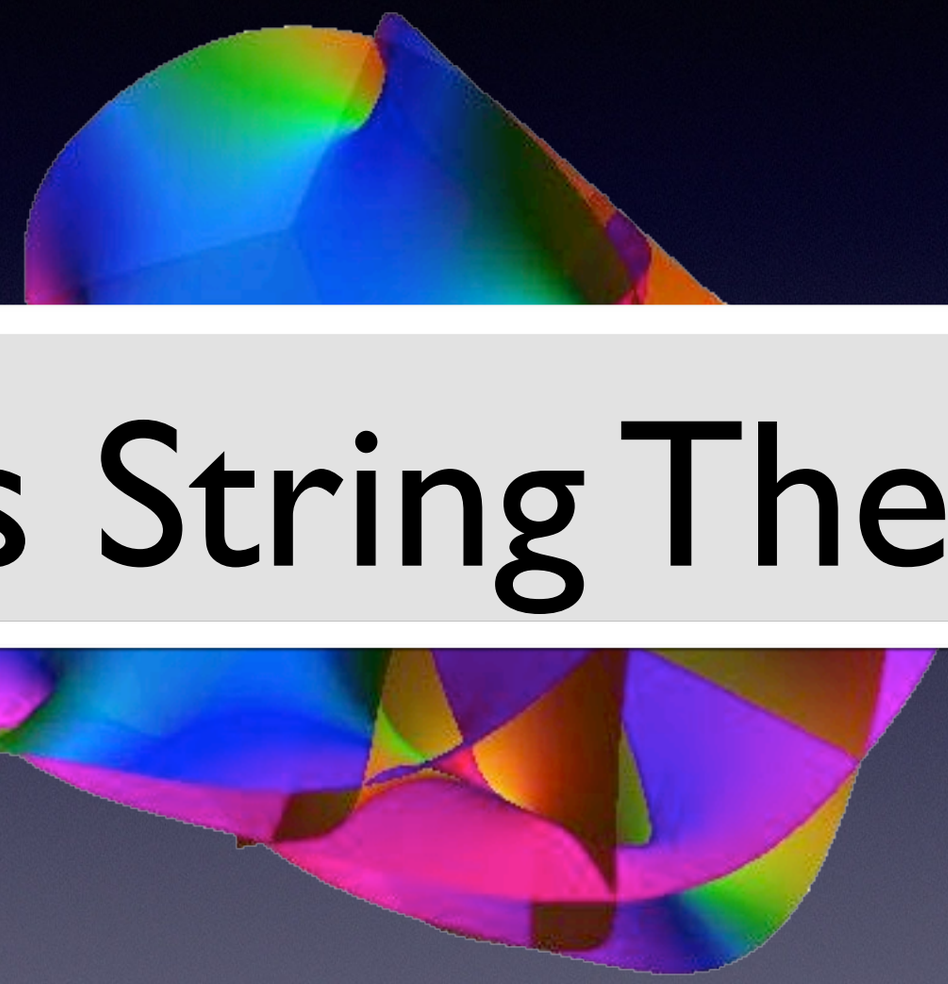
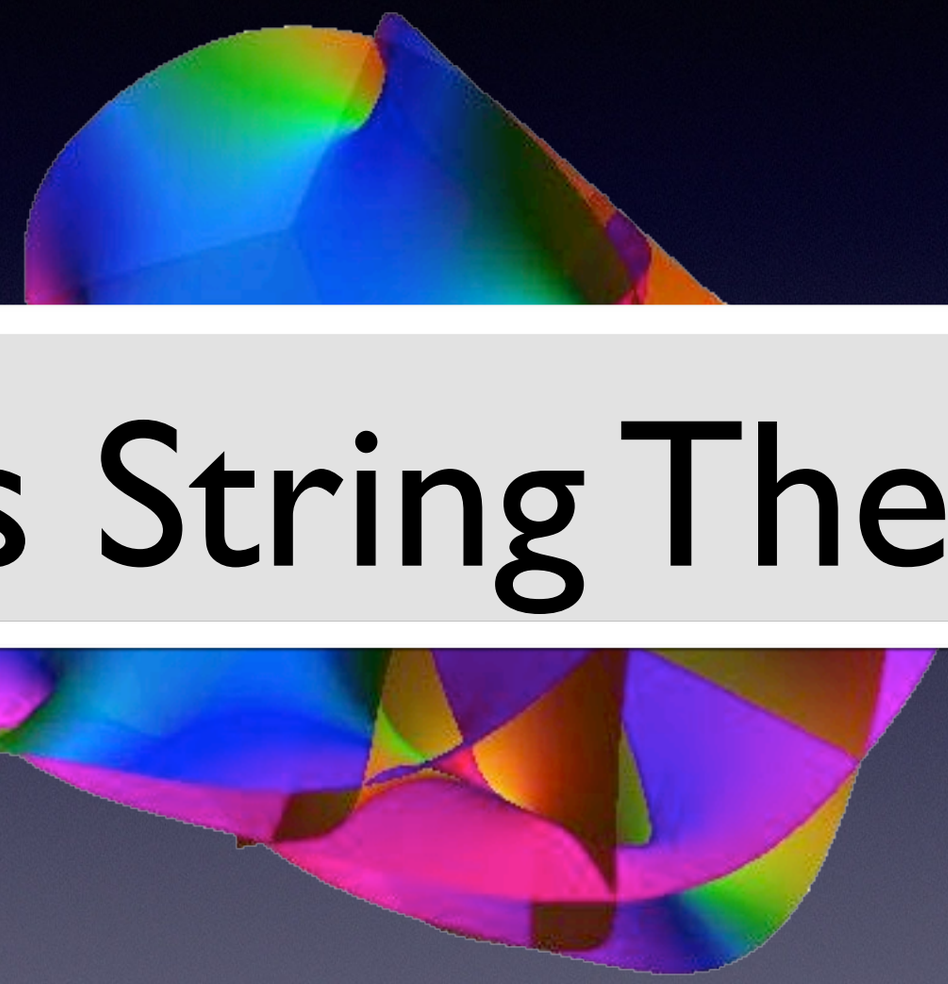


What's String Theory?



What's String Theory?

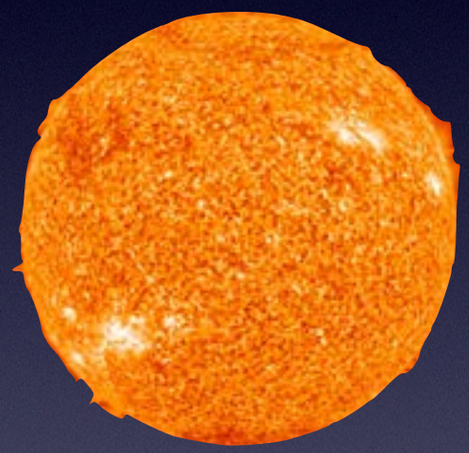
String theory is ...

- A branch of physics
  where we try to reconcile
- gravity and
- quantum mechanics.

- What's gravity?
- What's quantum mechanics?
- Why do we have to reconcile them?
- How do we reconcile them?

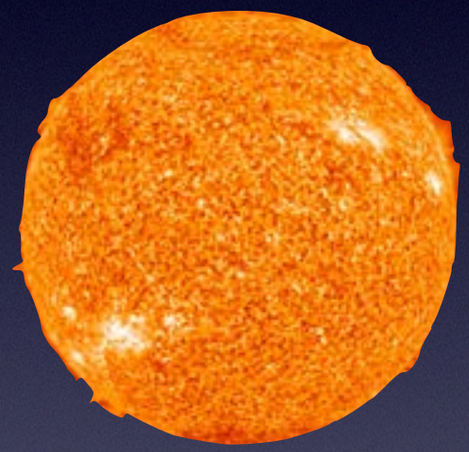
What's Gravity?

Mostly described by Newtonian mechanics!



https://commons.wikimedia.org/wiki/File:The_Sun_by_the_Atmospheric_Imaging_Assembly_of_NASA%27s_Solar_Dynamics_Observatory_-_20100819.jpg
https://commons.wikimedia.org/wiki/File:The_Earth_seen_from_Apollo_17.jpg
https://apod.nasa.gov/apod/ap130301.html

Mostly described by Newtonian mechanics!



https://commons.wikimedia.org/wiki/File:The_Sun_by_the_Atmospheric_Imaging_Assembly_of_NASA%27s_Solar_Dynamics_Observatory_-_20100819.jpg
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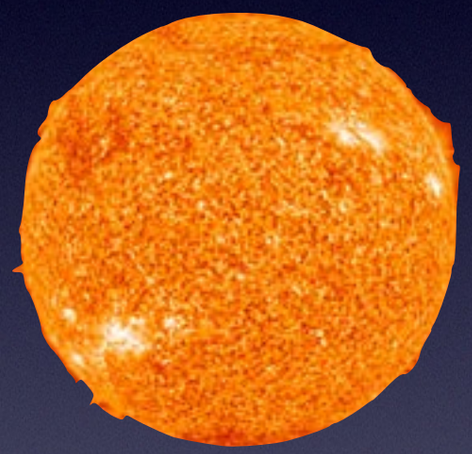
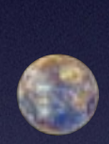
Mostly described by Newtonian mechanics!





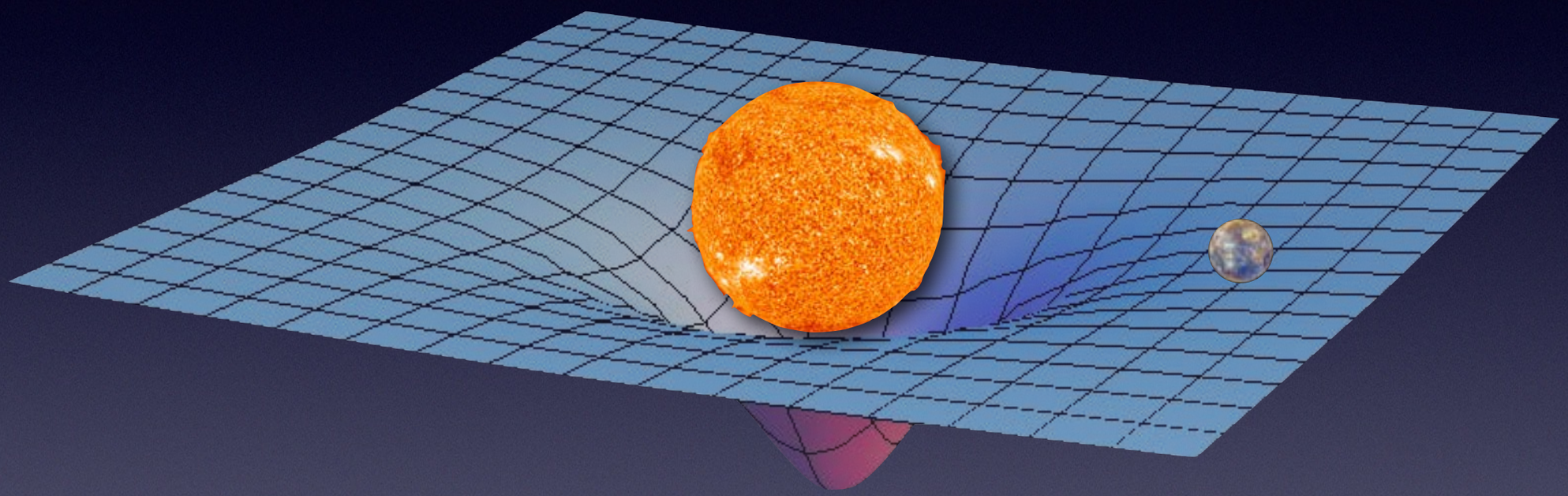
https://commons.wikimedia.org/wiki/File:The_Sun_by_the_Atmospheric_Imaging_Assembly_of_NASA%27s_Solar_Dynamics_Observatory_-_20100819.jpg
https://commons.wikimedia.org/wiki/File:The_Earth_seen_from_Apollo_17.jpg
https://apod.nasa.gov/apod/ap130301.html

Mostly described by Newtonian mechanics!

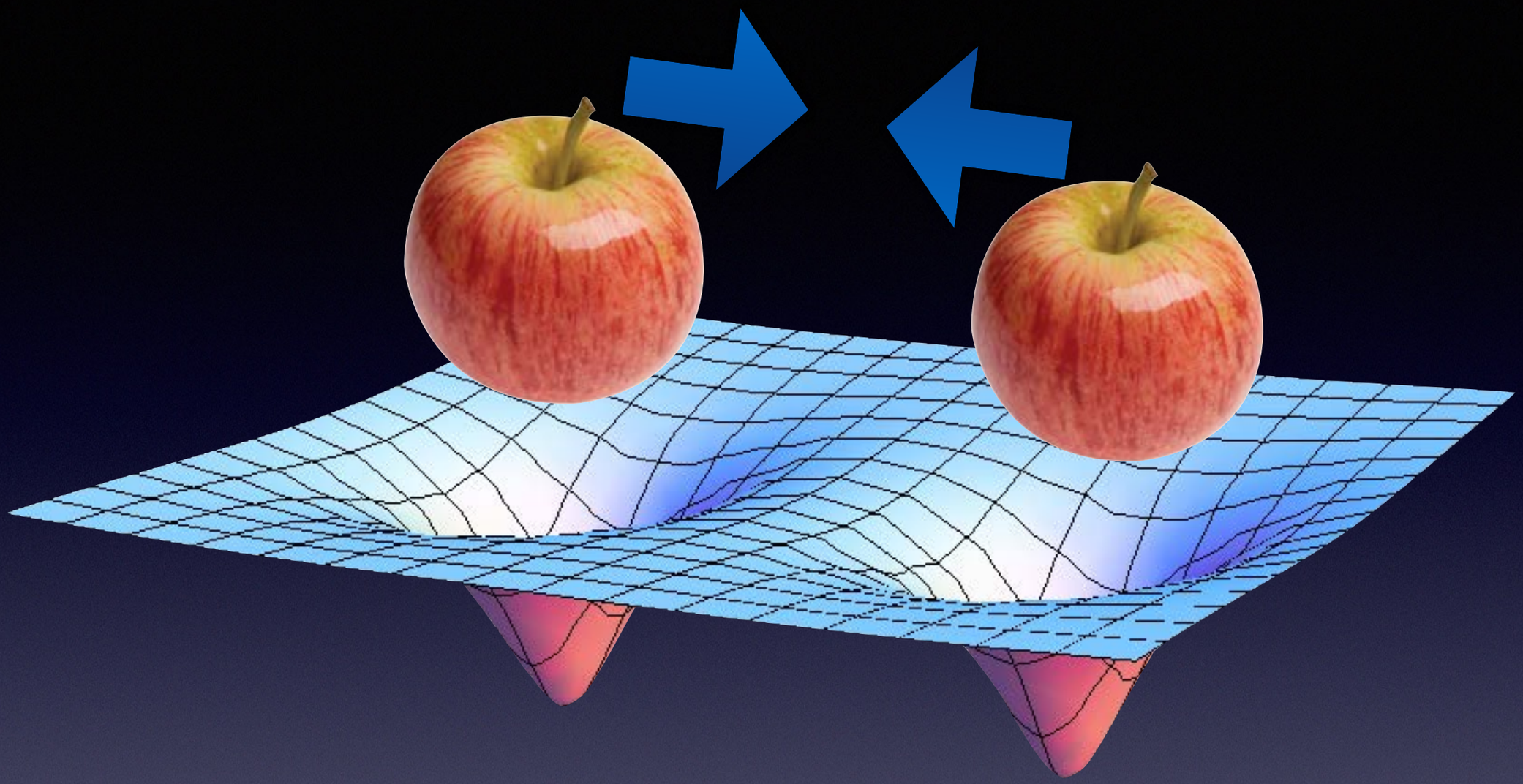
Motion of Mercury deviates significantly from it.

https://commons.wikimedia.org/wiki/File:The_Sun_by_the_Atmospheric_Imaging_Assembly_of_NASA%27s_Solar_Dynamics_Observatory_-_20100819.jpg
https://commons.wikimedia.org/wiki/File:The_Earth_seen_from_Apollo_17.jpg
https://apod.nasa.gov/apod/ap130301.html

Einstein says it's due to the warping
of the spacetime itself.



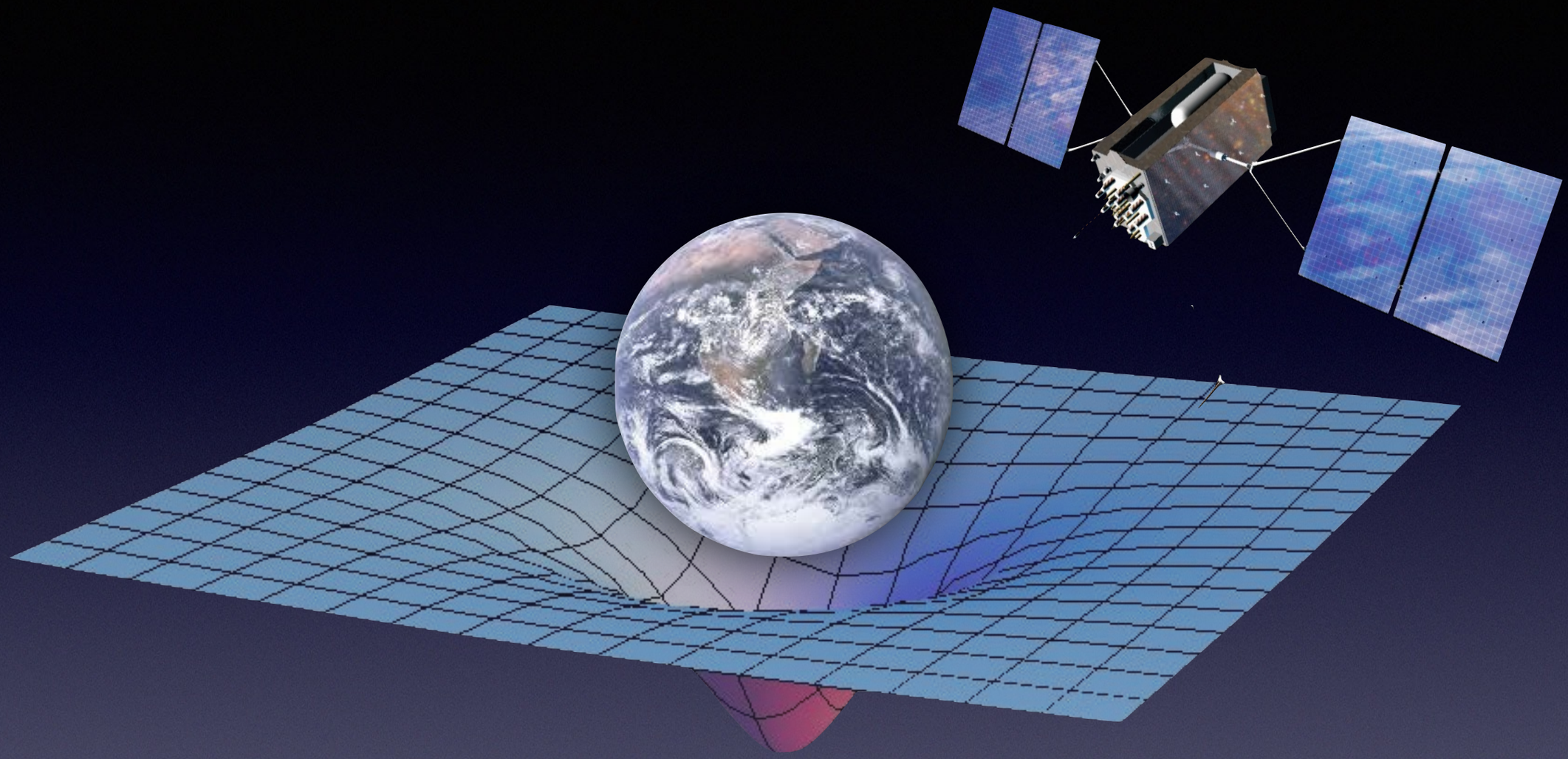
Called General Relativity.



Attraction between apples can also be
described by General Relativity
if you want, but it's probably overkill.

