathbb{Z}_2$
which does not allow objects in the vector representation of $\mathfrak{so}(32)$.

Witten called them **backgrounds without vector structure**.

[hep-th/9712028]

It is measured by an obstruction class $v_2 \in H^2(\text{spacetime}, \mathbb{Z}_2)$ such that

$$\int_C v_2 = 0 \text{ or } 1$$

depending on whether the $\mathbf{Spin(32)}/\mathbb{Z}_2$ configuration restricted on the 2-cycle C **lifts to** $\mathbf{Spin(32)}$ or **not**.

So it is a $\mathbb{Z}_2$ charge.

Is there a dynamic object which has this charge?

It should be a **6-brane** of the form

$$\mathbb{R}^{6,1} \times \mathbb{R}_{>0} \times S^2$$

where

$$\int_{S^2} v_2 = 1 \mod 2.$$

It is not supersymmetric.

A brane breaks translation, and therefore breaks some SUSY.
But the minimal SUSY in 7d has the same # of supercharges as the 10d bulk theory itself.

It is a $\mathbb{Z}_2$-torsion brane.

This is because its conserved charge is $\int_{S^2} v_2$, which is mod 2.

There are $\mathbb{Z}_2$-torsion non-BPS D-branes in type I $\mathfrak{so}(32)$ superstring.
[Witten, hep-th/9810188]

Is it one of those non-BPS D-branes? No.

Type I $\mathbb{Z}_2$-torsion branes are D8, D7 and D0, D-1.

More fundamentally, all type I D-branes come from $\mathbf{O(32)}$ gauge fields on the 9-branes.

And our brane is characterized exactly by the condition that it is *not* an $\mathbf{O(32)}$ gauge field.

Does it really exist? If it does, what properties does it have?

A simple $\mathfrak{so}(32)$ configuration on S^2 without vector structure is obtained as follows.

Consider $\mathbf{U(16)} \subset \mathbf{SO(32)}$,
and put a charge-$\mathbf{1/2}$ monopole in the $\mathbf{U(1)}$ part of $\mathbf{U(16)}$.

This kills $\mathbf{16}$ of $\mathbf{U(16)}$,
and therefore also kills $\mathbf{32}$ of $\mathbf{SO(32)}$. So it is without vector structure.

Adjoints and spinors of $\mathfrak{so}(32)$ becomes $\wedge^{\text{even}}\mathbf{16}$,
the even-indexed antisymmetric tensors, of $\mathbf{U(16)}$. So they are fine.

Let us take the S-wave approximation to analyze

$$\mathbb{R}^{6,1} \times \mathbb{R}_{>0} \times S^2.$$

By performing a KK reduction along S^2, one finds that
$\exists$ **an 8d chiral spinor** in $\wedge^2 \mathbf{16}$ of $\mathfrak{su}(16)$ on

$$\mathbb{R}^{6,1} \times \mathbb{R}_{>0}.$$

How can a chiral spinor be put on a half-space?

There seems no boundary condition you can write.

It is similar to the Callan-Rubakov problem, but in higher dimensions.

The anomaly polynomial I_{10} of an 8d chiral spinor in $\wedge^2 \mathbf{16}$ of $\mathfrak{su}(16)$ has the factorized form

$$I_{10} = -\left(\frac{p_1(R)}{4} + \frac{c_2(F)}{2}\right) c_3(F)$$

and is canceled by the Green-Schwarz mechanism with the B-field.

So **the total anomaly vanishes**, and there is **no fundamental obstruction to write a boundary condition to the combined system**, although no weakly-coupled one can be written (as far as I know).

Therefore, **the 6-brane at the boundary in the infrared limit would realize a strongly-coupled non-supersymmetric scale-invariant theory** if it really exists.

Does such a 6-brane really exist?

Let's start with the black brane solution in heterotic $\mathfrak{so}(32)$ theory [Gibbons-Maeda 1988], [Garfinkle-Horowitz-Strominger 1991]:

$$
\begin{aligned}
F &= Q\epsilon_{S^2}, \\
e^{-2\phi} &= 1 - r_-/r, \\
\mathrm{d}s^2 &= -\frac{(1 - r_+/r)}{(1 - r_-/r)}\mathrm{d}t^2 + \mathrm{d}x^i\mathrm{d}x^i \\
&\qquad + \frac{\mathrm{d}r^2}{(1 - r_+/r)(1 - r_-/r)} + r^2\mathrm{d}\Omega^2_{S^2}
\end{aligned}
$$

where

$$
Q \propto r_+ r_-, \qquad M \propto r_+ + r_-.
$$

For $r_+ \gg \ell_s$, this is a perfectly nice supergravity black hole solution at nonzero temperature. **Let it evaporate ...**

