What is Quantum Field Theory?

Yuji Tachikawa

IPMU 10th anniversary conference

But before getting into that ...

When IPMU started in 2007, I was a postdoc in the US.

Everybody there told me that “Japan still knows how to fund basic science!”

Since then I learned that the reality is much more nuanced and complicated.

Anyway, I visited IPMU for the first time on December 2008.

Back then we didn't have the nice building we now have...

From the blog by a former postdoc,
"Confessions of a classless American goon":

From the blog by a former postdoc,
"Confessions of a classless American goon":

The blog says: **The institute placard has scotch tape involved.**

Ten years later, I am a faculty member at the IPMU.

I've seen how IPMU grew over the years.

This makes me nostalgic.

And here we are having this 10th anniversary workshop.

Well, birthday parties are for children,
and indeed IPMU is still in its childhood.

I hope IPMU to grow up further,
without too much problems, to become middle-aged,
and nobody feels the need to do an anniversary workshop.

What is Quantum Field Theory?

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I don’t know.

THE END.

Thank you for listening.

What is a **quantum** **field** **theory**?

- It describes **quantum** properties of **fields**,
- where a field is **anything that is extended along the space and time**, e.g.

  electromagnetic field, electron field, vibration of a crystal, …

A prototypical **quantum field theory** (QFT) is
the **Quantum Electrodynamics**

- which describes the quantized electromagnetic field interacting with electrons etc., and
- was established around 1950.

Since then there has been a steady progress.

By now we know that **our world**, at the most microscopic level that is experimentally accessible, is described by a **quantum field theory** called the **Standard Model**, established theoretically in the 1970s.

Its final piece, the Higgs boson, was confirmed experimentally in 2012, and both high energy theorists and experimentalists are looking for **physics beyond the Standard Model**.

Quantum field theories also appear ubiquitously in **condensed matter physics**.

For example, physics at the second-order phase transition is often described by **conformal field theories**,
which form a certain subclass of quantum field theories.

These days, **symmetry protected topological phases** are studied both in cond-mat and in hep-th. They are also quantum field theories.

This ubiquity is not surprising, since **quantum fields** are just the **quantum version of anything that are extended along the space**.

Due to its ubiquity, many people have worked on quantum field theories over its history of more than half a century.

Theoretical predictions agree well with experimental results. One extreme example is the **anomalous magnetic moment of electron**:

$$\begin{aligned} \boldsymbol{a_e^{\text{experimental}}} &= \mathbf{0.01\,159\,652\,181\ldots} \\ \boldsymbol{a_e^{\text{theoretical}}} &= \mathbf{0.01\,159\,652\,181\ldots} \end{aligned}$$

Even with this impressive agreement, I say **we don't yet know what's the correct framework to study quantum field theories.**

This is in contrast to the situation for **quantum mechanics or general relativity, for which** I say **we know the correct frameworks**.

Why do I say so?

In the case of quantum mechanics or general relativity, I can explain at least one framework to mathematicians in a few sentences.

Quantum mechanics

It's a study of unitary operators acting on a Hilbert space.

General relativity

It's a study of a differential equation satisfied by
the Riemann tensor of a Lorentzian metric on a manifold.

Not bad.

But then, what is a quantum field theory?

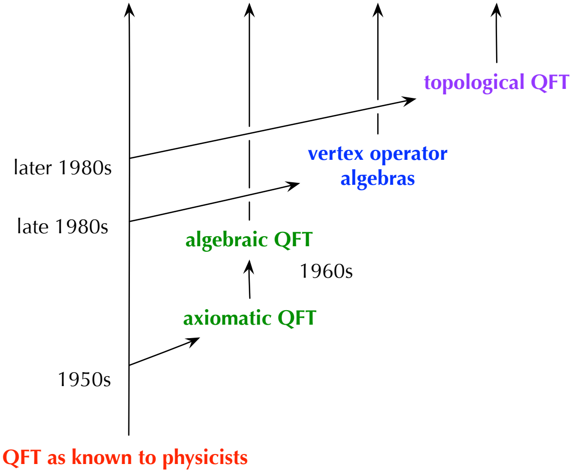
Quantum Field Theory

???