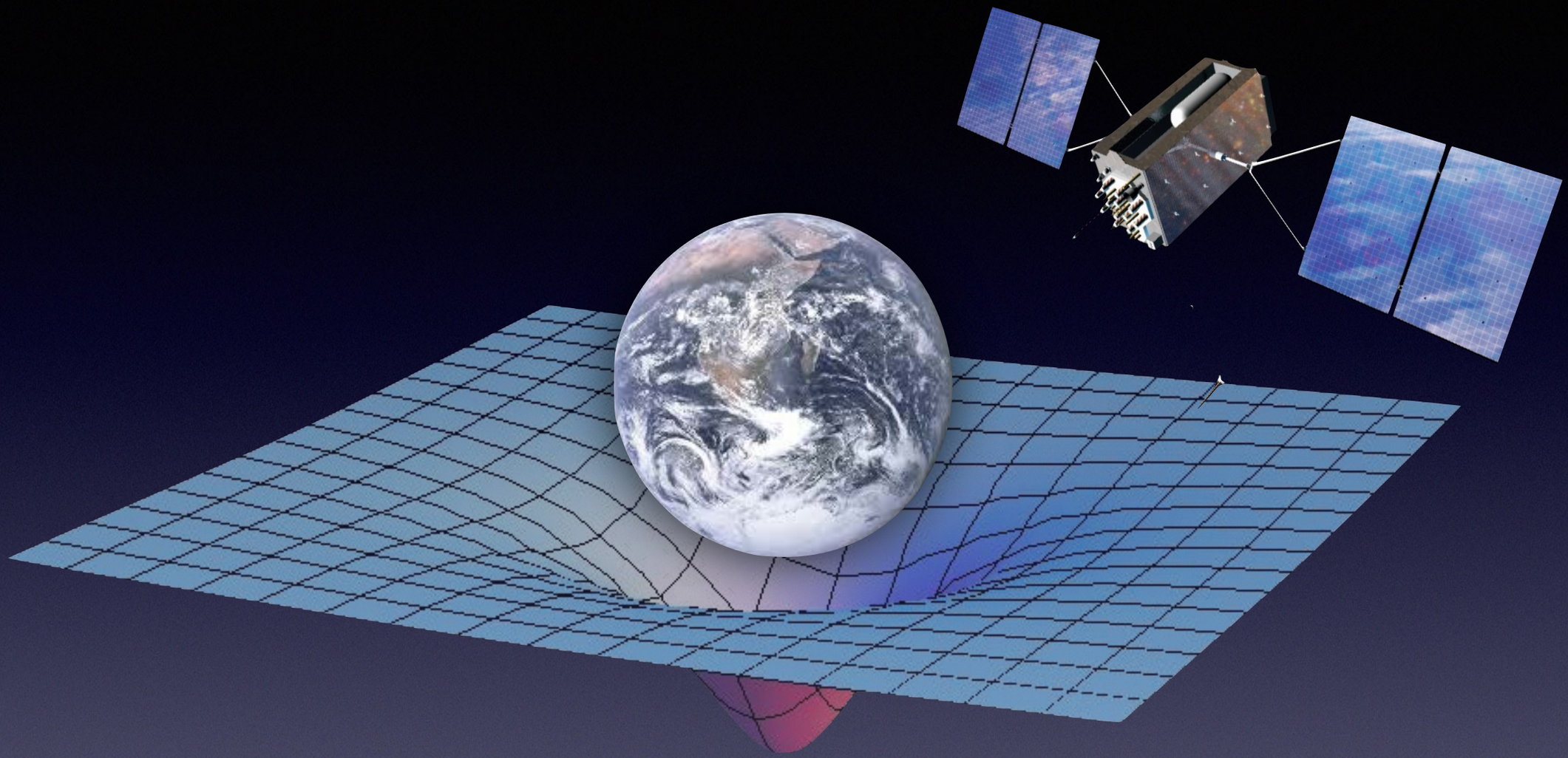
https://www.flickr.com/photos/applesnpearsau/12197650876

https://www.gps.gov/multimedia/images/GPS-III-A.jpg

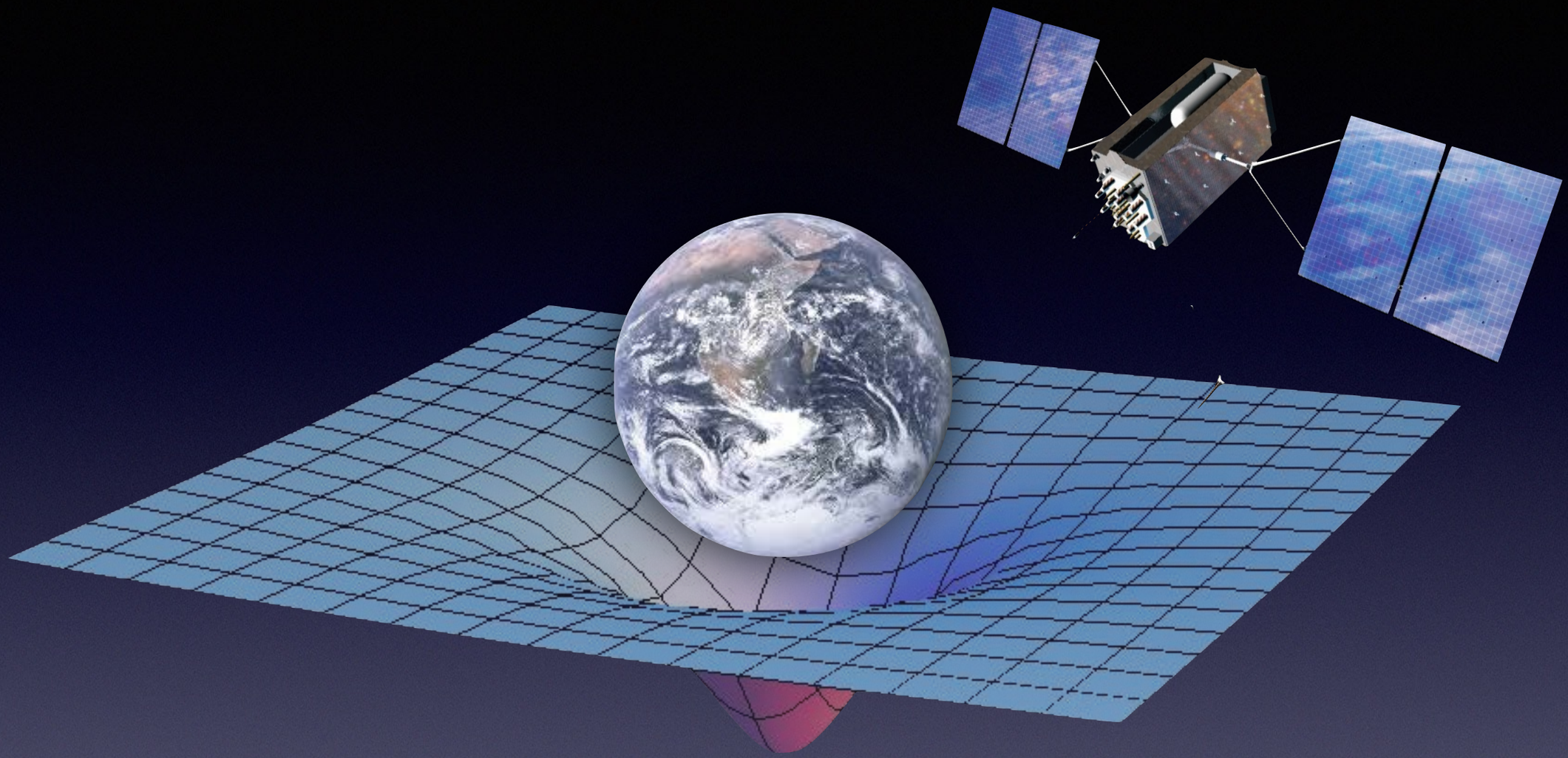


If you have a smartphone, it probably has GPS in it.

Due to relativistic effects,
the time inside the satellite runs faster
(around 40 μs per day).

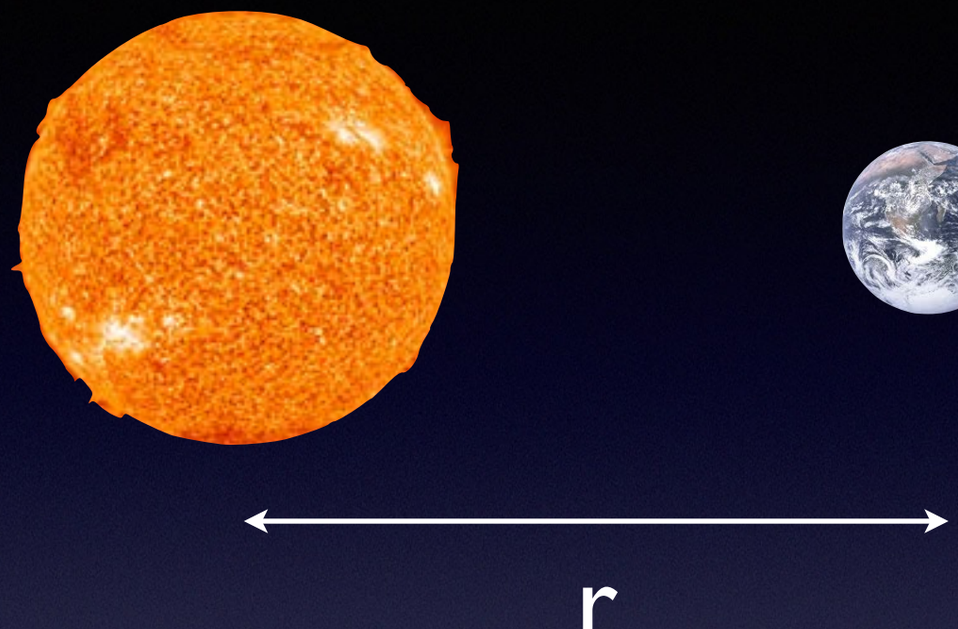


If uncorrected, this would totally ruin the accuracy
of your satellite navigation system!
People who designed GPS knew this, and implemented
precaution against it.

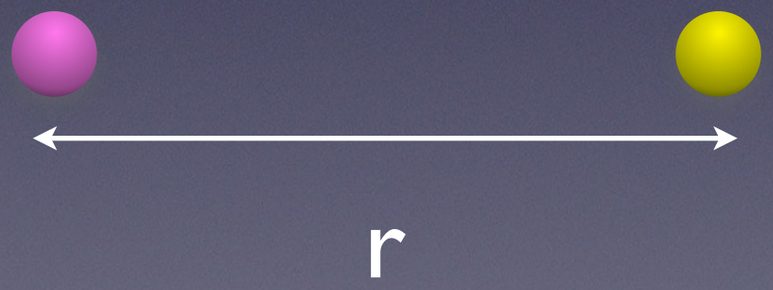


Even without Einstein, we would have known
the special and general relativity by now,
first as a mysterious source of error
in the GPS system.

Gravity

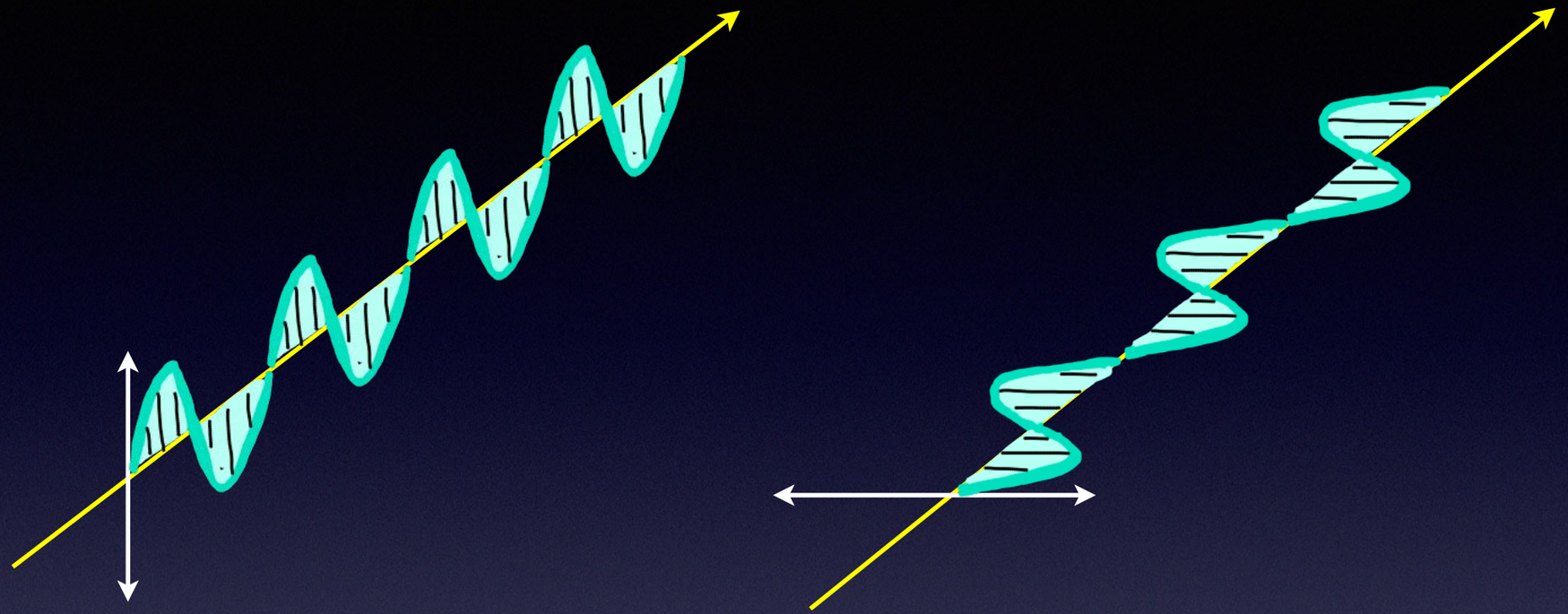


$$F = G\frac{Mm}{r^2}$$



$$F = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}$$

Electromagnetism



Light (electromagnetic wave) has two polarizations.

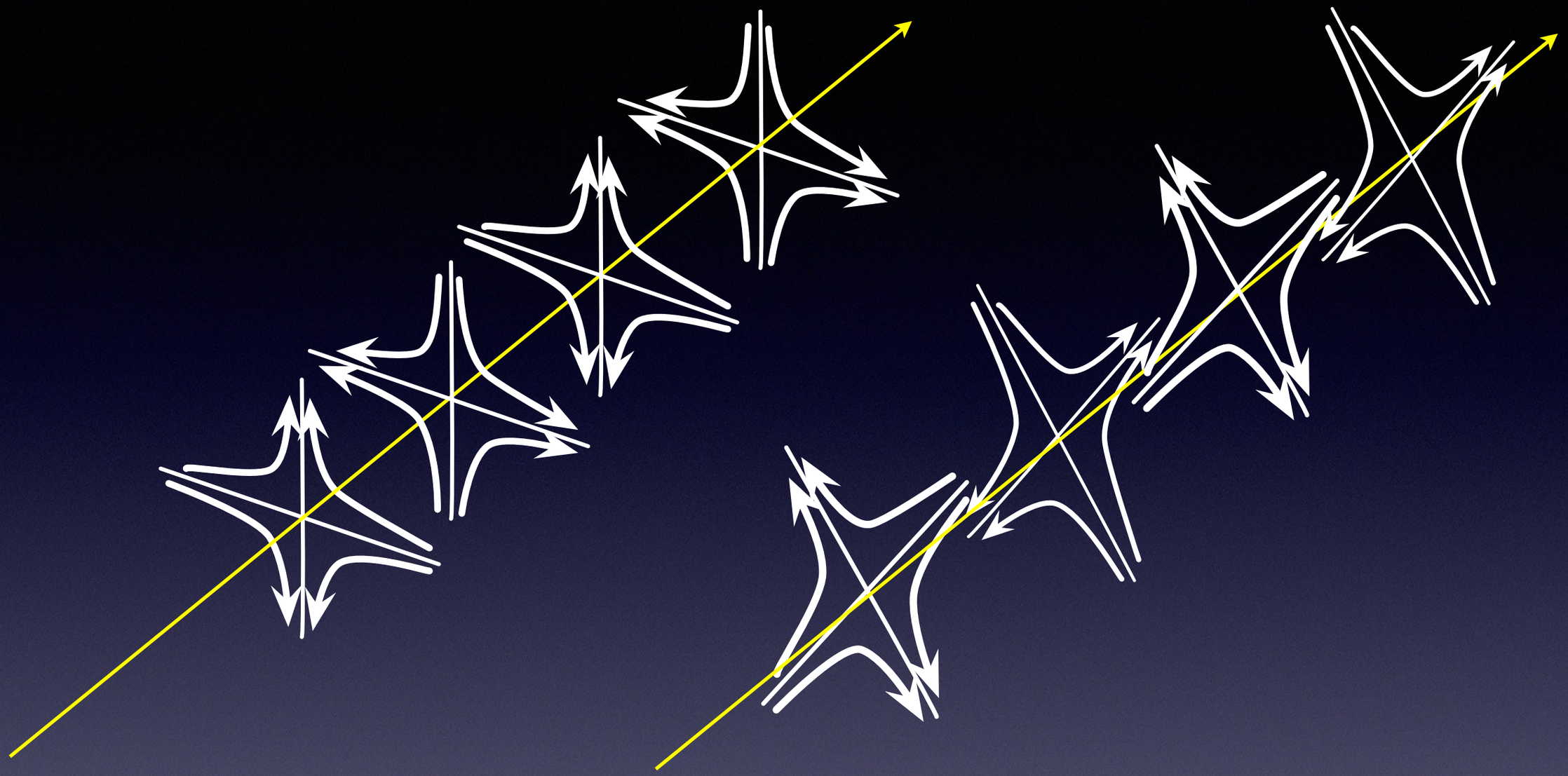
You rotate one 90 degrees, you get the other.

We can’t see polarization of light directly,
but mantis shrimps can.

https://www.flickr.com/photos/37707866@N00/2276197031

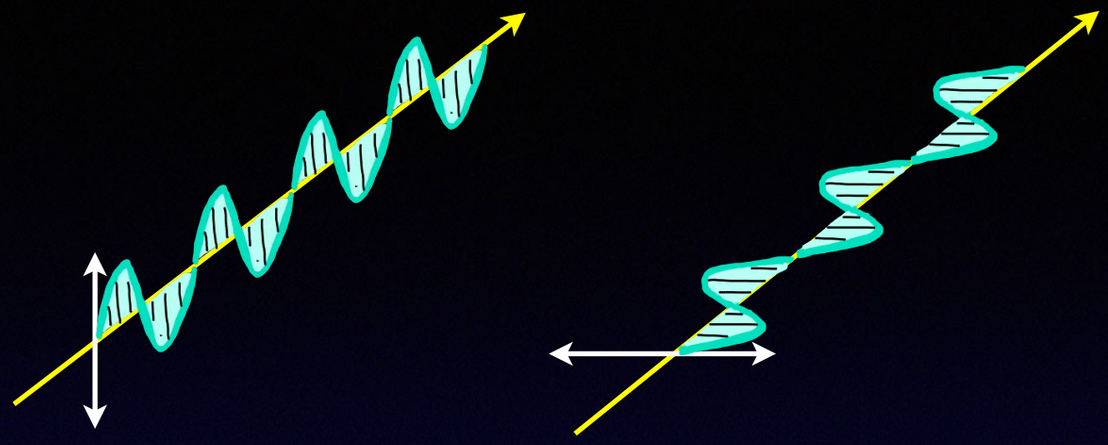
Some sunglasses have polarizers.

https://commons.wikimedia.org/wiki/File:Circularly_polarized_glasses.jpg



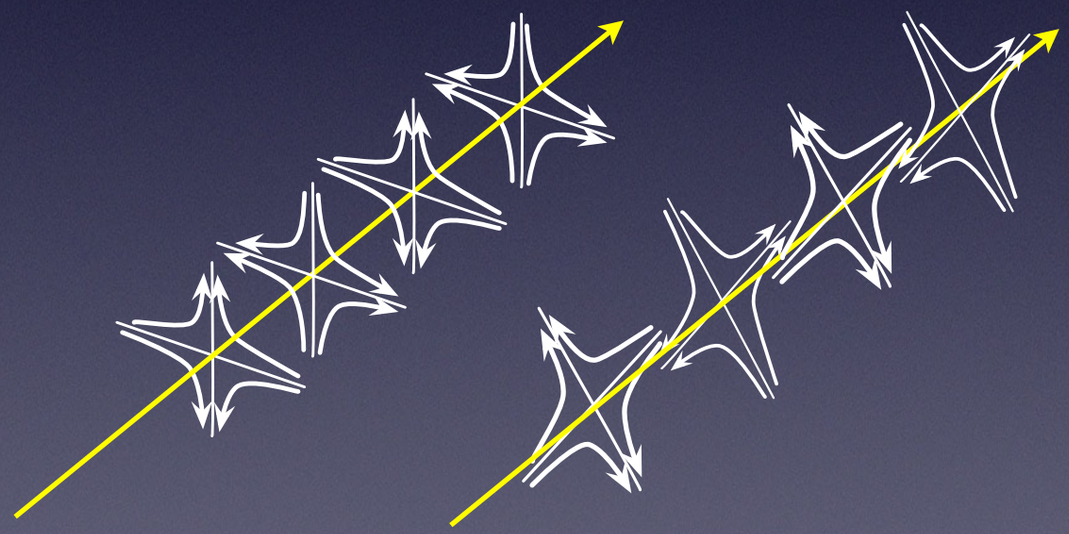
Gravitational wave also has two polarizations.

You rotate one 45 degrees, you get the other.



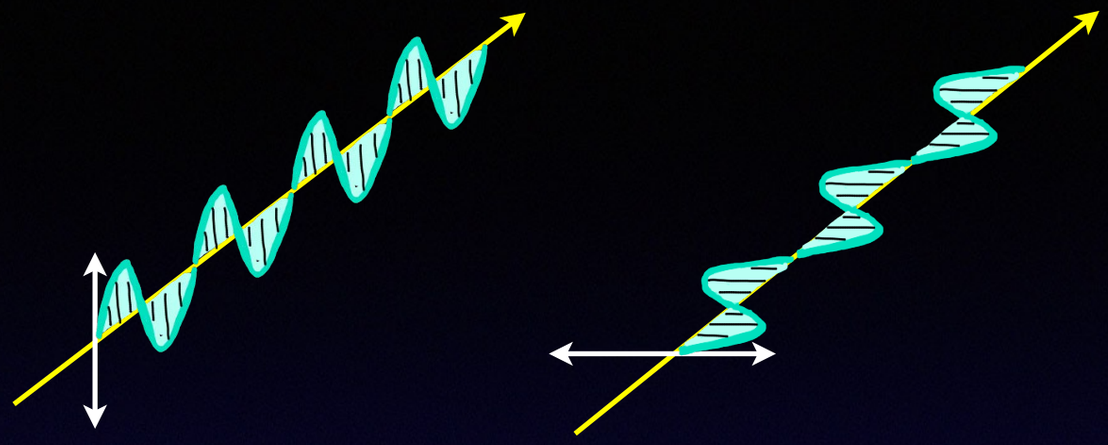
You rotate one 90 degrees, you get the other.

Called Spin 1.



You rotate one 45 degrees, you get the other.

Called Spin 2.



You rotate one 90 degrees, you get the other.

Called Spin 1.

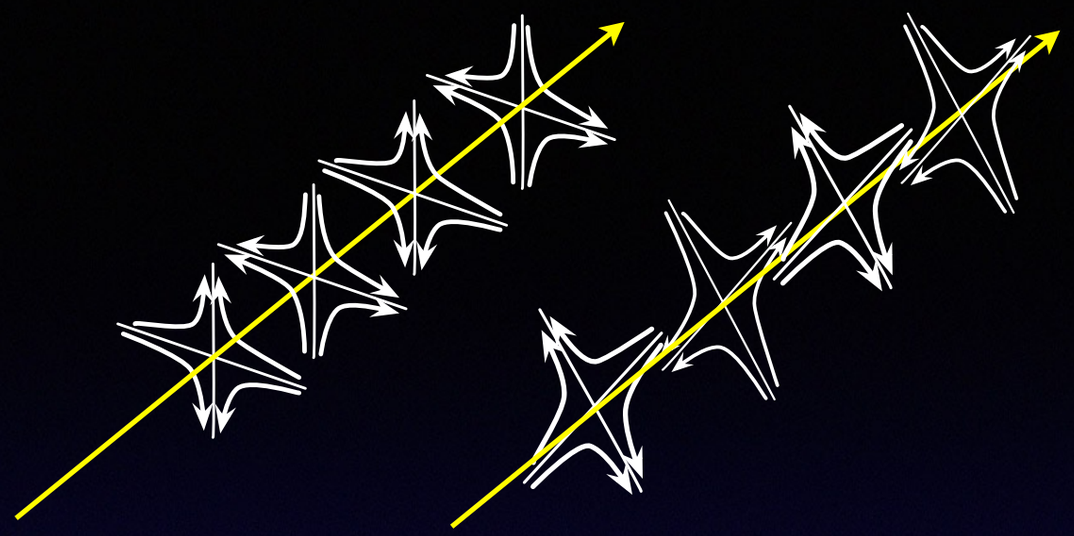
Light (electromagnetism) is spin 1.