We get to a very simple, extremal, zero-temperature solution with an infinitely-long throat region:

$$\begin{aligned} F &= Q\epsilon_{S^2}, \\ e^{-2\phi} &= e^{y/r_0}, \\ \mathrm{d}s^2 &= \mathrm{d}x^\mu \mathrm{d}x_\mu + \mathrm{d}y^2 + r_0^2 \mathrm{d}\Omega_{S^2}^2 \end{aligned}$$

where

$$r_0 \propto Q\ell_S.$$

All these were in the papers from 1991. The only new part is the particular choice of $\mathfrak{u}(1) \subset \mathfrak{u}(16) \subset \mathfrak{so}(32)$.

Comments

- Tension $\sim g_{\text{het}}^{-2}$ in heterotic, as in heterotic NS5-brane, but $\sim g_{\text{type I}}^{-3/2}$ in type I, in between NS5-brane and D-brane.
- Heterotic coupling diverges at the core, while Type I coupling goes to zero.
- Throat infinite in the heterotic frame but finite in length in the Type I frame. The origin singular.
- S^2 is of stringy size. Classical solution not reliable with $Q = 1$.

Let us have a more detailed look at the $\boldsymbol{S^2}$ part.
Luckily, the heterotic worldsheet theory can be analyzed exactly.

The worldsheet theory has the structure

$$\underbrace{(\psi^L_{1,\ldots,16}, \bar{\psi}^L_{1,\ldots,16})}_{\text{monopole flux}=1/2} \leftrightarrow S^2 \leftrightarrow \underbrace{(\Psi^R, \bar{\Psi}^R)}_{\text{monopole flux}=2}$$

It happens that

$$\psi^L_{1,\ldots,16}, \bar{\psi}^L_{1,\ldots,16} \sim (\mathrm{SU}(16)/\mathbb{Z}_4)_1 + (\Psi^L, \bar{\Psi}^L)$$

up to a finite gauging, so that the original theory is basically

$$(\mathrm{SU}(16)/\mathbb{Z}_4)_1 + \underbrace{(\Psi^L, \bar{\Psi}^L)}_{\text{monopole flux}=2} \leftrightarrow S^2 \leftrightarrow \underbrace{(\Psi^R, \bar{\Psi}^R)}_{\text{monopole flux}=2}$$

This is

$$(\mathbf{SU(16)}/\mathbb{Z}_4)_1 + (\mathcal{N}{=}(2,2)\ S^2 \text{ sigma model})$$

but the second term becomes gapped in the infrared, leaving just

$$(\mathbf{SU(16)}/\mathbb{Z}_4)_1,$$

a purely holomorphic spin-CFT of $c_L = 15$.

So, the extremal limit of our black 6-brane has
an exact heterotic worldsheet CFT description with

$$\mathbb{R}^{6,1} + (\text{linear dilaton}) + (\mathbf{SU(16)}/\mathbb{Z}_4)_1$$

which is very similar to that of an NS5-brane.

Will that convince you/me that this brane really exists? I don't know...

NB. This is the same solution as the stable background to the 10d tachyonic heterotic $\mathfrak{su}(16)$ theory found in [Kaidi 2010.10521].

Nature of the $\mathbb{Z}_2$ charge

We started by saying that we consider $\displaystyle\int_{\text{2-cycle}} \boldsymbol{v_2}$,
measuring the absence of the vector structure for $\mathfrak{so}(\mathbf{32})$ on the 2-cycle.

But this statement is only applicable
for geometric heterotic compactifications.

We saw that

$$\mathfrak{so}(\mathbf{32})\ \boldsymbol{S^2} \text{ without vector structure} \xrightarrow{\text{RG}} \boldsymbol{(SU(16)/\mathbb{Z}_4)_1}.$$

Therefore, an abstract internal CFT $\mathbf{(SU(16)/\mathbb{Z}_4)_1}$ should
equally have this $\mathbb{Z}_2$ charge.

What is this $\mathbb{Z}_2$ charge, in the worldsheet CFT language?

We want a $\mathbb{Z}_2$-valued invariant for worldsheet $\mathcal{N}=(0,1)$ SQFTs with $2(c_R - c_L) = -30$, suitable for a compactification to 8d.

The best-known invariant of 2d SQFTs is the elliptic genus = superconformal index, taking values in modular forms with integer q-expansion coefficients.

It is nonzero only for $2(c_R - c_L) \equiv 0$ or 4 mod 8.