As a physicist, I think I know,
but I don't have a way to explain it to mathematicians.

I believe it's not just a problem about me.

Many people tried to formulate mathematically some small part of what they knew about quantum field theories. Some from the 20th century are:



In none of the frameworks from the 20th century we can express e.g. the computations that led to the impressive prediction

$$\boldsymbol{a_e = 1\,159\,652\,181.7... \times 10^{-12}}$$

for the anomalous magnetic moment of the electron.

- The framework **axiomatic QFT** is **too broad and generic** to carry out this particular computation
- The frameworks **vertex operator algebras** and **topological QFT** are **too narrow** and exclude the real Quantum Electrodynamics

That said, the method to compute this value

$$\boldsymbol{a_e = 1\,159\,652\,181.7... \times 10^{-12}}$$

using quantum electrodynamics is **explained in quantum field theory textbooks for physicists**.

So it's natural to mathematically formulate what are written there.

This mathematical reformulation is actually being done
by Costello and collaborators under the name of **factorization algebras**.

Will this answer my question, **what is a quantum field theory**?

I don't think so.

I'd like to illustrate why using the rest of my talk,
with my own research in the last few years as an example.

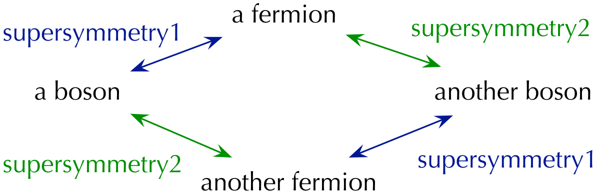
A big topic in my area is **supersymmetric** quantum field theories.

Supersymmetry allows us to relate
the **bosons** (photons, etc.) and the **fermions** (electrons, etc.):

$$\text{a boson} \xleftrightarrow{\text{supersymmetry}} \text{a fermion}$$

I'm interested in its theoretical structure.

There can be quantum field theories that can have **multiple supersymmetries**:



If there are two, it is called an $\boldsymbol{\mathcal{N}=2}$ supersymmetry, for historical reasons.

How many supersymmetries can a quantum field theory have?

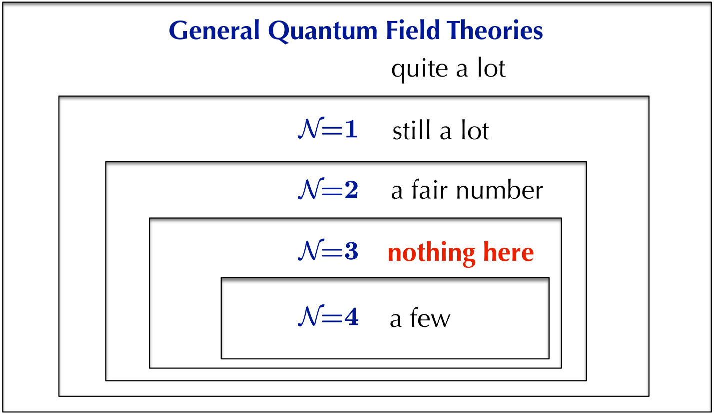
That's a topic covered often in the **first chapter** of a textbook on supersymmetry, and the standard answer **has been** as follows:

$$\mathcal{N} = 1, \qquad \mathcal{N} = 2, \qquad \mathcal{N} = 4$$

and that's it.

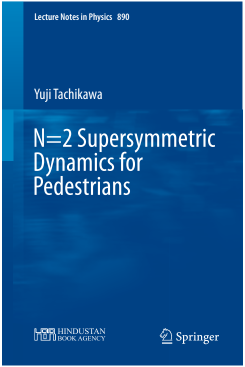
I don't have time to explain why there can't be more than four.

You might ask, why not $\mathcal{N}=3$?