"Weak nuclear force" is also spin 1.

"Strong nuclear force" is also spin 1.

Theoretically, you can have as many spin-1 forces as you want.

Experimentally, there are three.



You rotate one 45 degrees, you get the other.
Called Spin 2.

Gravity is spin 2.

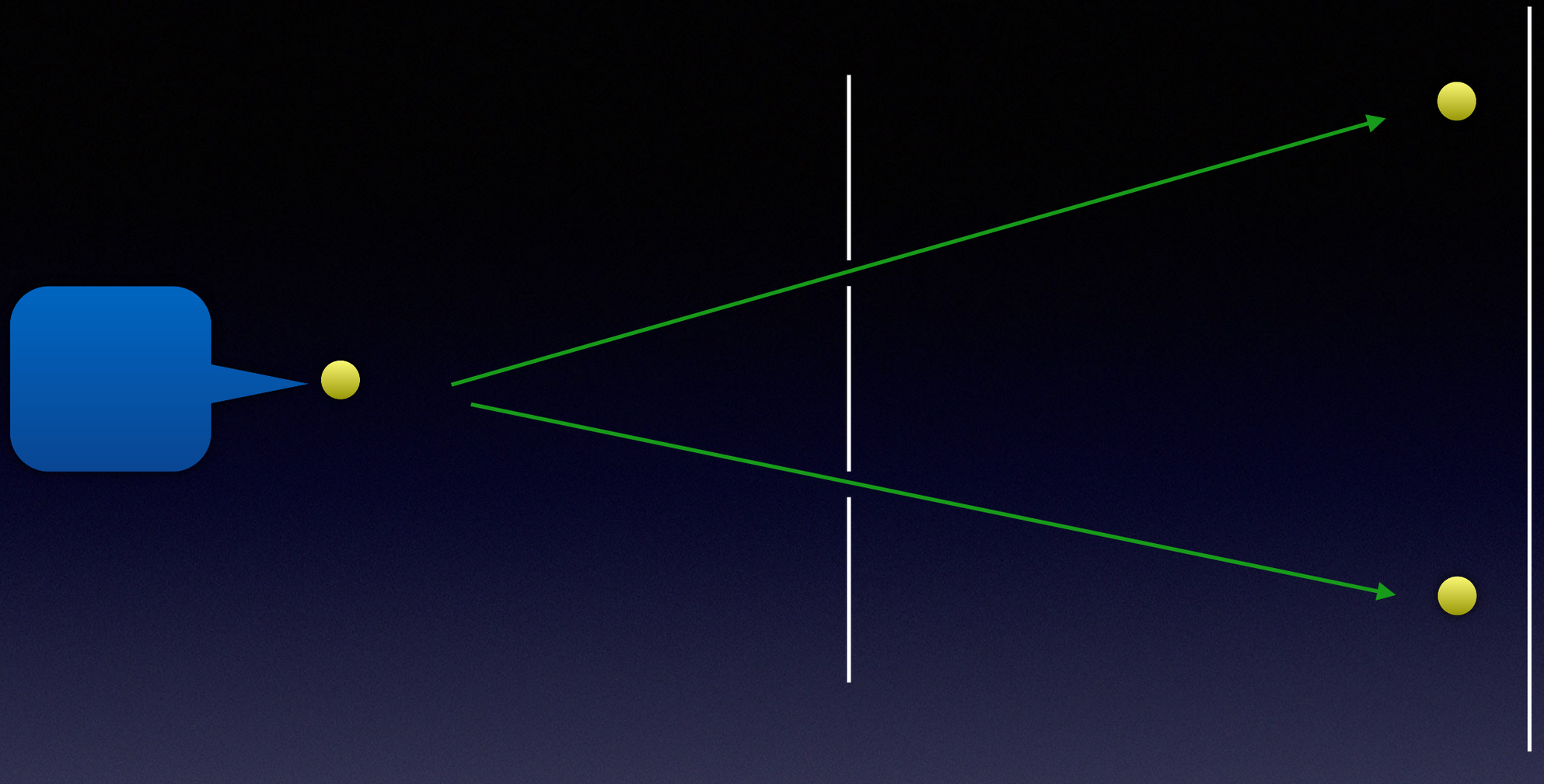
Experimentally, there is only one spin 2 force.

Theoretically, physicists even don't know how to write a theory with more than one spin-2 force.
It's simply impossible.

- There are four forces in the world:
  - Electromagnetism (light) spin 1
  - "Weak nuclear force" spin 1
  - "Strong nuclear force" spin 1
  - Gravity spin 2
- Gravity is rather different !

- ✓ What's gravity?
- What's quantum mechanics?
- Why do we have to reconcile them?
- How do we reconcile them?

What's Quantum Mechanics?



Electron Source

Double Slits

Screen

(c) Hitachi / Prof. Tonomura

This used to be available from https://www.hitachi.com/rd/research/em/movie.html ,
but alas, no longer. It is now available at https://www.youtube.com/watch?v=_oWRI-LwyC4 (without narration)
and at https://www.youtube.com/watch?v=jvO0P5-SMxk (with narration)

(c) Hitachi / Prof. Tonomura

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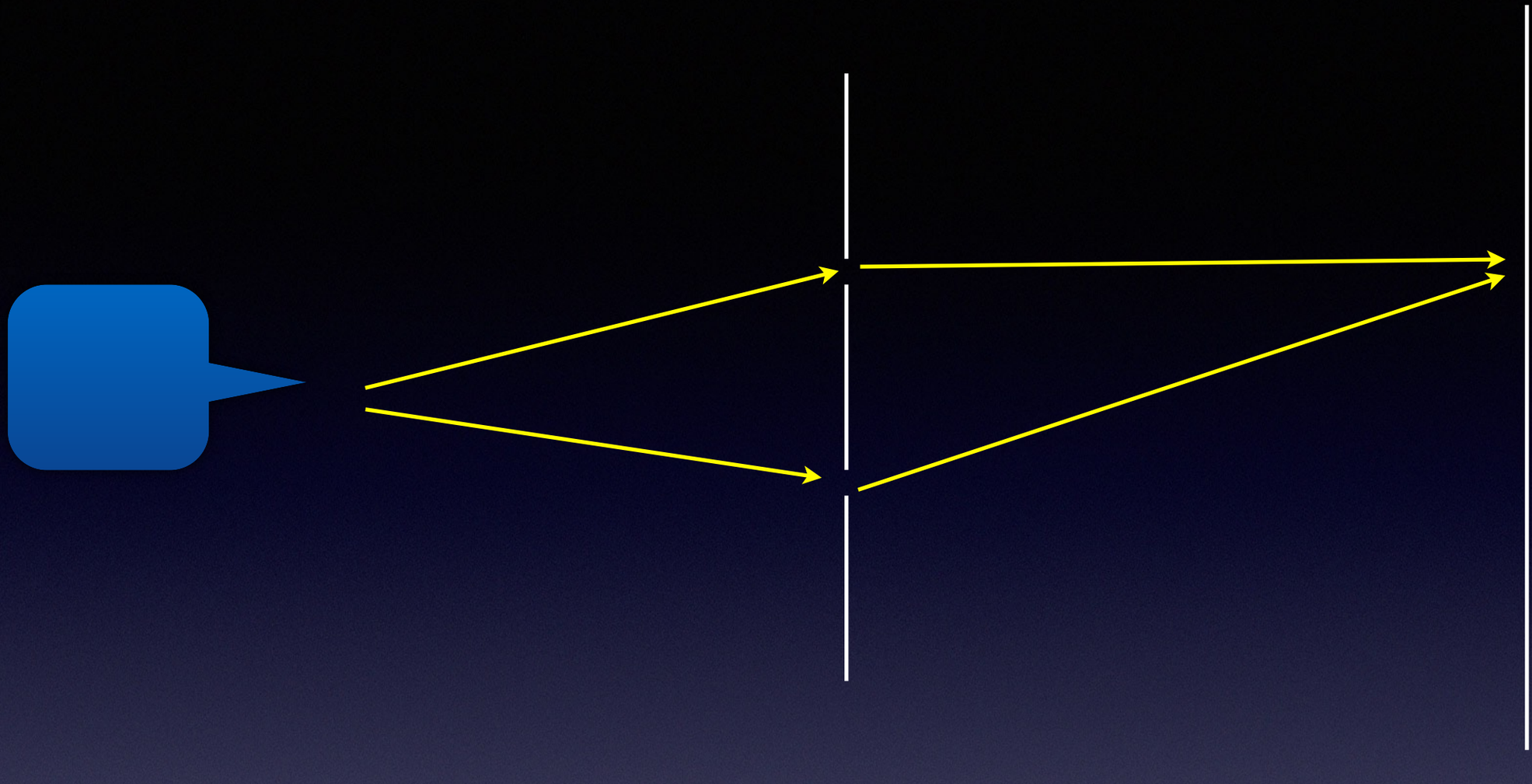
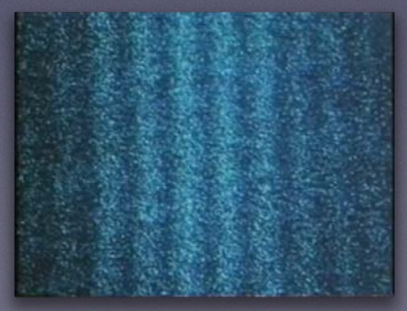


One electron passes the two slits
at the same time...
Its “wavefunction” interfere,
and causes



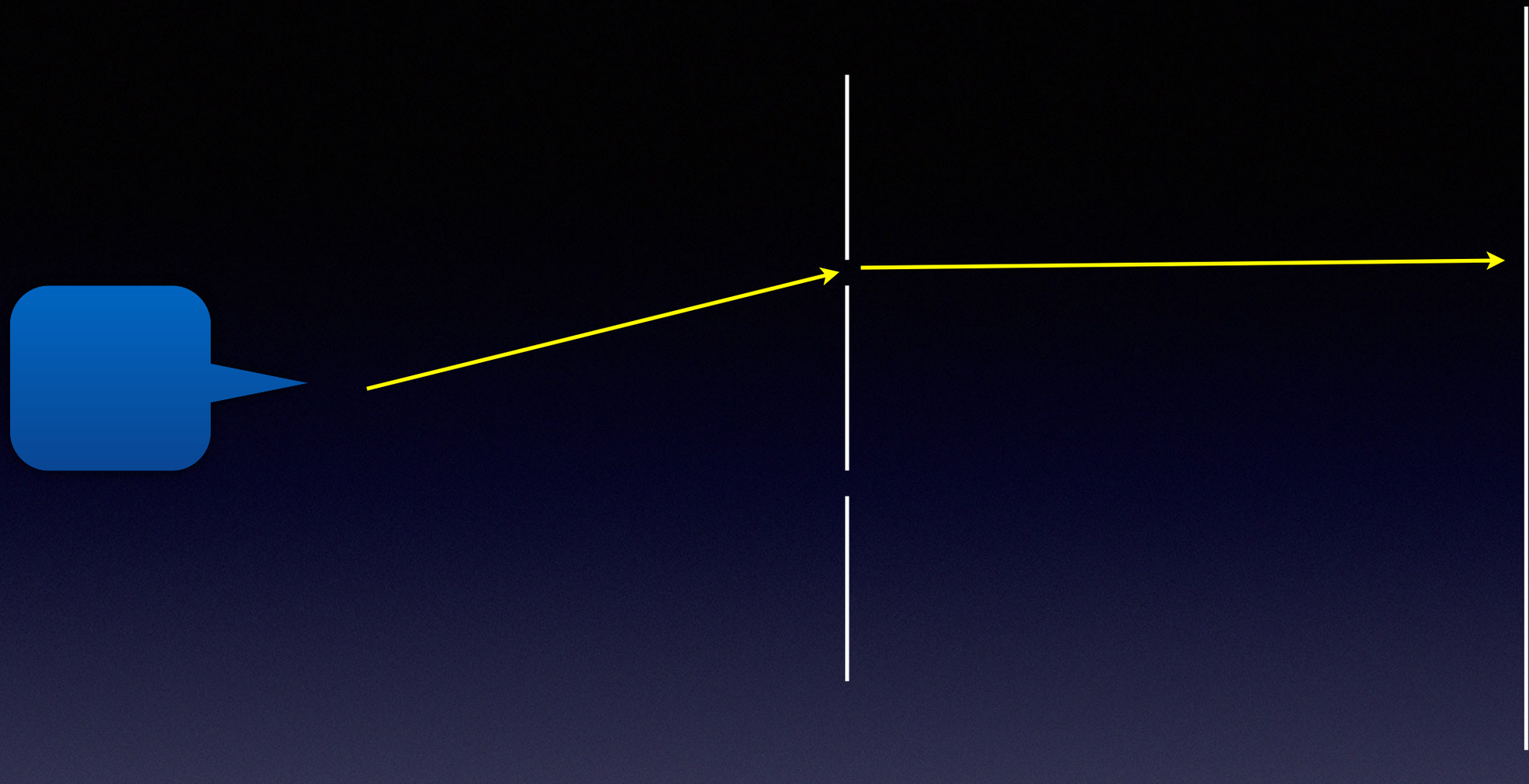
If nobody is watching
the moon,
does the moon exist?

If nobody is watching
the moon,
does the moon exist?

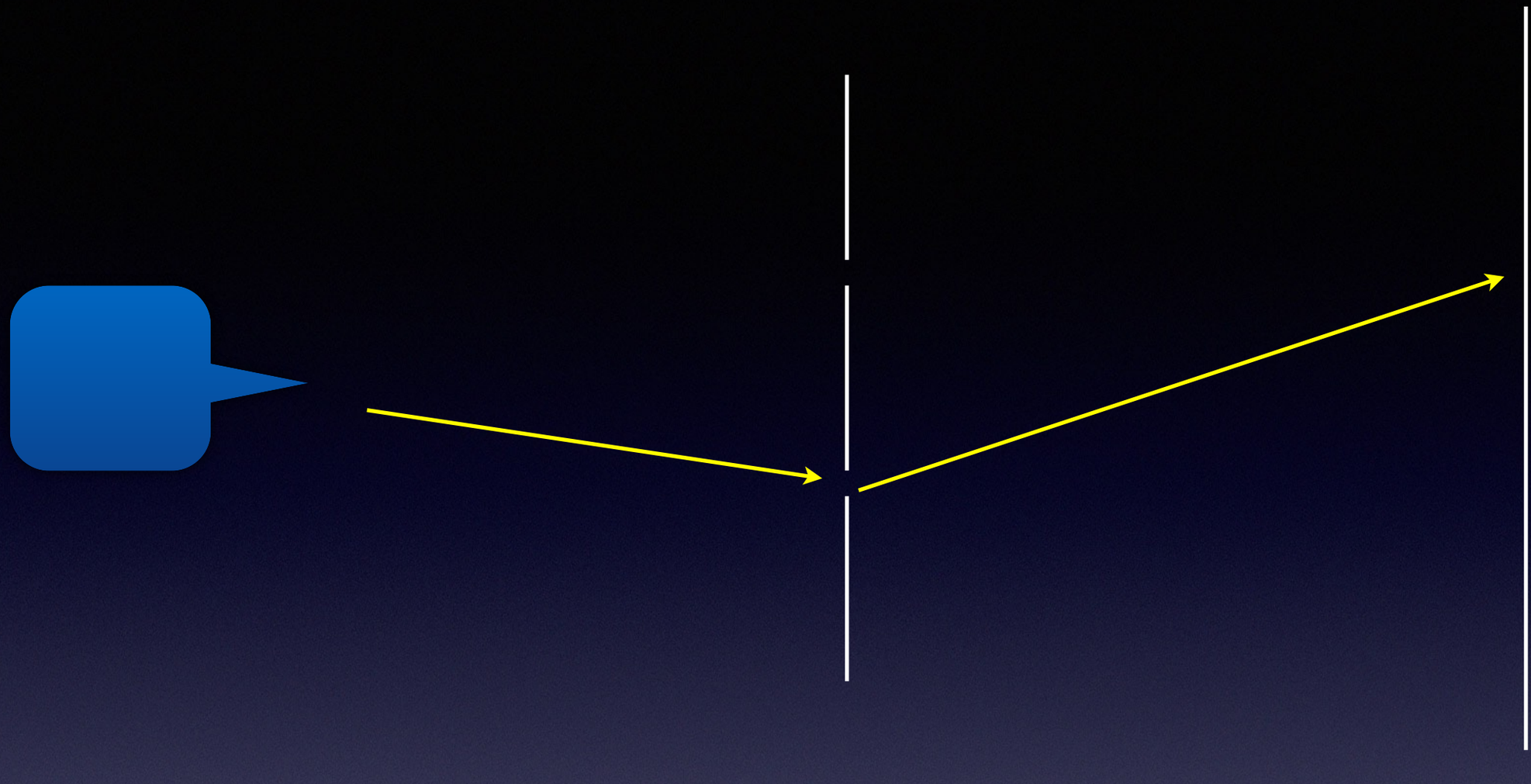
Is it even a physics
question?

If nobody is watching
the moon,
does the moon exist?

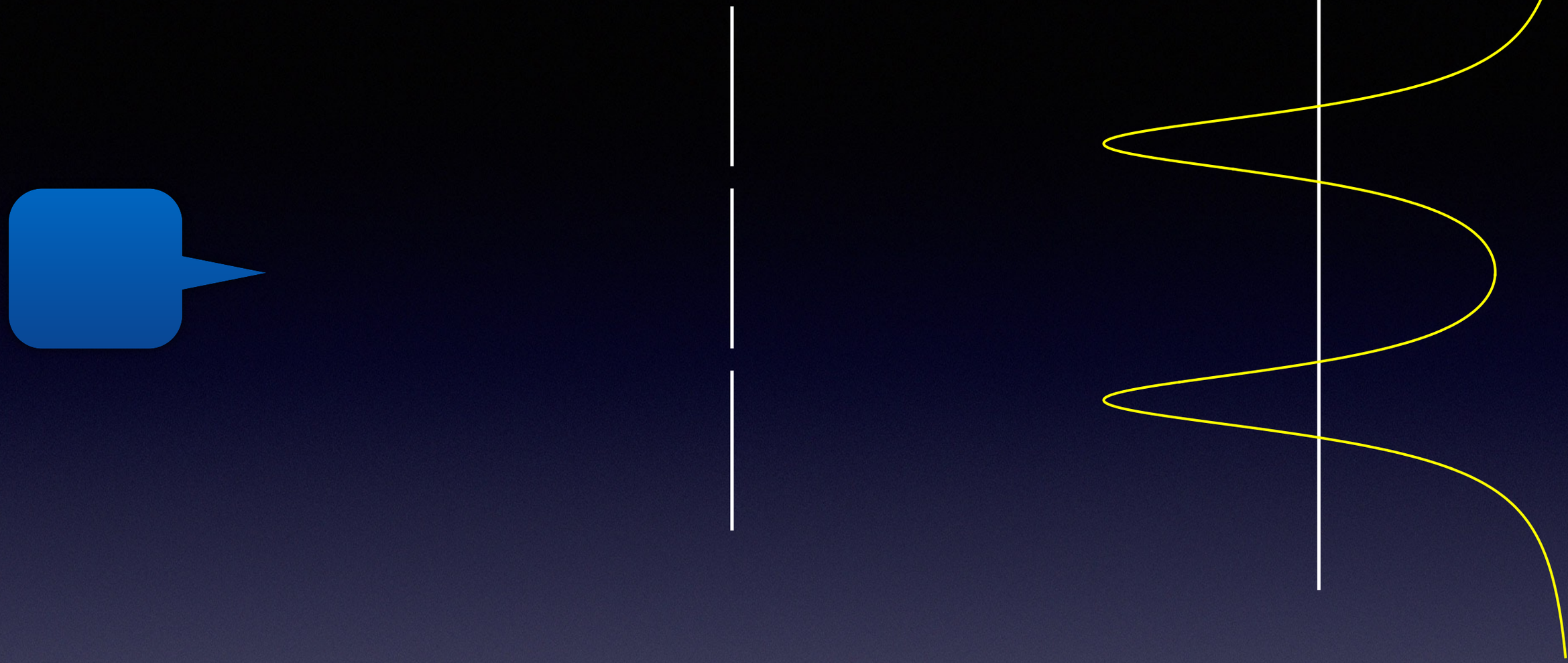
Yes it is a physics
question.



If you always watch an electron,
it goes through one particular path.

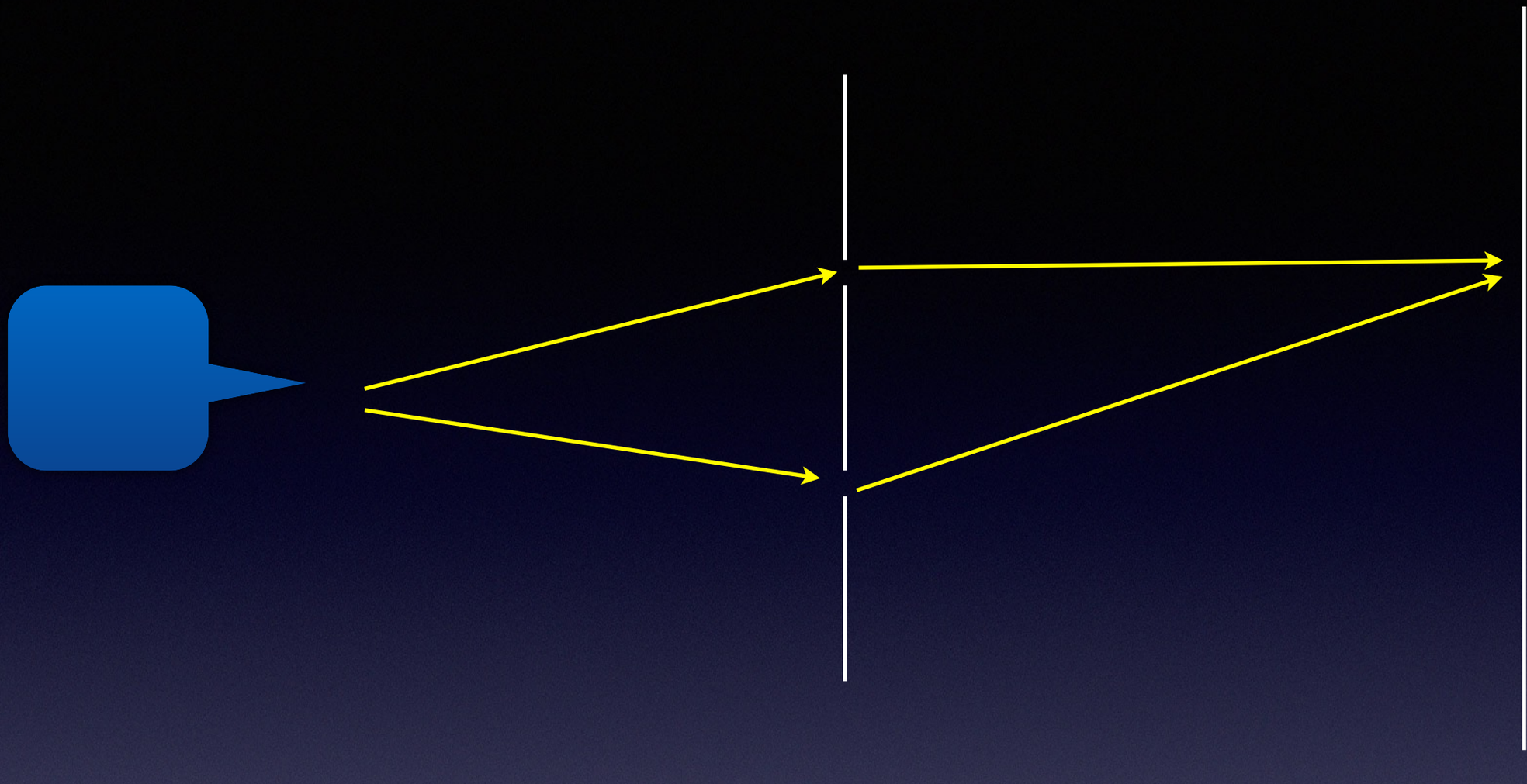


If you always watch an electron,
it goes through one particular path.