This is due to $\mathrm{KO}_i(pt) = \mathbb{Z}$ only for $i \equiv 0$ or 4 mod 8.

Corresponding to $\mathbf{KO}_i(pt) = \mathbb{Z}_2$ for $i \equiv 1, 2$ mod 8,
we can consider **mod-2 elliptic genera** of $\mathcal{N}{=}(0, 1)$ SQFTs
when $2(c_R - c_L) = 1, 2$ mod 8.

We have $2(c_L - c_R) = -30 = 2$ mod 8, so this might be it,
we thought.

Surprisingly, there's no paper on the hep-th side,
and just two papers on the math side [Ochanine 1991], [Liu 1992].

Kazuya and I developed the theory of mod-2 elliptic genera
from hep-th point of view (unpublished).

Unfortunately, they turned out to be zero for $(\mathrm{SU}(16)/\mathbb{Z}_4)_1$.

So our efforts are wasted.

What is this $\mathbb{Z}_2$ charge, then?

Well, the Stolz-Teichner conjecture says that:

$$\mathbf{TMF}_n(pt) = \frac{\left\{ \begin{array}{c} \text{2d } \mathcal{N}=(0,1) \text{ supersymmetric theory} \\ \text{with } n = 2(c_R - c_L) \end{array} \right\}}{\text{continuous deformation}}$$

where $\mathbf{TMF}_n(pt)$ is the **group of degree-n topological modular forms**.

Our case is $n = -30$, for which mathematicians have found

$$\mathbf{TMF}_{-30}(pt) = \underbrace{\mathbb{Z}_2}_{\substack{\text{has zero} \\ \text{mod-2 elliptic genus}}} \oplus \underbrace{\mathbb{Z}_2[J]}_{\substack{\text{has nonzero} \\ \text{mod-2 elliptic genus}}}$$

So, our $\mathbb{Z}_2$ charge measuring the absence of vector structure is a good candidate for this $\mathbb{Z}_2$ part.

And indeed it is the case;
Mayuko Yamashita kindly confirmed it by a heroic computation.

Let me reiterate:

The **absence of vector structure on a geometric 2-cycle**,
when stated abstractly in terms of heterotic worldsheet theory,
is **a subtle TMF charge**.

But the 8d spacetime is still geometric:

$$\begin{array}{cccc} 10\mathrm{d} = & \mathbf{8d} & + & \mathbf{2d} \\ & \textbf{geometric} & + & \textbf{abstract SCFT} \end{array}$$

Is there an 8d interpretation to this charge?

Is there an 8d interpretation to this charge?

Mod-2 elliptic genus counts the number mod-2 of R-ground states.
R-ground states of the worldsheet theory give massless fermions.

So, **mod-2 elliptic genus of the worldsheet theory**
= the number mod-2 of massless spacetime fermions.

Are there other natural $\mathbb{Z}_2$-valued quantities in 8d?

In 2d $\mathbf{U}(\boldsymbol{N})$ gauge theory, we can consider the continuous theta angle

$$\boldsymbol{i\theta} \int \mathbf{tr}\, \frac{\boldsymbol{F}_{\mathbf{U}(\boldsymbol{N})}}{\boldsymbol{2\pi}}$$

where $\boldsymbol{\theta} \sim \boldsymbol{\theta} + \boldsymbol{2\pi}$. When the gauge group is $\mathbf{SO}(\boldsymbol{N})$, we have the discrete theta angle

$$\boldsymbol{i\theta} \int \boldsymbol{w}_{\mathbf{2}}(\mathbf{SO}(\boldsymbol{N}))$$

where $\boldsymbol{\theta} = \mathbf{0}$ or $\boldsymbol{\pi}$, instead.

When the topological class is $\mathbb{Z}$ valued, the theta angle is continuous.

When the topological class is $\mathbb{Z}_{\mathbf{2}}$ valued, the theta angle is discrete.

In 8d with gravity and B-field with the condition $dH \propto \operatorname{tr} R^2$, available topological classes are [Giambalvo 1971]

$$\Omega_8^{\text{string}}(pt) = \mathbb{Z}_2 \oplus \mathbb{Z}$$

where $\mathbb{Z}$ part is

$$\propto \int_{8\text{d}} \operatorname{tr} R^4$$

so a continuous theta angle is possible:

$$\propto i \int_{8\text{d}} \theta \operatorname{tr} R^4 .$$

This continuous theta angle is actually present in heterotic strings. Indeed, in 10d, we have

$$\propto i \int_{10d} B \wedge \mathrm{tr}\, R^4$$

required from the Green-Schwarz cancellation.