If you always watch an electron,
it goes through one particular path.

You won’t get this.



By *not* watching the electron during the way,
it creates

- Many famous physicists didn't like it.
- But nature doesn't care if famous physicisits like it or not.
- For example, consider the "Schrödinger's cat" gedanken experiment.

https://www.kirin.co.jp/alcohol/beer/ichiban/
https://commons.wikimedia.org/wiki/File:Danger_radiation.svg
https://commons.wikimedia.org/wiki/File:Geigerz%C3%A4hler2.jpg

https://www.kirin.co.jp/alcohol/beer/ichiban/
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https://commons.wikimedia.org/wiki/File:Geigerz%C3%A4hler2.jpg

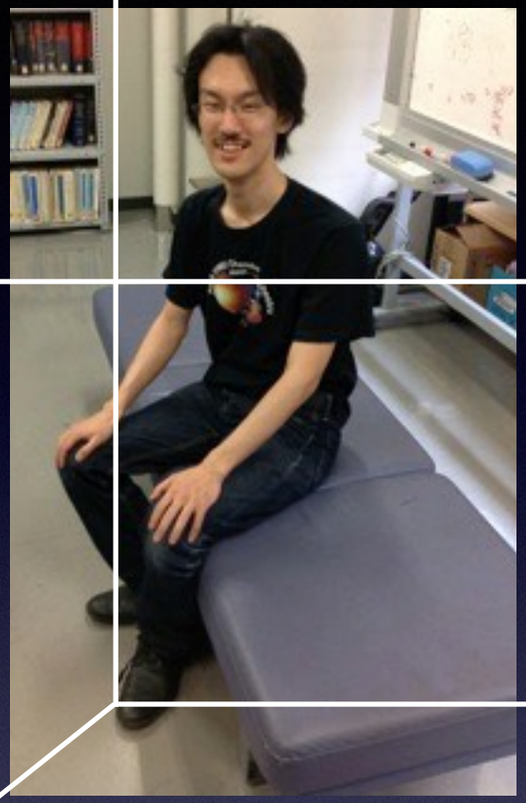
“Schrödinger’s me” experiment

https://www.kirin.co.jp/alcohol/beer/ichiban/

https://commons.wikimedia.org/wiki/File:Danger_radiation.svg

https://commons.wikimedia.org/wiki/File:Geigerz%C3%A4hler2.jpg

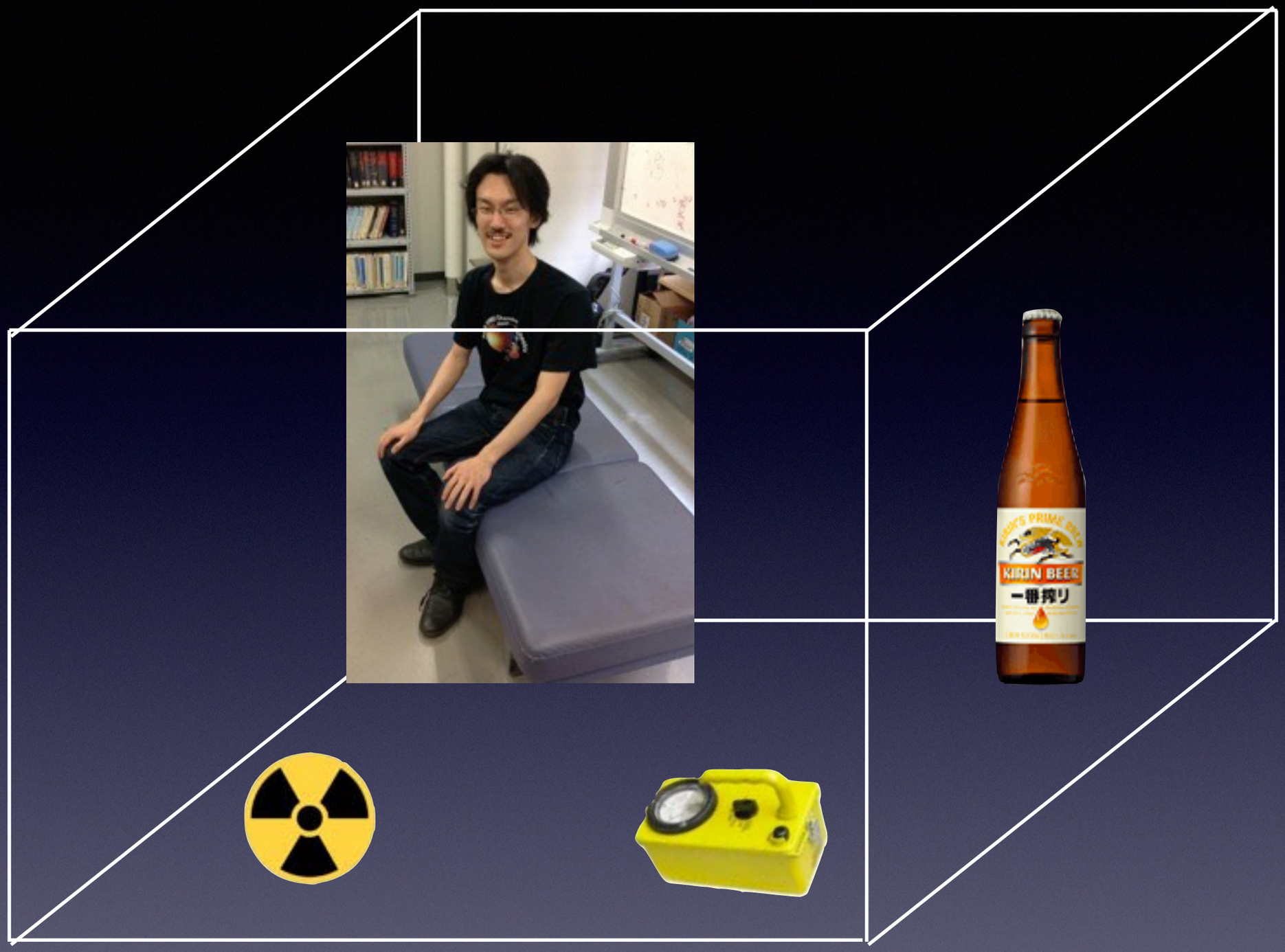
In an hour, the Geiger counter
“clicks” with 50% probability.



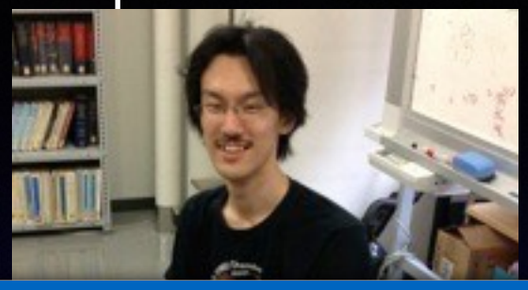
If it clicks, I drink and get drunk.

If it clicks, I drink and get drunk.

If it doesn’t click,
I don’t drink and stay sober.

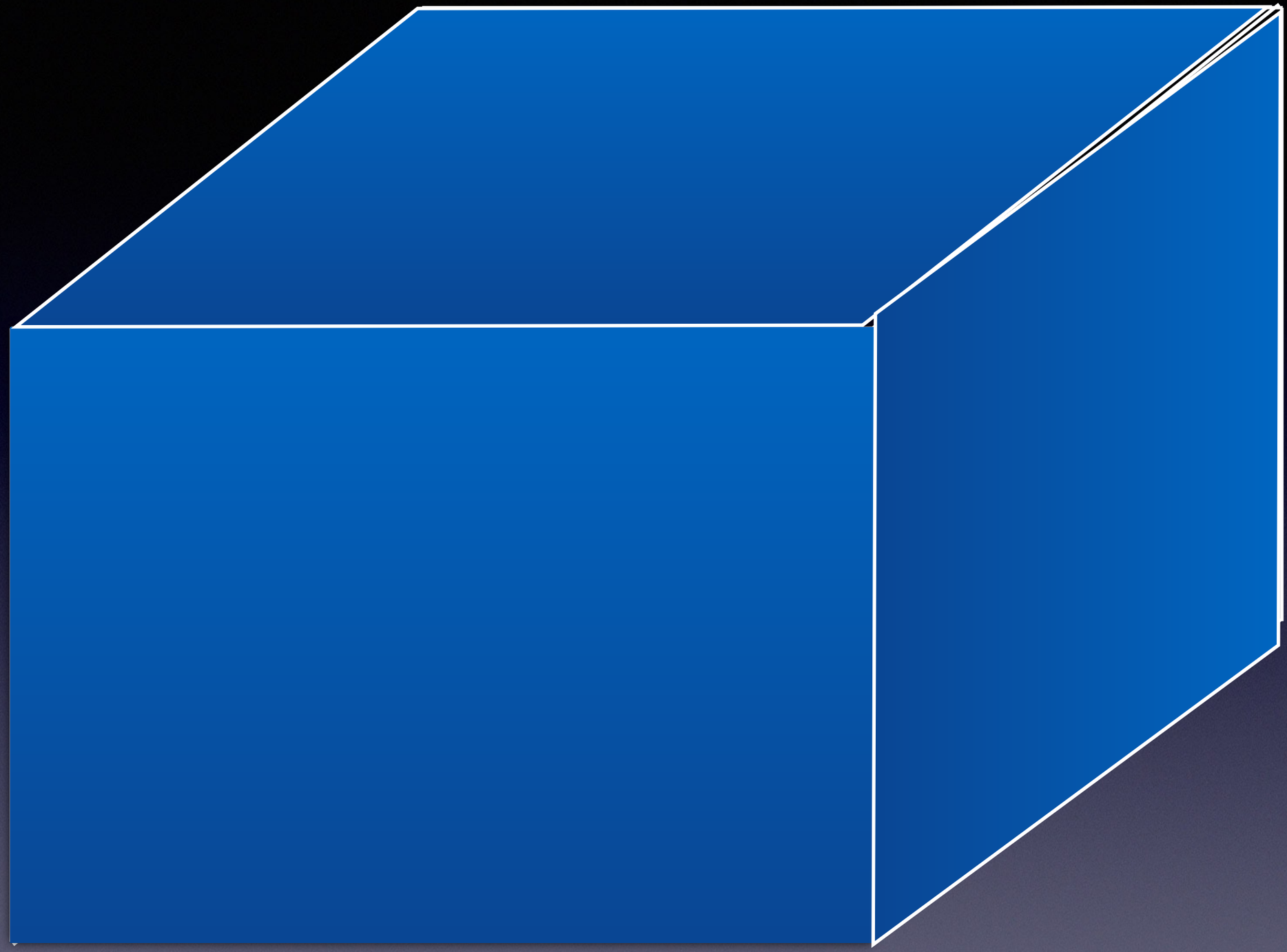


Suppose you keep me in a sealed box
and do this experiment.

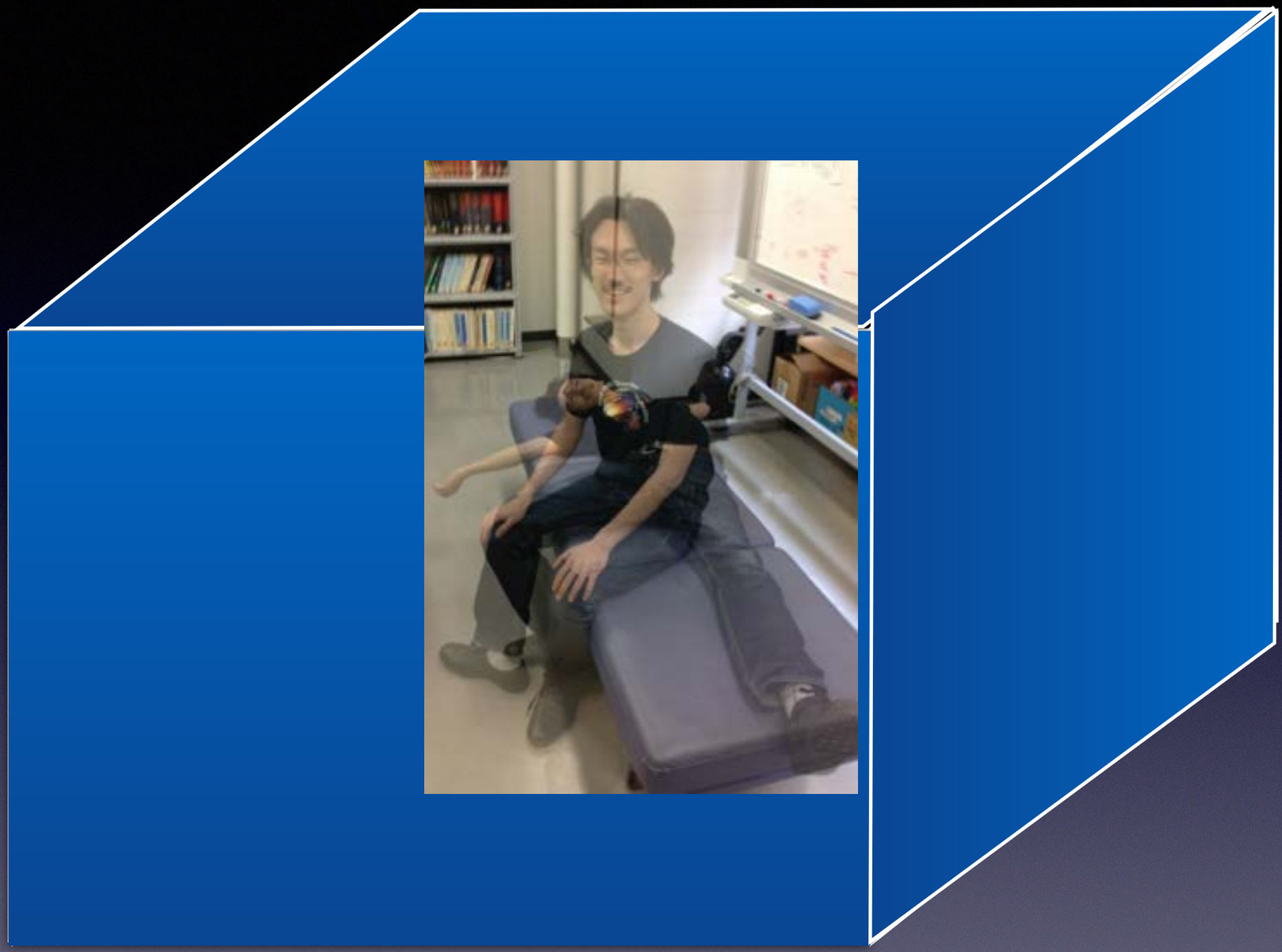


Suppose you keep me in a sealed box
and do this experiment.

Suppose you keep me in a sealed box
and do this experiment.



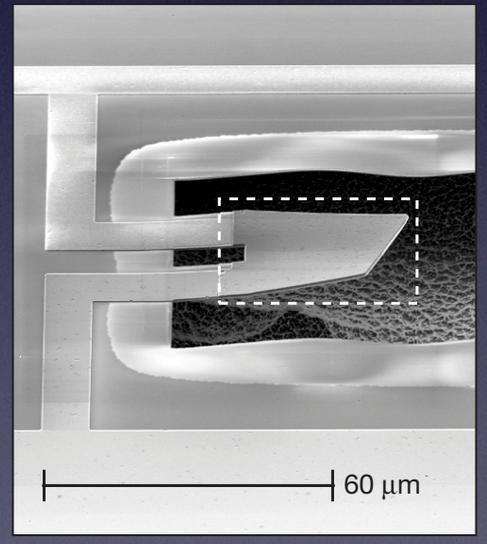
Suppose you keep me in a sealed box
and do this experiment.



Before you open the box after an hour,
am I in a superposition?

I think I will be, from your point of view
from outside of the box.

Bigger and bigger things are put into
quantum mechanical superposition
experimentally.
There doesn't seem to be any upper bound
in principle.



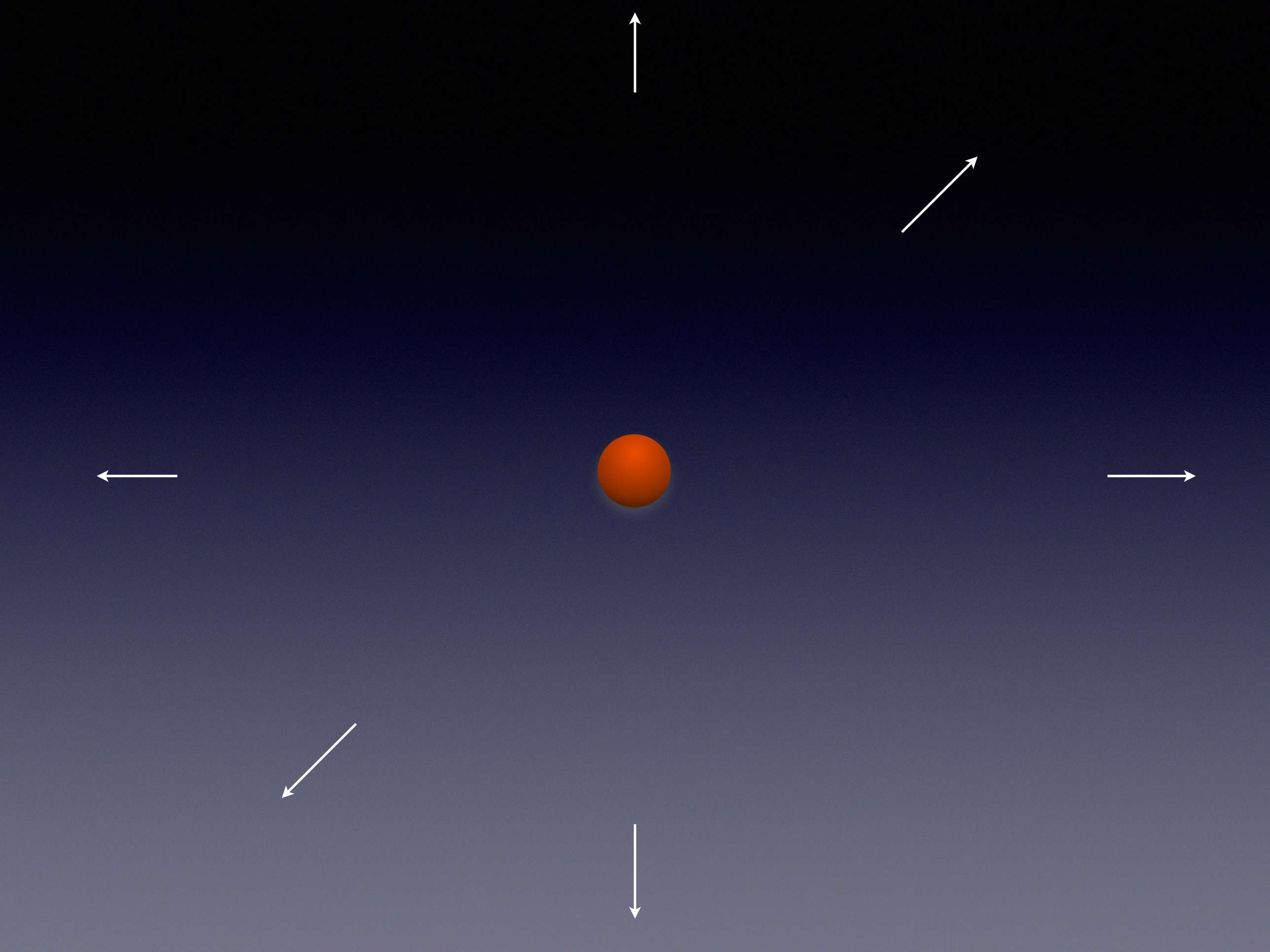
O'Connel et al., Nature 464 (2010) p. 697

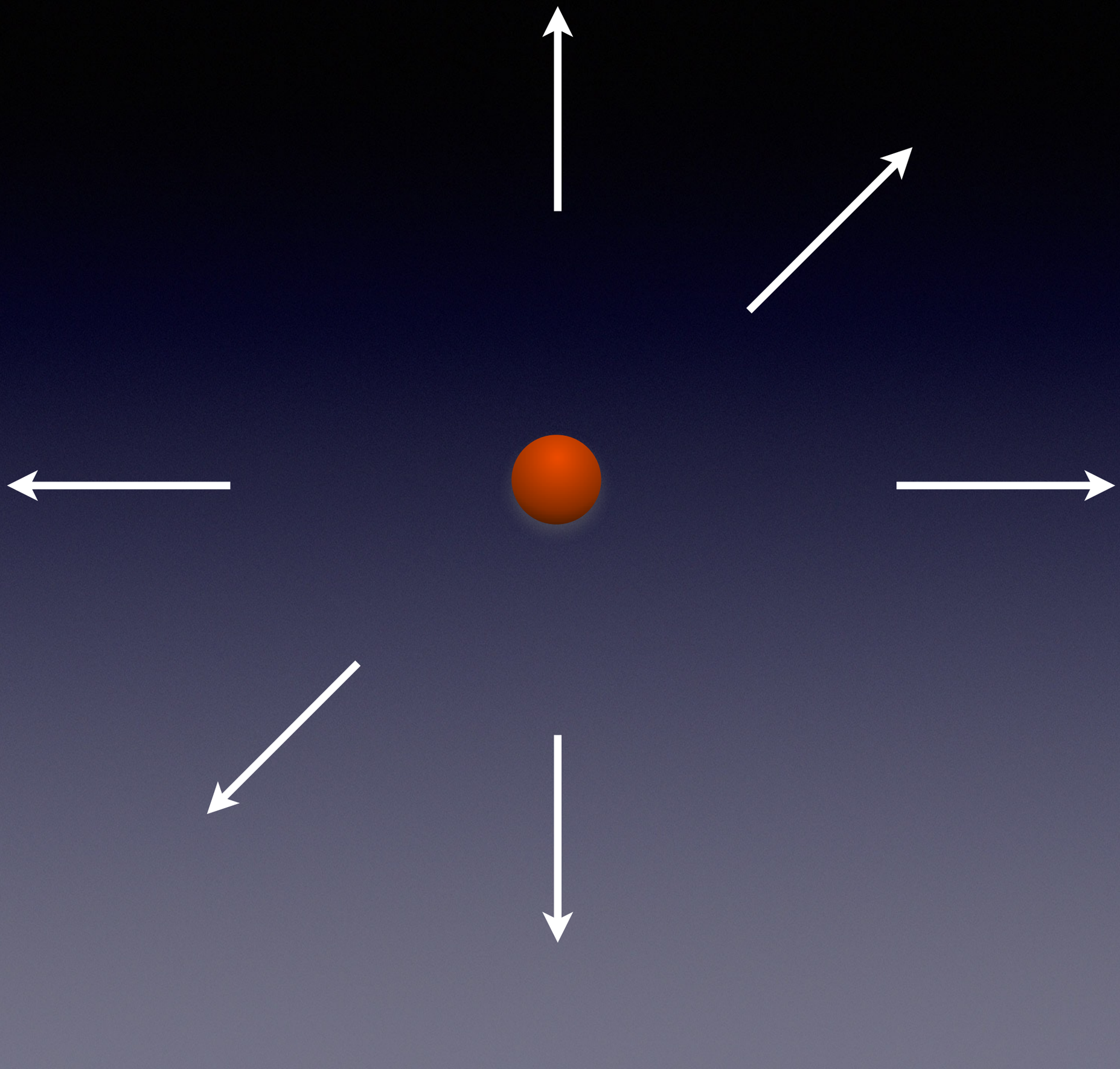
✓ • What's gravity?

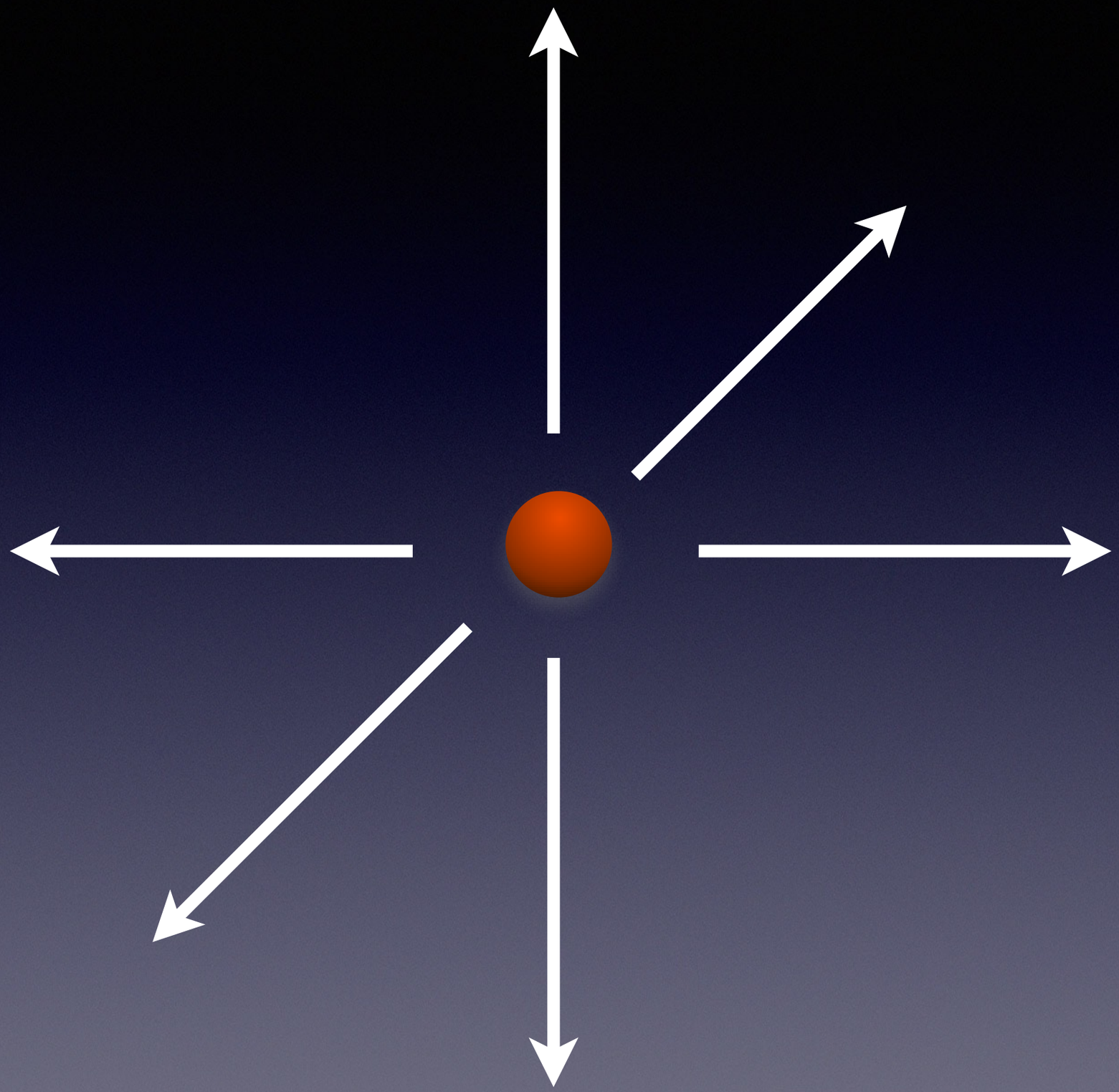
✓ • What's quantum mechanics?

- Why do we have to reconcile them?
- How do we reconcile them?

Why do we have to reconcile them?







- Electric field ~ $1/r^2$
- Energy per volume ~ | electric field |^2
- Energy carried by the electric field ~

$$\int_0^\infty \left(\frac{1}{r^2}\right)^2 r^2 dr = \left[-\frac{1}{r}\right]_{r=0}^{r=\infty} = \infty$$

- An electron, if pointlike, always carry infinite amount of energy.

- Energy = mass × (speed of light)^2

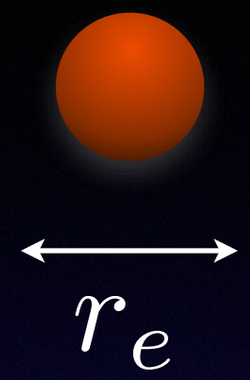
- Electron is infinitely massive!

- Of course the electron has finite mass.

- This puzzled physicists at the turn of 20th century greatly.
- One idea which didn't work:

$$\int_0^\infty \left(\frac{1}{r^2}\right)^2 r^2 dr = \left[-\frac{1}{r}\right]_{r=r_e}^{r=\infty} = m_e$$

- Maybe all the mass is due to the electric field.
- Maybe electron has a finite radius.



- But it doesn't work with relativity.

- Basically, relativity doesn't like finite-sized rigid body.

- In relativity, nothing can exceed the speed of light.

- This can’t happen.
- Finite size = bad in relativity.

- In relativity, nothing can exceed the speed of light.

- This can’t happen.
- Finite size = bad in relativity.

- This electron mass problem was solved around in 1950.

- So it took almost half a century to solve.

- I can’t easily say how it was resolved... as it is rather involved.

- It uses Quantum Mechanics in an essential way.

Naively, we expect something like

Real Electron Mass = $-\infty$

"Naked Electron Mass"

\+ Energy of the Electric Field

$\infty + m_e$

But you never see a naked electron
without its electric field.
So you don't and can't think about them.

Naively, we expect something like

Real Electron Mass

But you never see a naked electron
without its electric field.
So you don’t and can’t think about them.

- This theory is called
  the Quantum ElectroDynamics.

- It works extremely well.

- The most amazing example:
  the electron anomalous magnetic moment

Experiment: 0.00115965218...
Theory: 0.00115965218...

They agree up to the expected
experimental & theoretical errors.

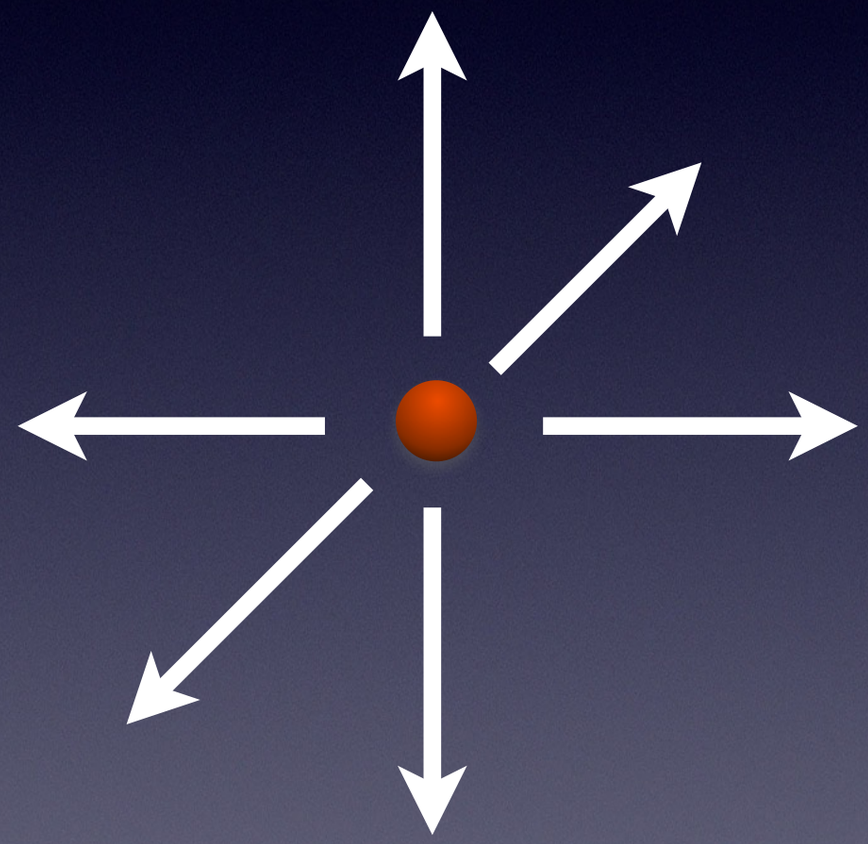
The same problem of infinite energy arises for all four forces:

electromagnetism

"strong nuclear force"

"weak nuclear force"

gravity



The same problem of infinite energy arises for all four forces:

electromagnetism

“strong nuclear force”

“weak nuclear force”

gravity

Problems solved.

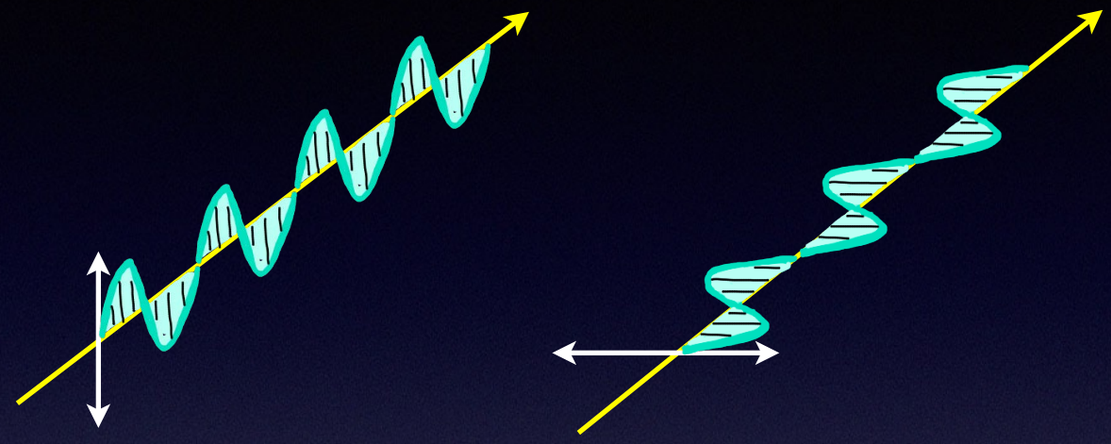
Problems unsolved.

electromagnetism

“strong nuclear force”

“weak nuclear force”

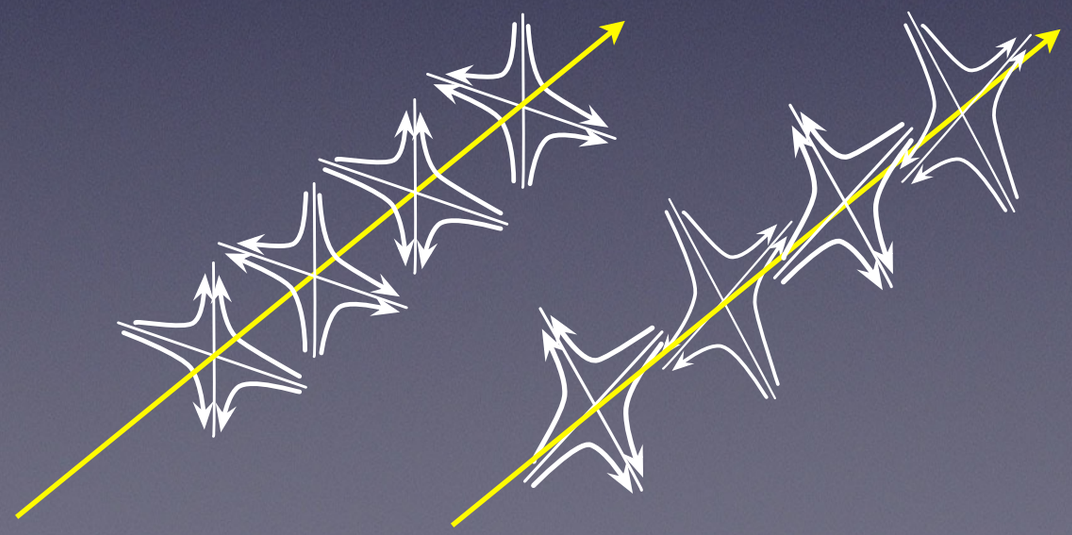
Problems solved.



Called Spin 1.

gravity

Problem unsolved.



Called Spin 2.

Gravity is different.

- Anyway, we learned by the late 70s how to treat quantum mechanically
  - electromagnetism,
  - "strong nuclear force," and
  - "weak nuclear force."
- So it was unsatisfactory that we didn't know how to treat gravity quantum mechanically.

- ✓ What's gravity?
- ✓ What's quantum mechanics?
- ✓ Why do we have to reconcile them?
- How do we reconcile them?

How do we reconcile them?

- So far I told you
  - The world is quantum mechanical.
  - There is gravity in the world.
- So we want to treat gravity quantum mechanically.

There are two known methods:

Loop Quantum Gravity

String Theory

Which is correct? Nobody knows yet.

This is because the quantum mechanical effect of gravity tends to be very, very tiny and very, very hard to observe.

- A theory is correct if it describes some aspect of this world we live in.

- Neither string theory nor loop quantum gravity is known to be correct in this sense.

- At least both are “logically consistent.”

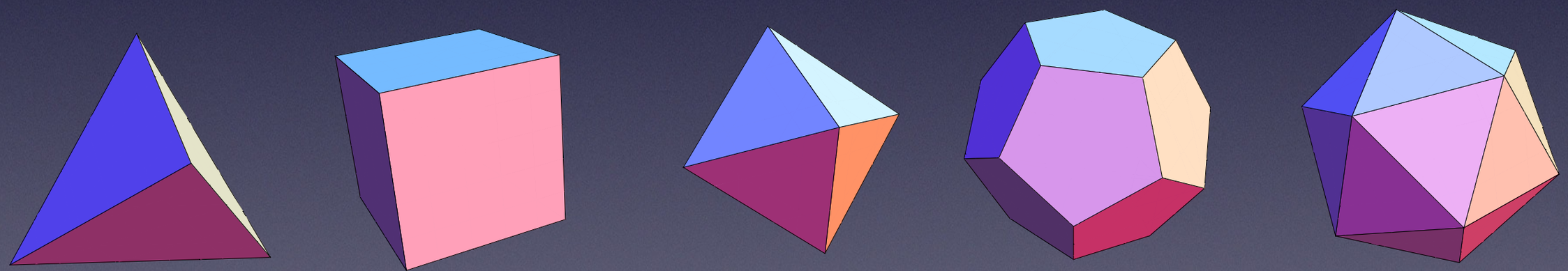
- So, I'm studying a logically consistent entity, called String Theory.

- I'm doing it mostly disregarding whether it describes the world or not. The structure of the theory itself is interesting to me.

- This makes me a non-scientist.

- Mathematicians deal with "logically consistent idealized entities" rigorously.

- For example, ancient Greeks have found that there are five and only five regular polyhedra:



- This is the last proposition of Euclid's *Elements*!

- Unfortunately, string theory is not quite rigorous yet.

- So mathematicians don't consider string theorists mathematicians.

- Scientists don't consider string theorists scientists either.

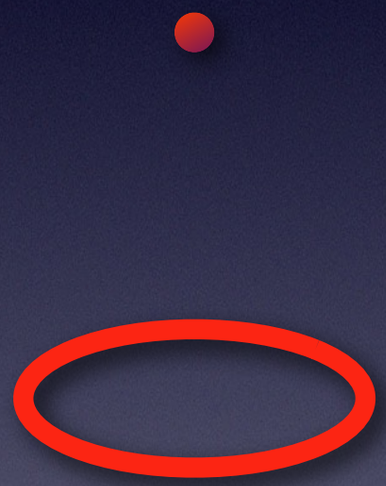
- So I'm stuck.

What's String Theory?

- It was not invented to treat gravity quantum mechanically.
- Instead, it came from the idea of a few crazy physicists:

They thought, around early 1970s,

"It's too boring to always treat
zero-sized particles quantum mechanically.

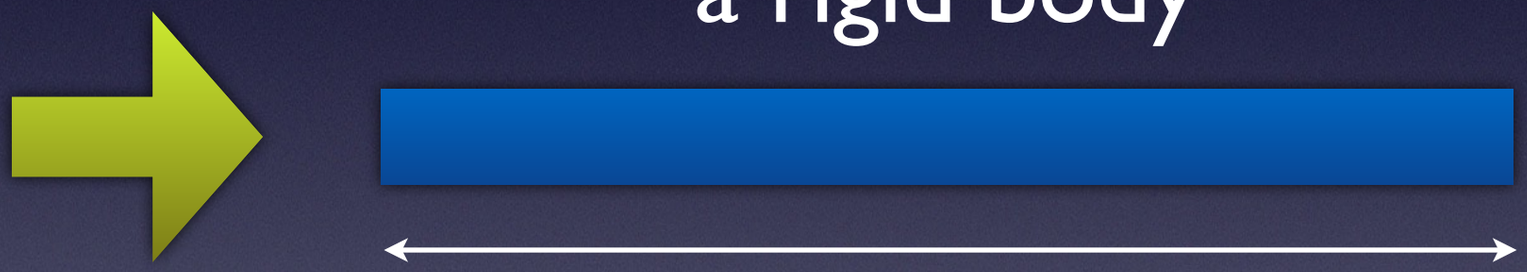


particle	zero size
string	finite size

What happens if we treat
finite-sized strings quantum mechanically?"

- As I told you, finite sized objects are bad in relativity.

a rigid body

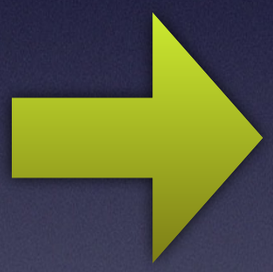


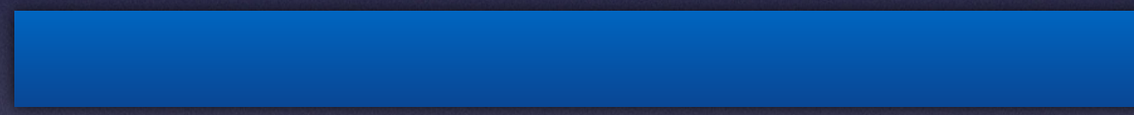
Push!

one light year

- As I told you, finite sized objects are bad in relativity.

a rigid body





Push!

one light year

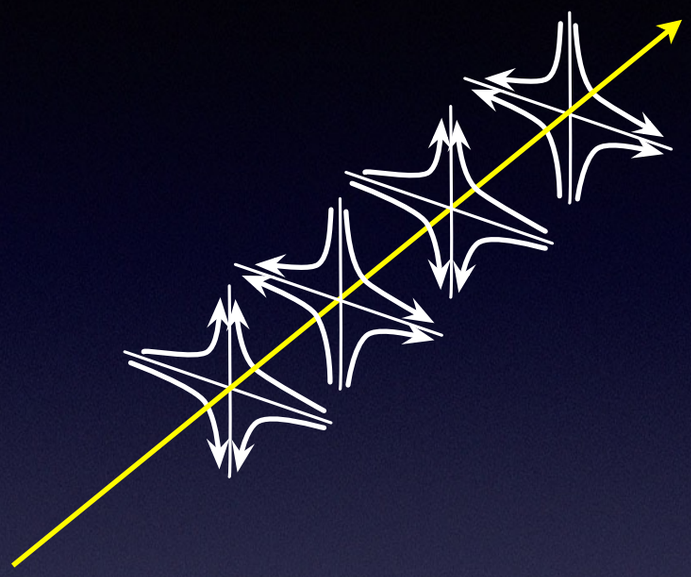
- So, those physicists tried hard to treat strings quantum mechanically,
- but they failed and failed. And then failed.