Its 8d compactification with

$$\theta := \int_{2d} B$$

gives

$$\propto i \int_{8\mathrm{d}} \theta\, \mathrm{tr}\, R^4.$$

where θ is **dynamical**, as it should/would be in a quantum gravity theory.

How about the $\mathbb{Z}_2$ part in

$$\Omega_8^{\text{string}}(pt) = \mathbb{Z}_2 \oplus \mathbb{Z} \quad ?$$

This is known to be generated by
the exotic 8-sphere $\sim$ the group manifold $\mathbf{SU(3)}$ with a unit $\boldsymbol{H}$ flux.

Is there a discrete theta angle for it in heterotic strings?

Yes, exactly when the 2d internal space is without vector structure.

More abstractly, a **heterotic internal worldsheet theory giving the element in the $\mathbb{Z}_2$ part in $\mathbf{TMF}_{-30}(pt)$** **leads to this nontrivial discrete $\mathbb{Z}_2$ theta angle in 8d**, detecting the exotic 8-sphere $\sim$ the group manifold $\mathbf{SU(3)}$ with a unit $\boldsymbol{H}$ flux:

$$
\begin{array}{rcccccc}
\text{internal 2d `space':} & \mathbf{TMF}_{-30}(pt) = & \mathbb{Z}_2 & \oplus & \mathbb{Z}[J] \\
\text{(represented by abstract SCFT)} & & \updownarrow & & \\
\text{8d spacetime:} & \Omega_8^{\text{string}}(pt) = & \mathbb{Z}_2 & \oplus & \mathbb{Z}
\end{array}
$$

There should be a physics derivation of this fact.

At present, we only have a very abstract method using the Anderson self-duality of $\mathbf{Tmf}$, again thankfully provided by M. Yamashita.

Summary of the physics part

We described some properties of heterotic 6-brane
carrying the *without-vector-structure* $\mathbb{Z}_2$ charge.

It is non-supersymmetric, but has a very simple worldsheet description

$$\mathbb{R}^{6,1} + \text{(linear dilaton)} + (\mathbf{SU(16)}/\mathbb{Z}_4)_1.$$

It gives a mysterious strongly-coupled boundary condition
for an 8d chiral fermion whose anomaly is canceled
via Green-Schwarz mechanism.

It strongly suggests that it provides us a strongly coupled
non-supersymmetric theory in 7d.

Summary of the math part

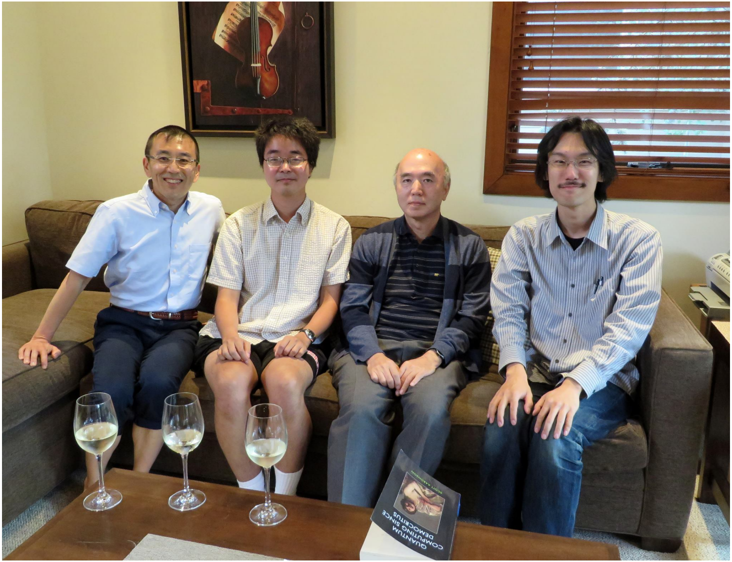
This *without-vector-structure* $\mathbb{Z}_2$ charge,
when stated abstractly in terms of heterotic worldsheet theory,
is the $\mathbb{Z}_2$ part of

$$\mathbf{TMF}_{-30}(pt) = \mathbb{Z}_2 \oplus \mathbb{Z}_2[J].$$

In 8d spacetime language, it is a $\mathbb{Z}_2$ theta angle for the $\mathbb{Z}_2$ part of

$$\Omega_8^{\text{string}}(pt) = \mathbb{Z}_2 \oplus \mathbb{Z}$$

measuring the exotic 8-sphere
$\sim$ the group manifold $\mathbf{SU(3)}$ with a unit H flux.



Aspen, August, 2013