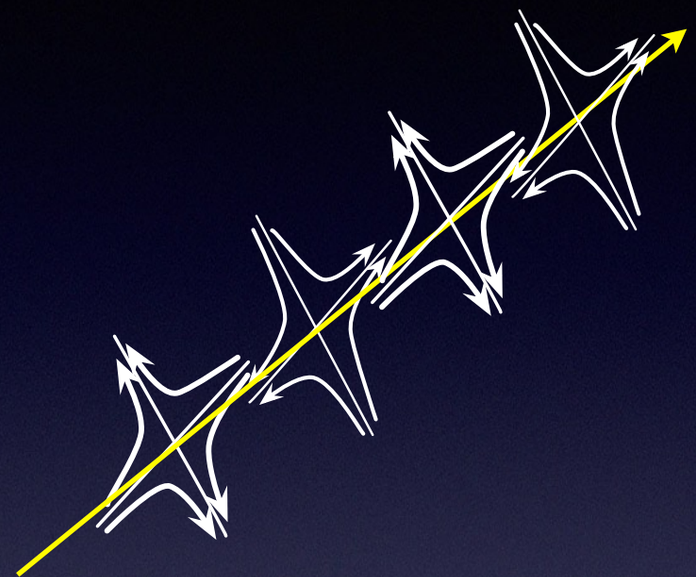
- They only succeeded in 1984,
  with a bad news and a good news.

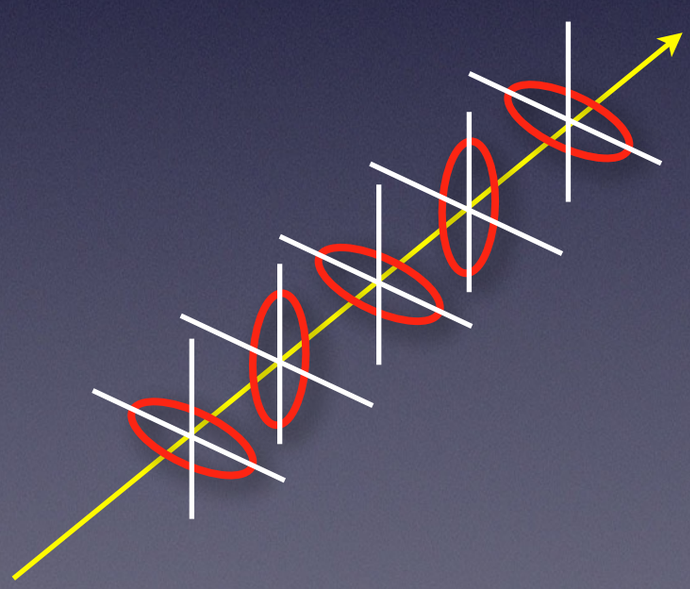
- The BAD: Strings need to move
  in 9+1 dimensional spacetime.

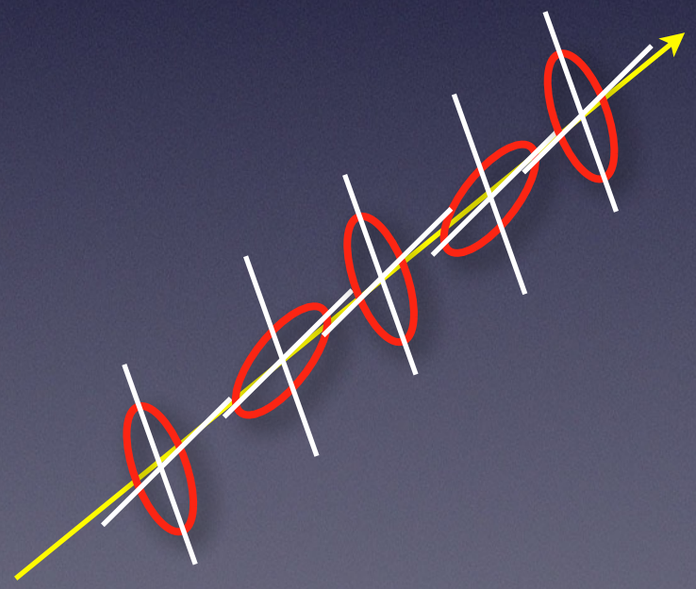
- The GOOD:
  It contained quantum gravity.

- Let's start with the good news.









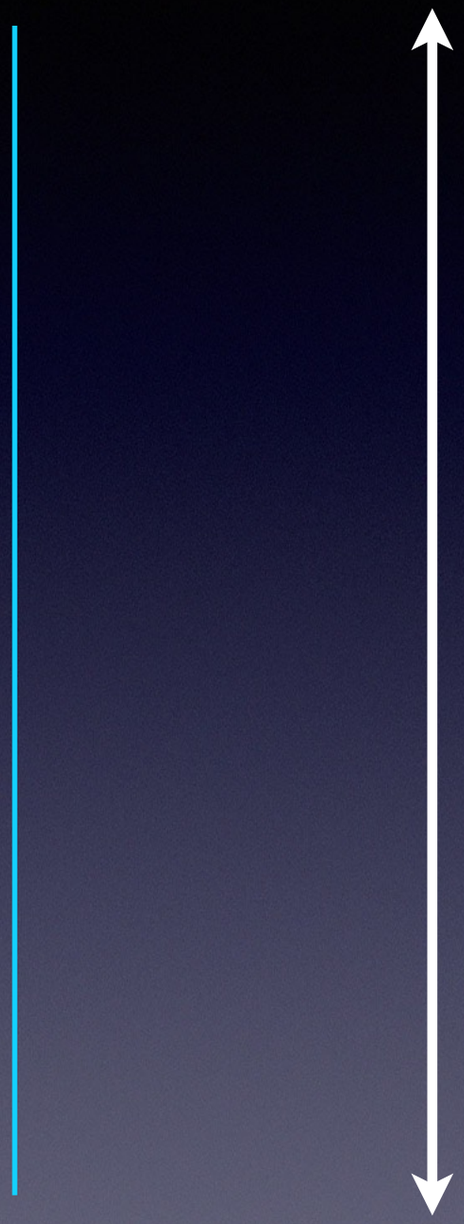
- Strings can vibrate just as gravitational waves would.

- The bad news: 9+1 dimensions?

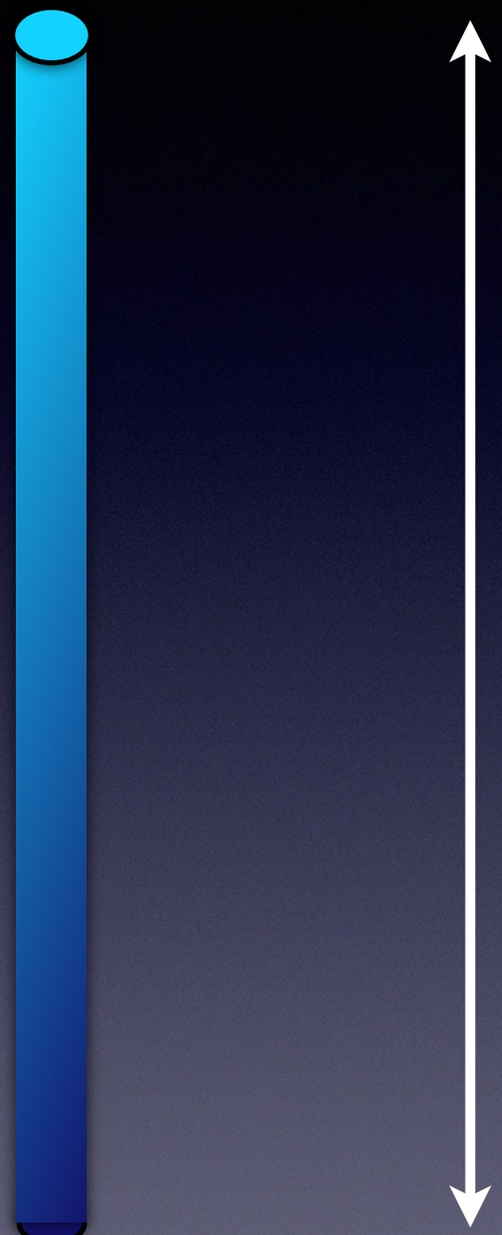
Space

Time

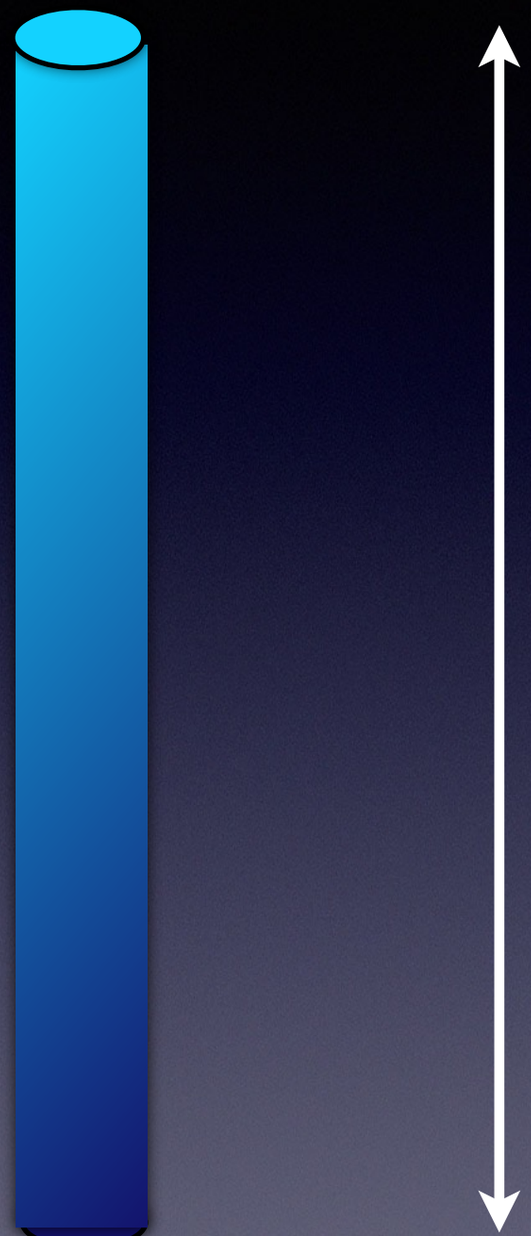
3+1 dimensions !



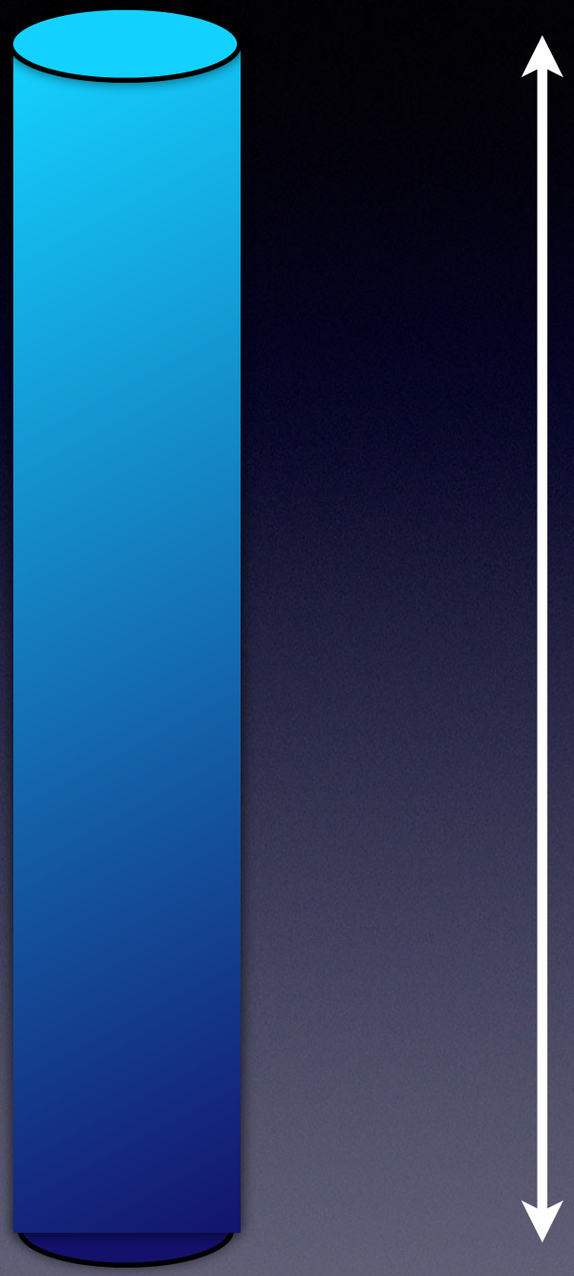
1 dimension ?



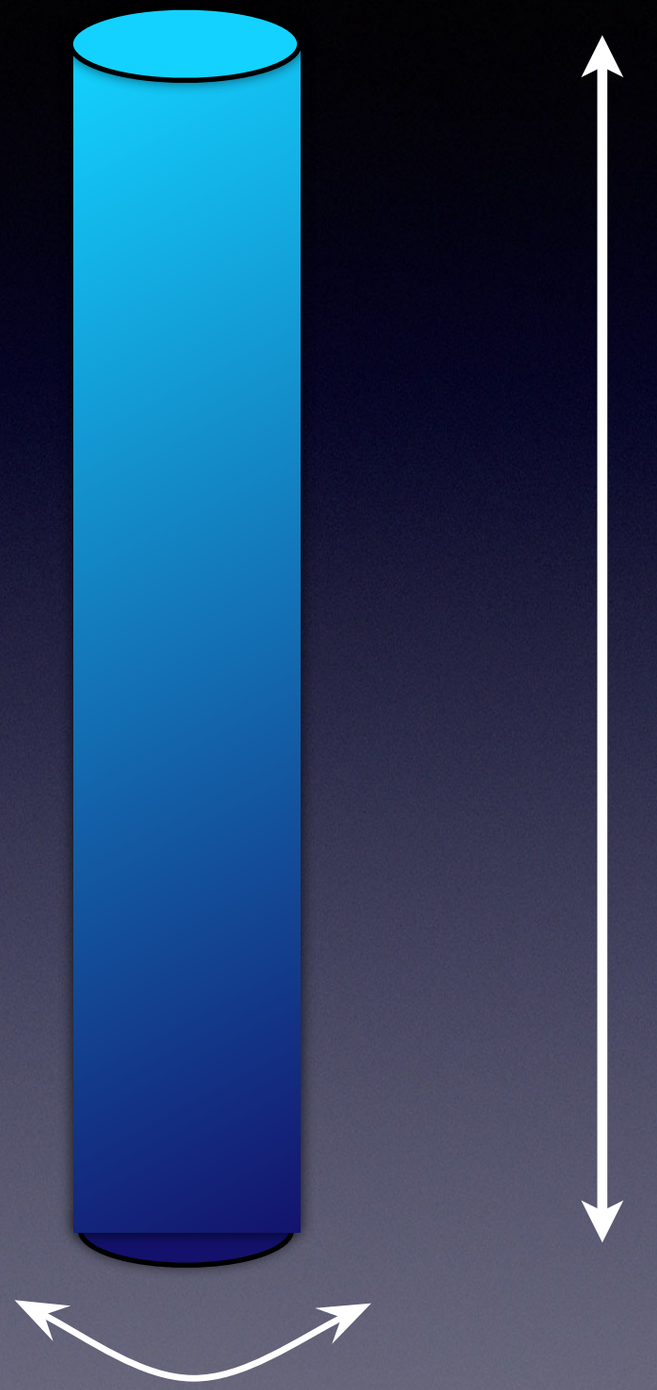
1 dimension ?



1 dimension ?



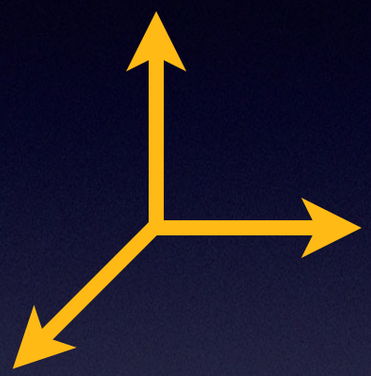
1 dimension ?



1 dimension ?

Another dimension !

needs to be
very very tiny.

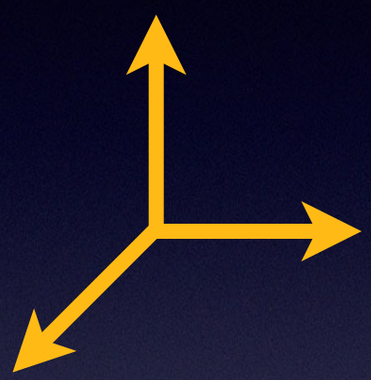


Space Time extra 6 dimensions

3+1 dimensions

- If the real world is really like this,

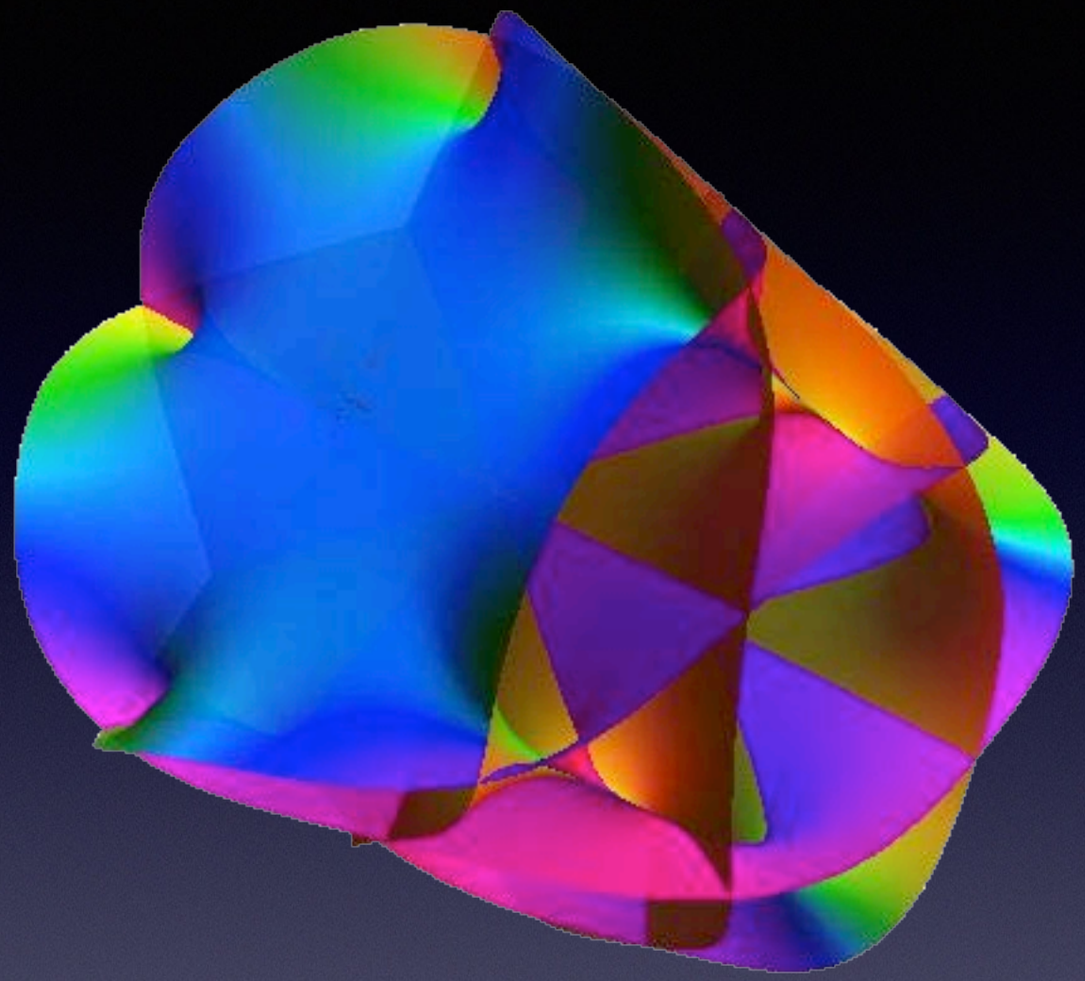
very very tiny.



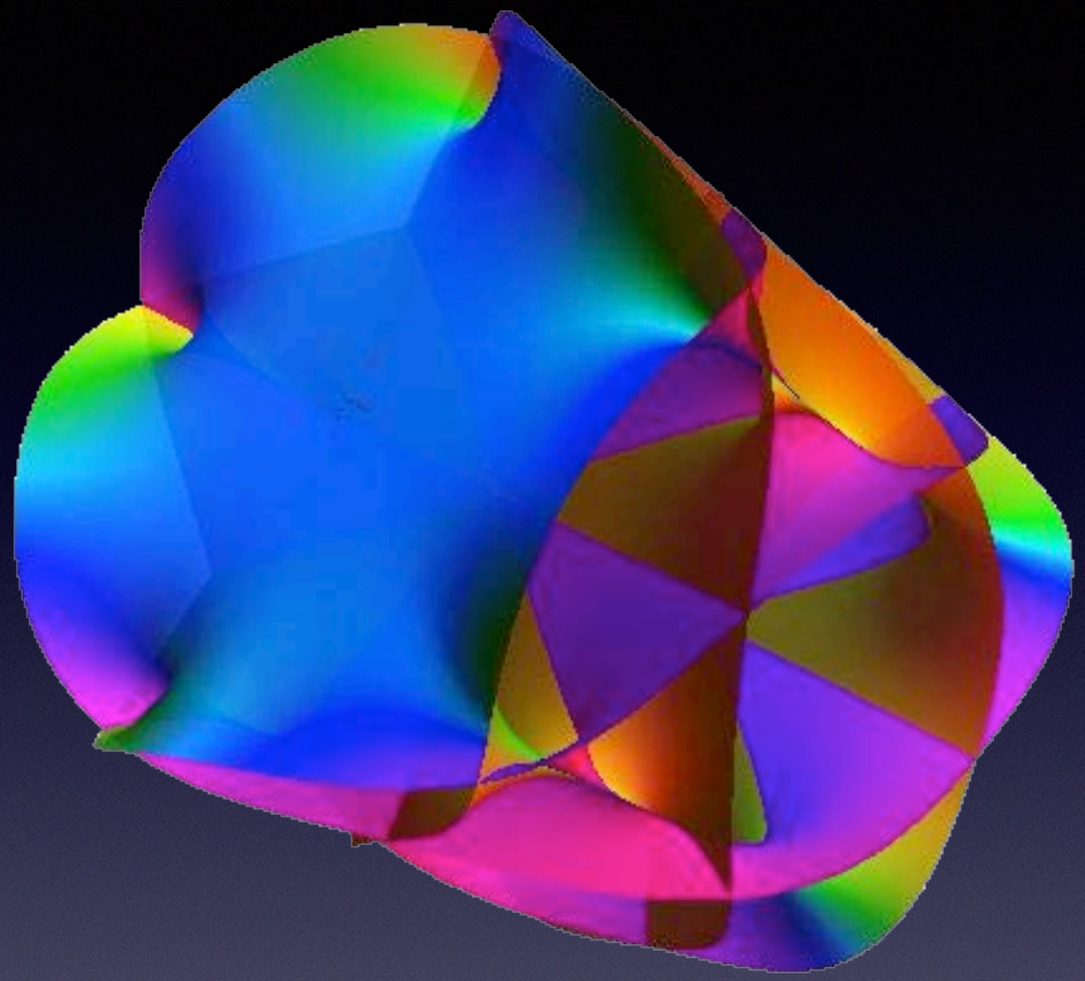
Space Time extra 6 dimensions

- The shape of the extra 6 dimensions determine the physics of elementary particles.

- E.g. how many kinds of electron-like particle there is.



- This shows 2d slices of very-well studied six-dimensional space called the quintic Calabi-Yau...



- This shows 2d slices of very-well studied six-dimensional space called the quintic Calabi-Yau...

9+1 dimensional world is described by String Theory

The extra 6d space has this particular shape:

(specified by a math equation.)

The 3+1d physics is such and such.

If this agrees with experiments, claim victory!

9+1 dimensional world is described by String Theory

The extra 6d space has this particular shape:

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If not,

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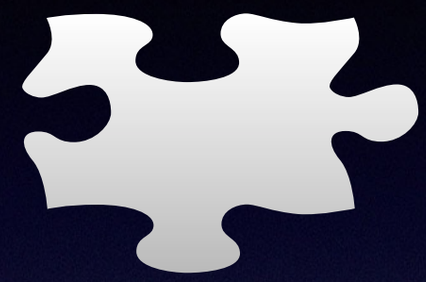

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9+1 dimensional world is described by String Theory

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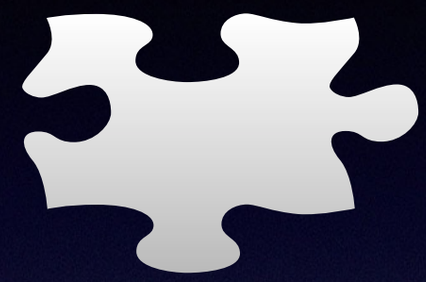


(specified by yet another equation.)

The 3+1d physics is such and such.

If not, ad infinitum.

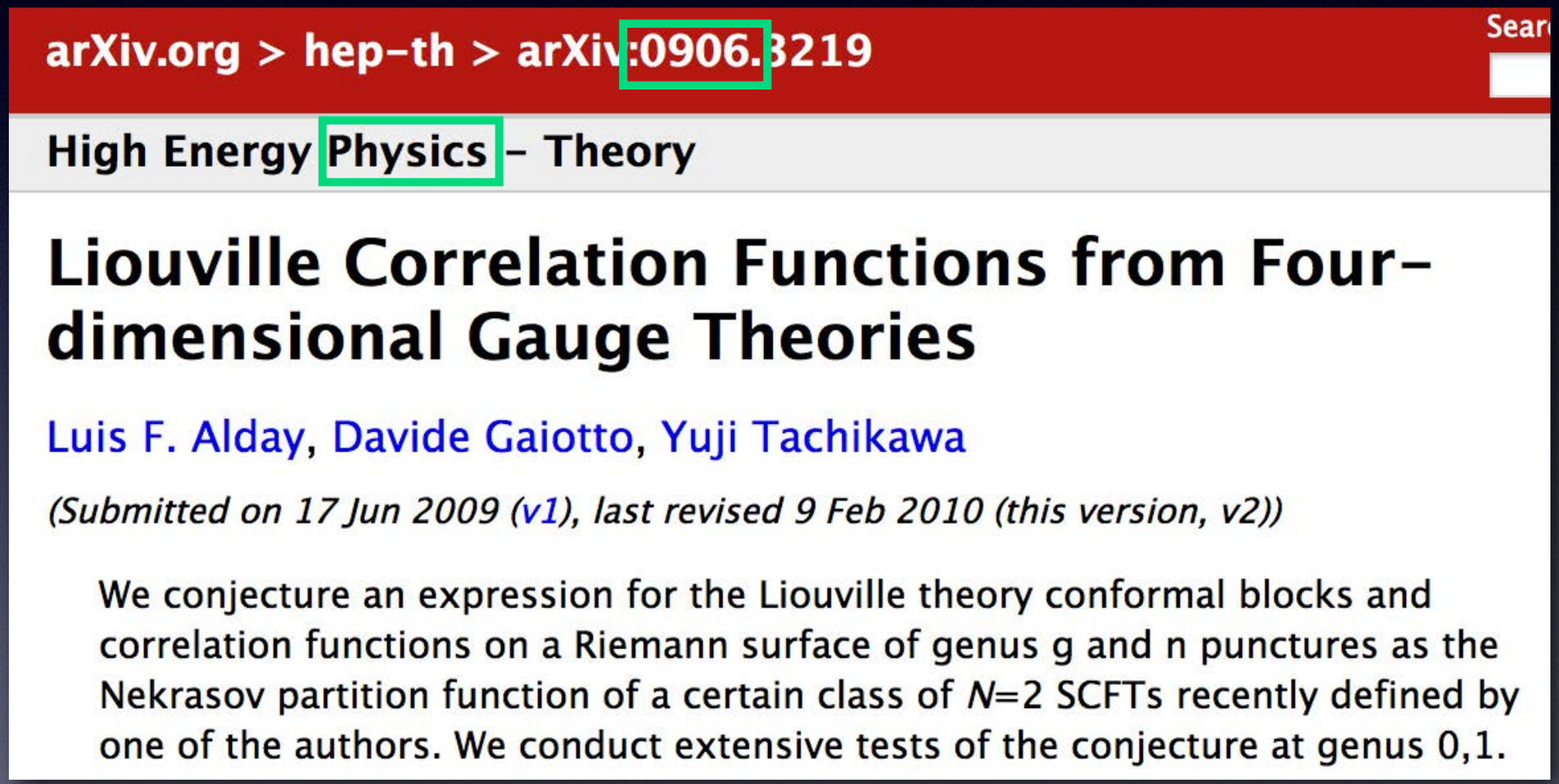
- So far, we haven't found *the 6d extra space* which gives this world we live in.
- There's no proof there isn't either.
- It's not that every string theorist is involved this so-far infinite process either.

- At least, string theorists learned a lot about the geometry of the six-dimensional spaces.

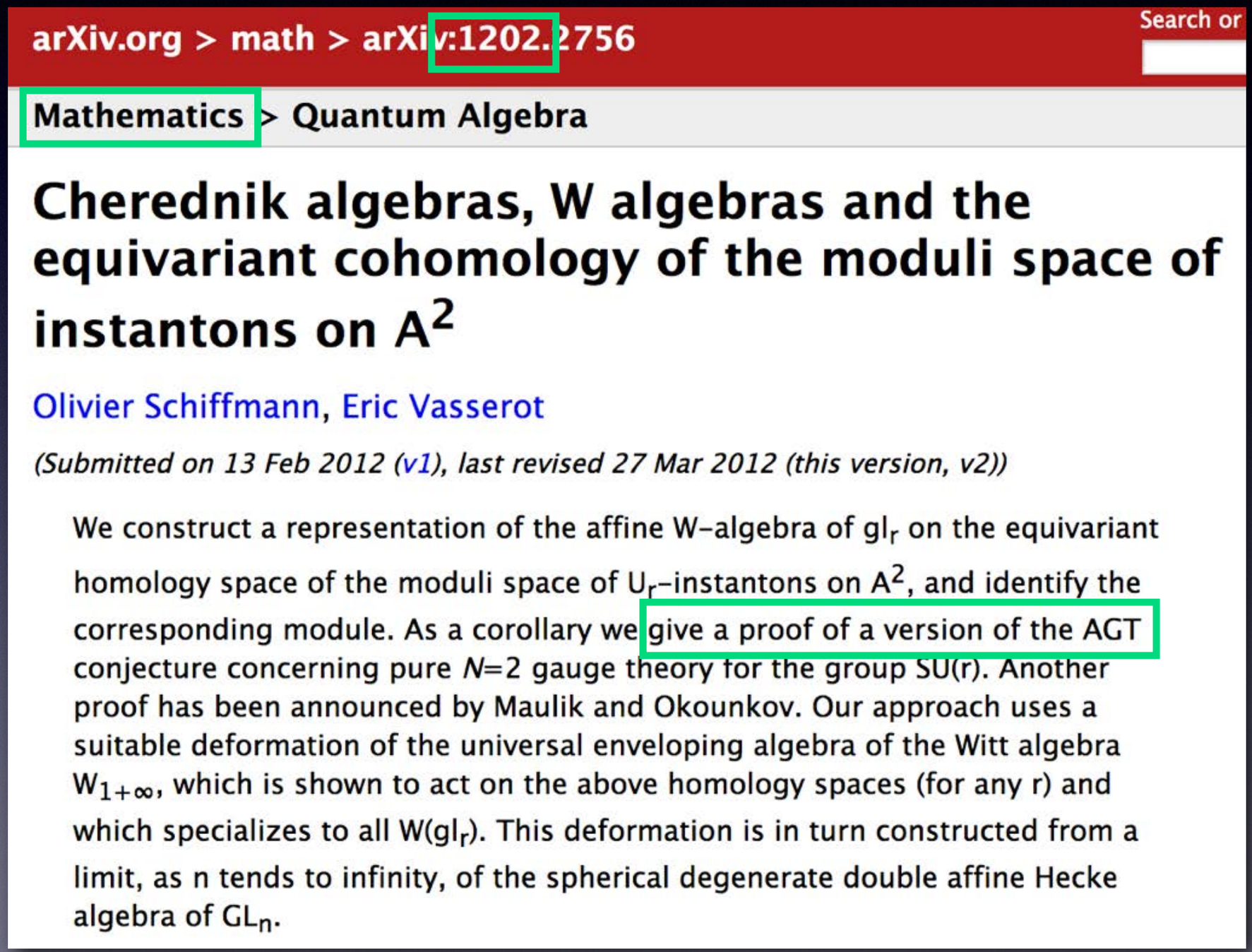
- We learned so much about them, and made tons of mathematical conjectures.

- And a lot of mathematicians work on these conjectures now.

- I also made one:



- which was later proved:



arXiv.org > math > arXiv:1202.2756

Search or

Mathematics > Quantum Algebra

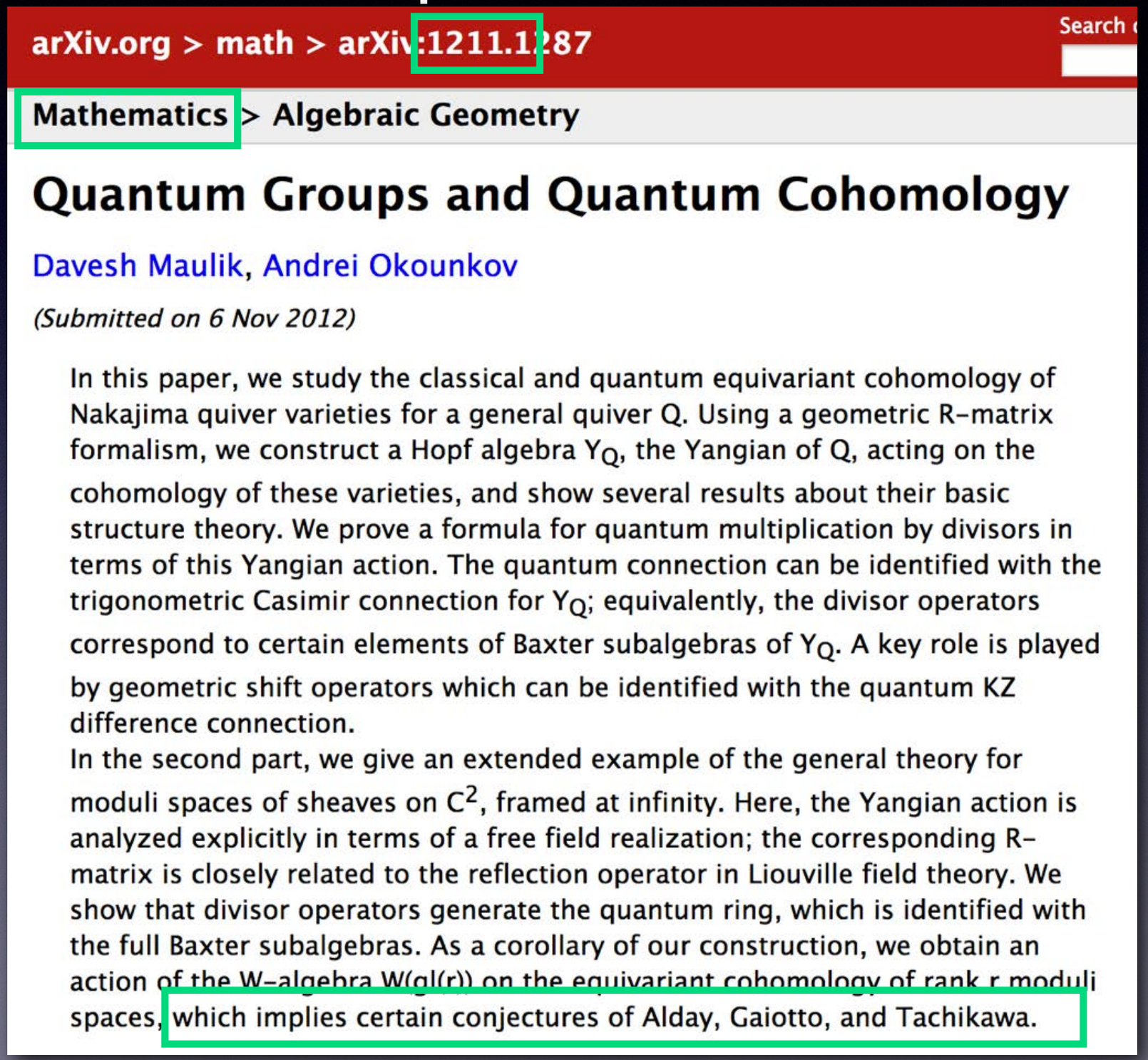
Cherednik algebras, W algebras and the equivariant cohomology of the moduli space of instantons on A^2

Olivier Schiffmann, Eric Vasserot

(Submitted on 13 Feb 2012 (v1), last revised 27 Mar 2012 (this version, v2))

We construct a representation of the affine W-algebra of gl_r on the equivariant homology space of the moduli space of U_r-instantons on A^2, and identify the corresponding module. As a corollary we give a proof of a version of the AGT conjecture concerning pure N=2 gauge theory for the group SU(r). Another proof has been announced by Maulik and Okounkov. Our approach uses a suitable deformation of the universal enveloping algebra of the Witt algebra $W_{1+\infty}$, which is shown to act on the above homology spaces (for any r) and which specializes to all $W(gl_r)$. This deformation is in turn constructed from a limit, as n tends to infinity, of the spherical degenerate double affine Hecke algebra of GL_n.

- which was later proved:



arXiv.org > math > arXiv:1211.1287

Search

Mathematics > Algebraic Geometry

Quantum Groups and Quantum Cohomology

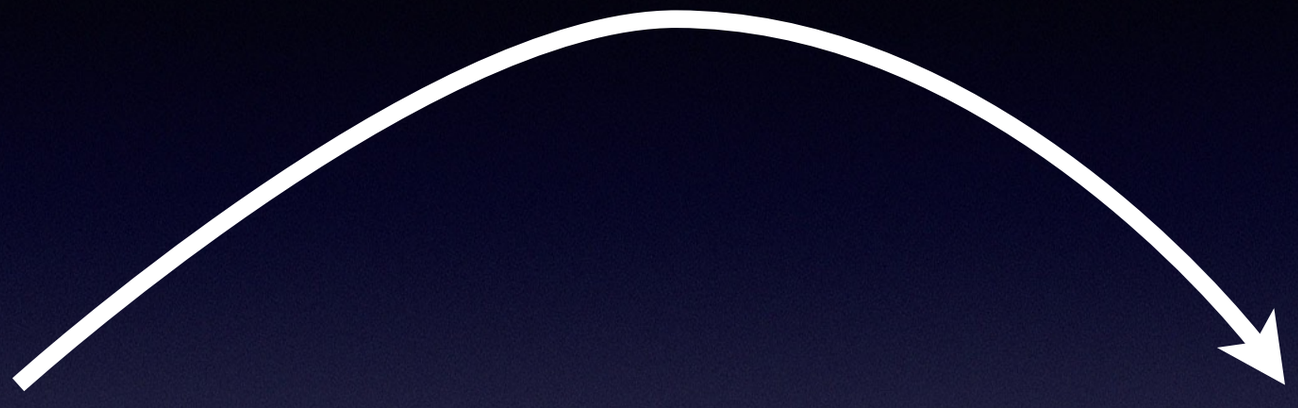
Davesh Maulik, Andrei Okounkov

(Submitted on 6 Nov 2012)

In this paper, we study the classical and quantum equivariant cohomology of Nakajima quiver varieties for a general quiver Q. Using a geometric R-matrix formalism, we construct a Hopf algebra Y_Q, the Yangian of Q, acting on the cohomology of these varieties, and show several results about their basic structure theory. We prove a formula for quantum multiplication by divisors in terms of this Yangian action. The quantum connection can be identified with the trigonometric Casimir connection for Y_Q; equivalently, the divisor operators correspond to certain elements of Baxter subalgebras of Y_Q. A key role is played by geometric shift operators which can be identified with the quantum KZ difference connection.

In the second part, we give an extended example of the general theory for moduli spaces of sheaves on C^2, framed at infinity. Here, the Yangian action is analyzed explicitly in terms of a free field realization; the corresponding R-matrix is closely related to the reflection operator in Liouville field theory. We show that divisor operators generate the quantum ring, which is identified with the full Baxter subalgebras. As a corollary of our construction, we obtain an action of the W-algebra W(gl(r)) on the equivariant cohomology of rank r moduli spaces, which implies certain conjectures of Alday, Gaiotto, and Tachikawa.

Suggest new conjectures

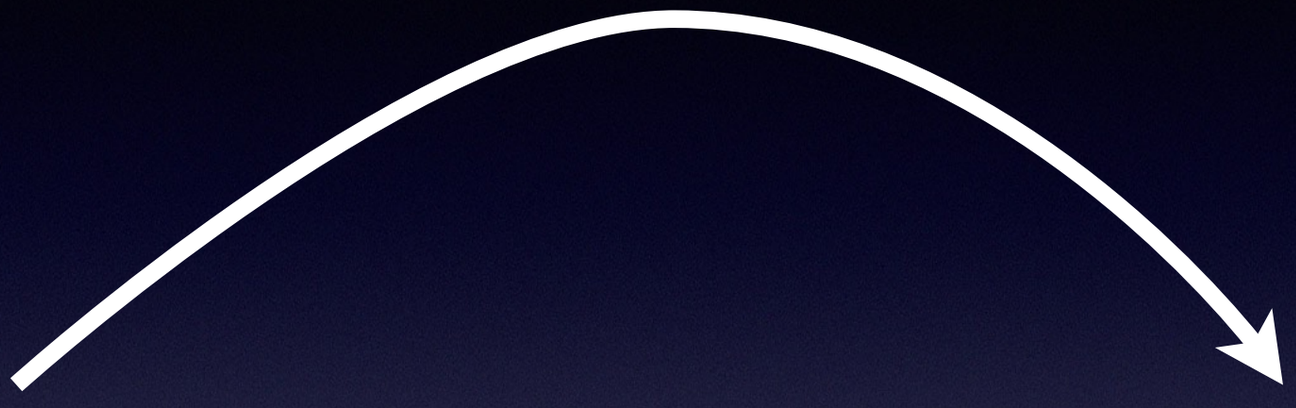


String theory

Mathematics

Present new mathematical results

Suggest new phenomena



Theoretical Science

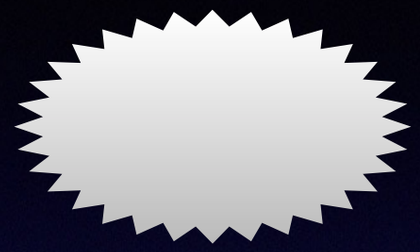
Experimental Science

Present new experimental results

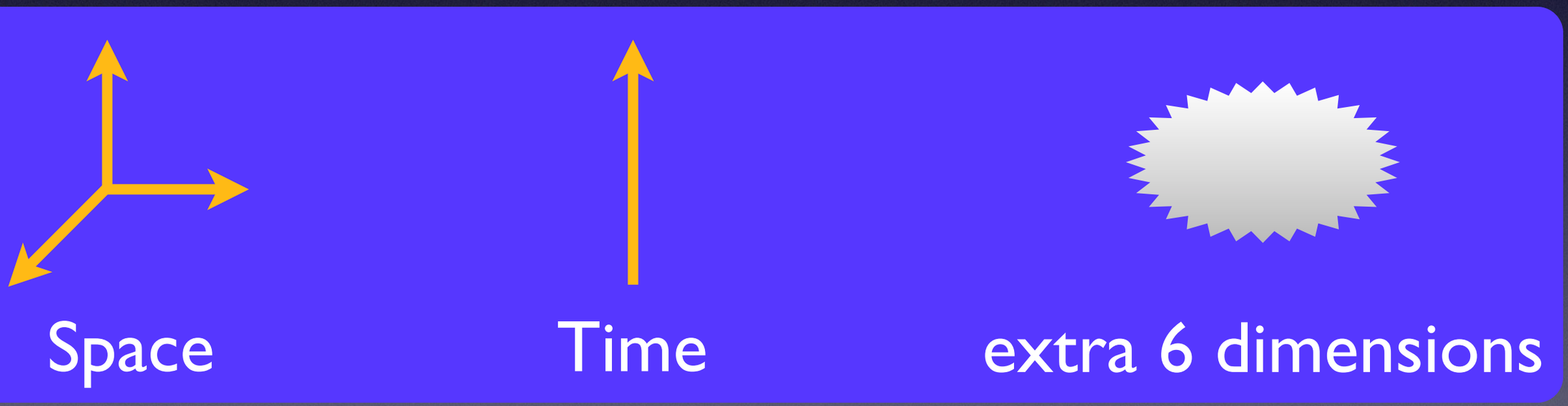
Theoretical Science : Experimental Science

= String theory : Mathematics

Of course, it can happen that somebody has found
the 6d space



specified by a very nice equation, such that

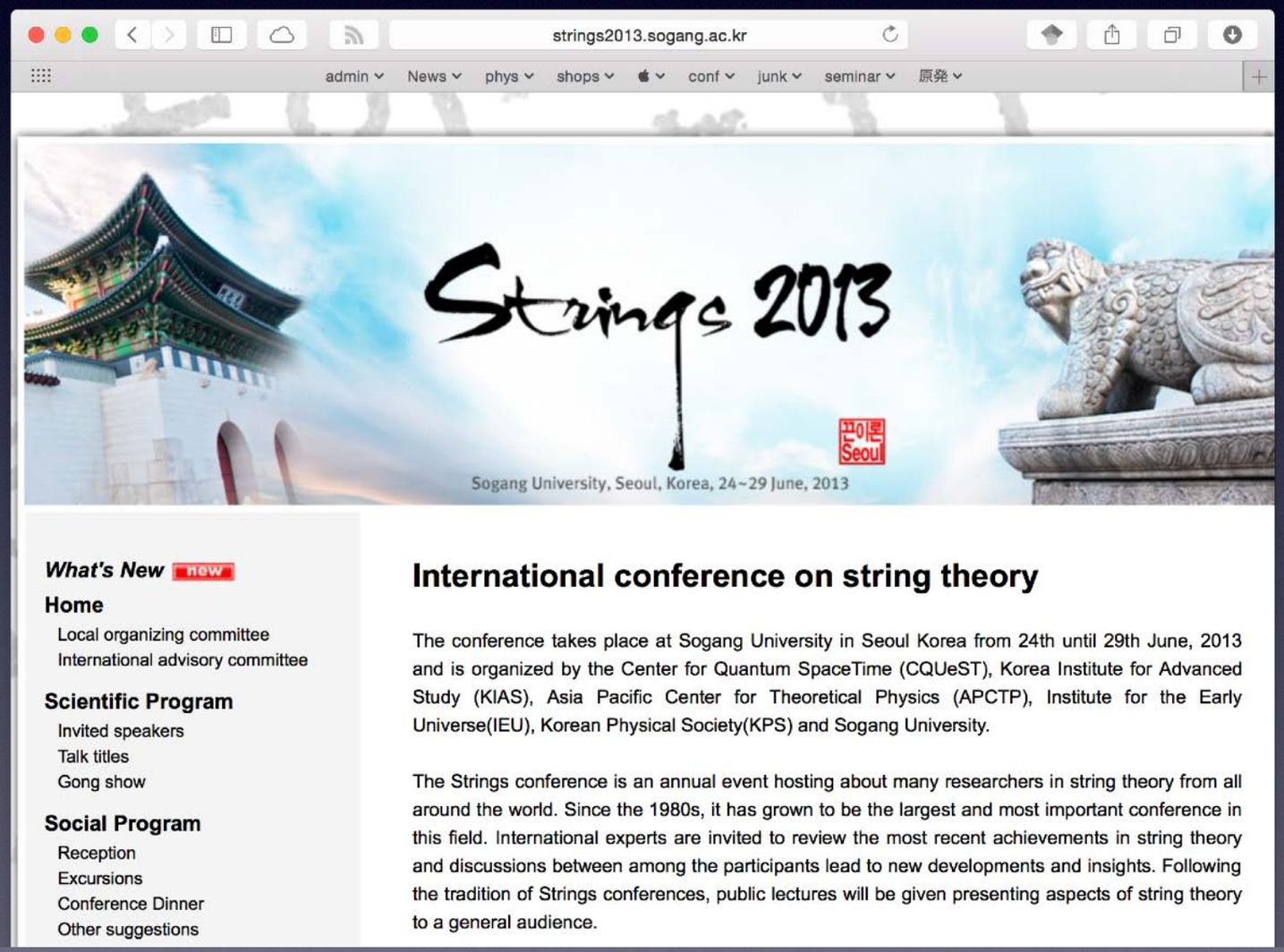


describes this world, and now is preparing a paper.

- Then string theory becomes a theoretical science.
- Until then, what I do is not really a science.
- That was what I wanted to say today!

- There’s an annual international String Theory conference every year

- There's an annual international String Theory conference every year



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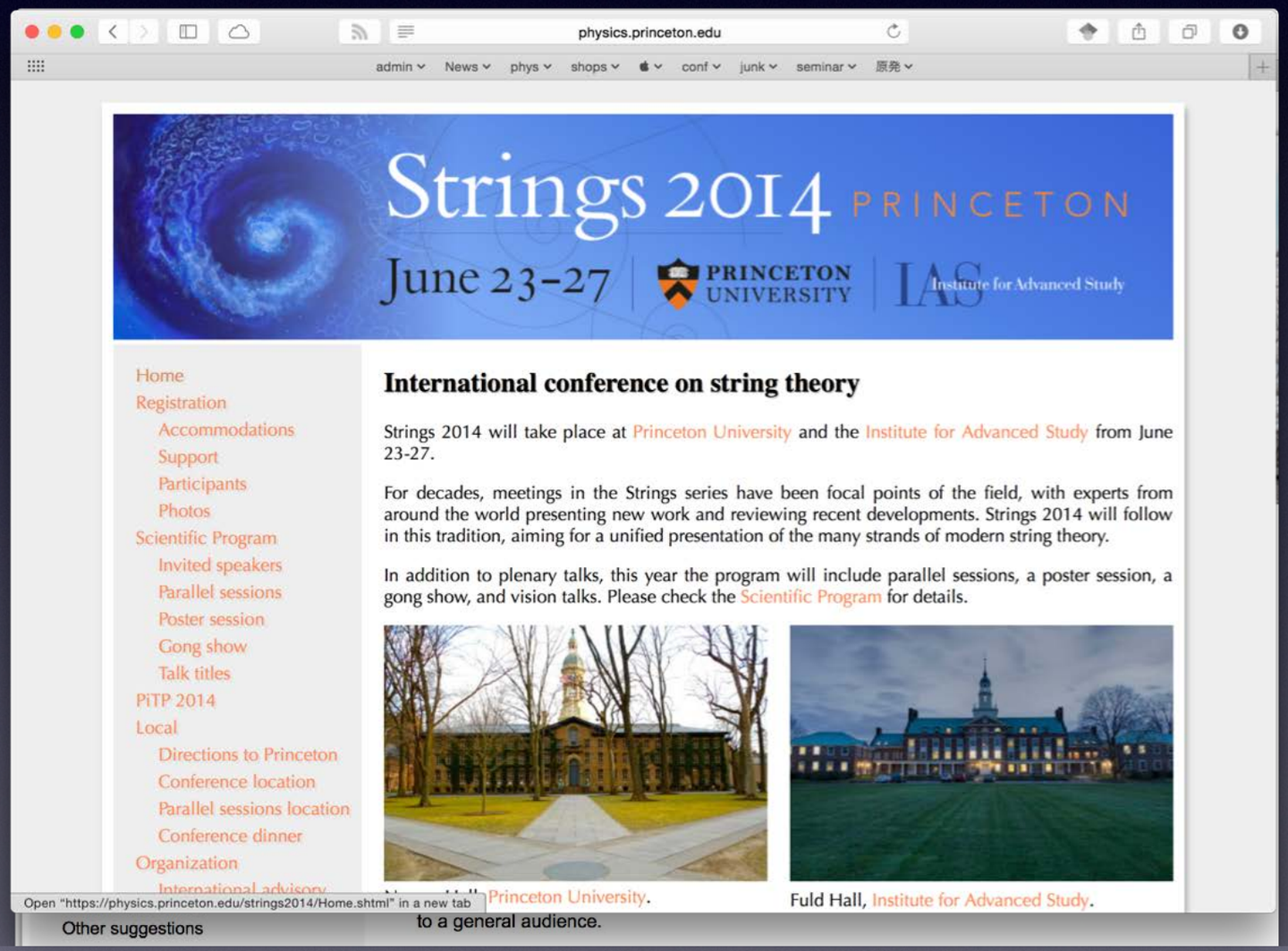

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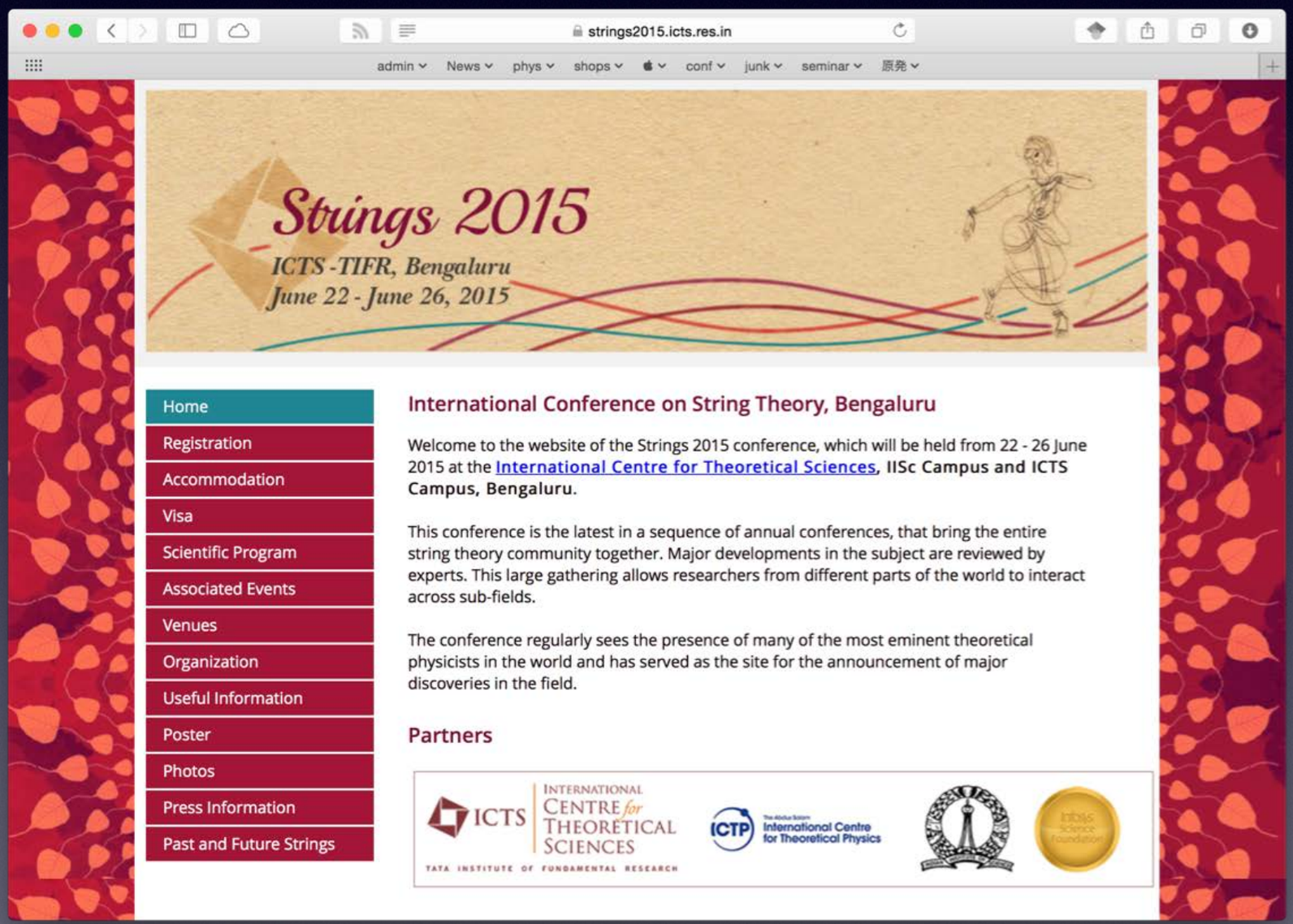
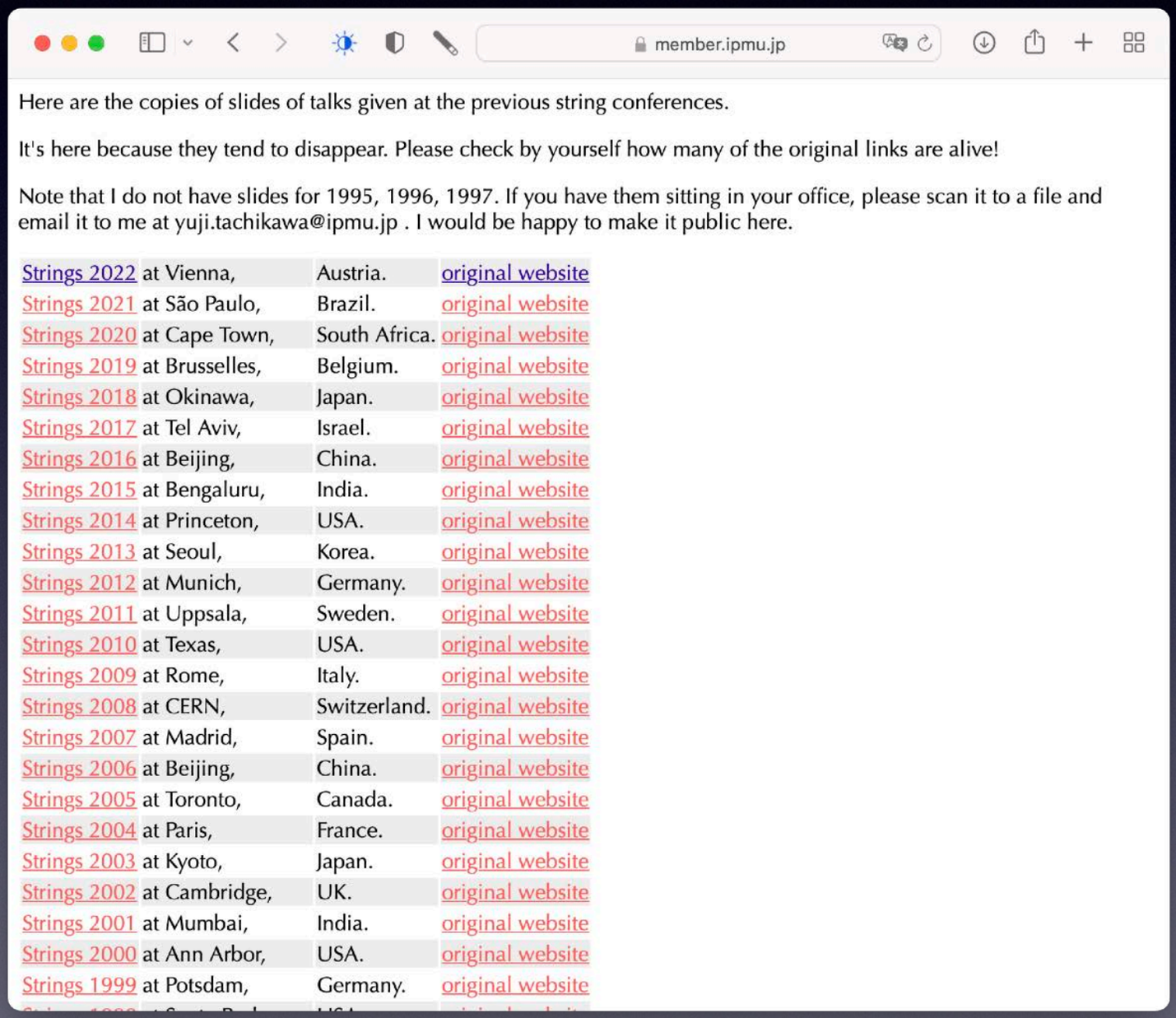
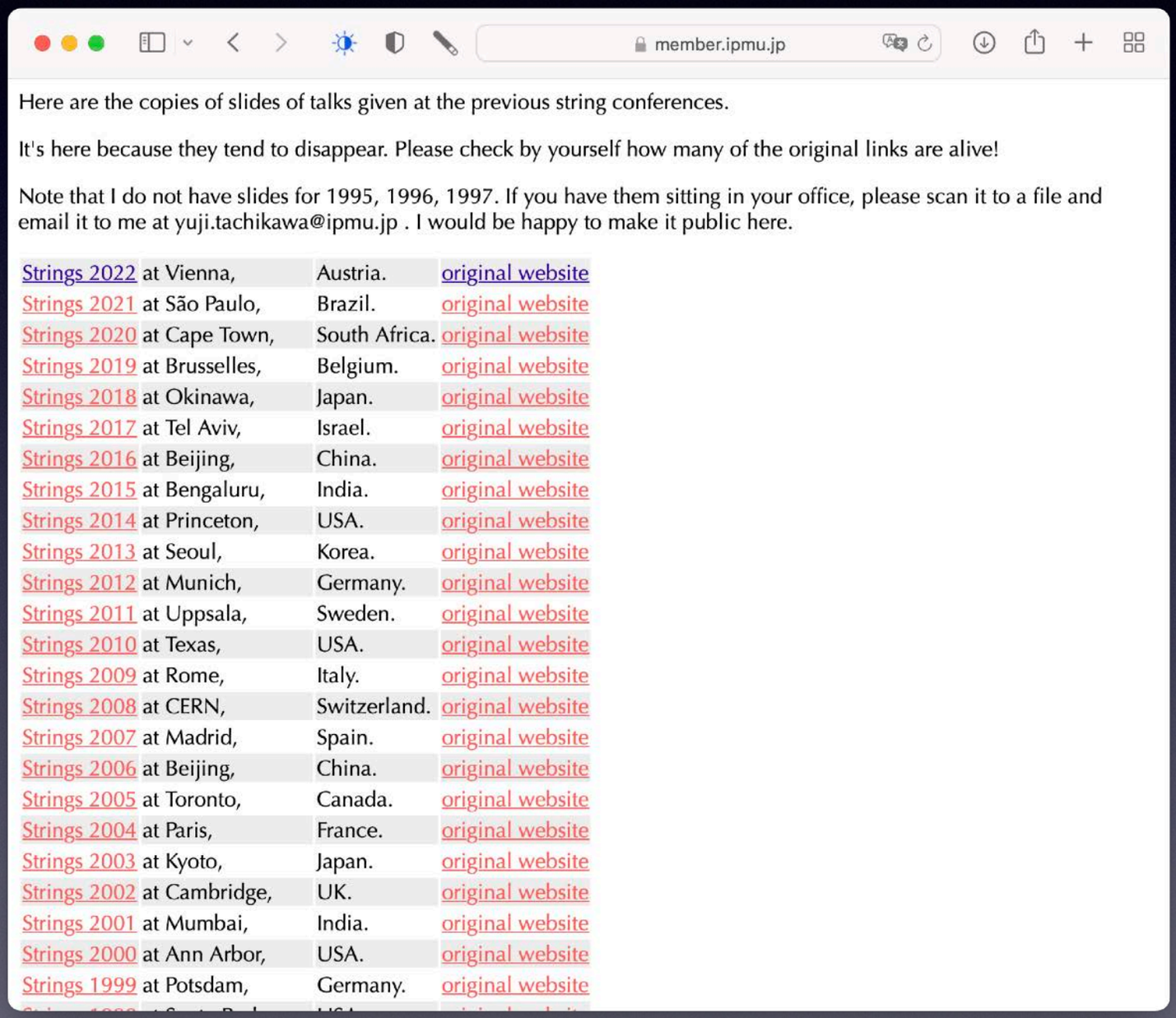


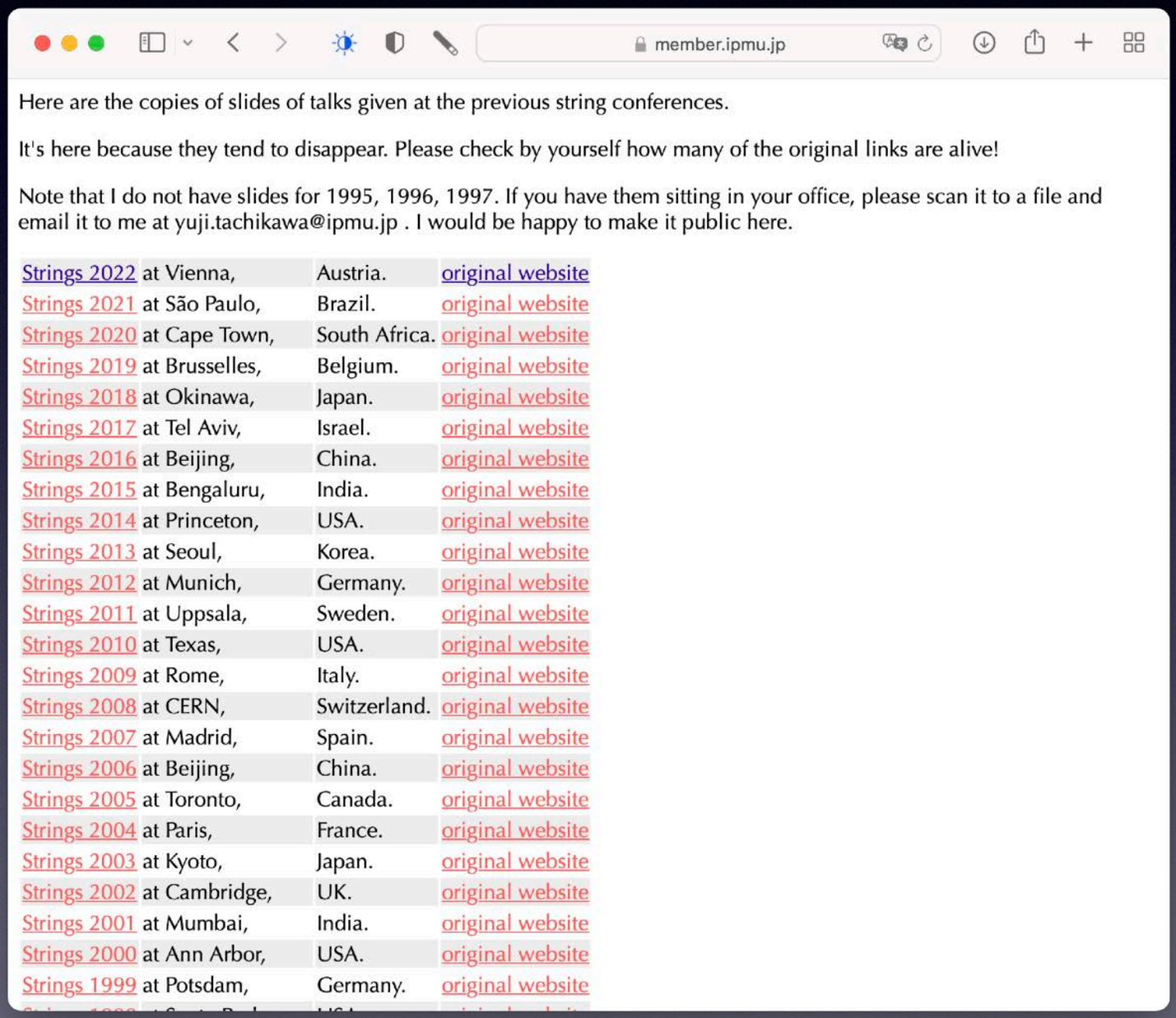
- I maintain a website collecting slides from the annual String Theory conferences.



- It contains roughly 30 talks x 25 years, and gives a great overview.



- If you are interested in string theory, you might want to have a look.



- I also wrote an iPad app to read slides at the String Theory conferences. It's free.