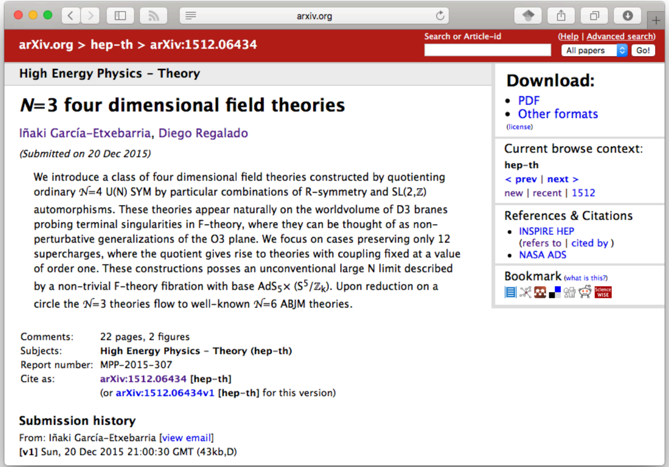


Well, if a quantum field theory has $\mathcal{N}=3$,
you can easily check it has in fact $\mathcal{N}=4$, or so people said.

I myself explained thus in a lecture note published **in 2014** from Springer:



And still, this paper appeared on **December 20, 2015** !



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High Energy Physics – Theory

$\mathcal{N}$=3 four dimensional field theories

Iñaki García-Etxebarria, Diego Regalado

(Submitted on 20 Dec 2015)

We introduce a class of four dimensional field theories constructed by quotienting ordinary $\mathcal{N}$=4 U(N) SYM by particular combinations of R-symmetry and SL(2,$\mathbb{Z}$) automorphisms. These theories appear naturally on the worldvolume of D3 branes probing terminal singularities in F-theory, where they can be thought of as non-perturbative generalizations of the O3 plane. We focus on cases preserving only 12 supercharges, where the quotient gives rise to theories with coupling fixed at a value of order one. These constructions posses an unconventional large N limit described by a non-trivial F-theory fibration with base $AdS_5 \times (S^5/\mathbb{Z}_k)$. Upon reduction on a circle the $\mathcal{N}$=3 theories flow to well-known $\mathcal{N}$=6 ABJM theories.

Comments: 22 pages, 2 figures
Subjects: **High Energy Physics – Theory (hep-th)**
Report number: MPP-2015-307
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where **genuinely $\mathcal{N}$=3 theories were found, using string theory**.

General Quantum Field Theories

quite a lot

$\mathcal{N}$=**1** still a lot

$\mathcal{N}$=**2** a fair number

$\mathcal{N}$=**3** **a few**

$\mathcal{N}$=**4** a few

So, why were all the textbooks (including mine) wrong?

In the case of an object attached to a mechanical spring:

Equation of motion: $$m\frac{d^2}{dt^2}x = -kx$$

In the case of an object attached to a mechanical spring:

Hamiltonian: $\boldsymbol{H_t = \frac{1}{2m}p^2 + \frac{k}{2}x^2}$

$\downarrow$

Equation of motion: $\boldsymbol{m\frac{d^2}{dt^2}x = -kx}$

In the case of an object attached to a mechanical spring:

$$\text{Lagrangian:} \qquad S = \int dt\left(\frac{m}{2}\left(\frac{dx}{dt}\right)^2 - \frac{k}{2}x^2\right)$$

$$\updownarrow$$

$$\text{Hamiltonian:} \qquad H_t = \frac{1}{2m}p^2 + \frac{k}{2}x^2$$

$$\downarrow$$

$$\text{Equation of motion:} \qquad m\frac{d^2}{dt^2}x = -kx$$

We are then taught that **Hamiltnian and Lagrangian frameworks are equivalent.**

In textbooks on quantum field theories, it's often said:

Equation**s** of motion: complicated.

In textbooks on quantum field theories, it's often said:

Hamiltonian**s**:		$\boldsymbol{H_t}$, $\boldsymbol{H_x}$, $\boldsymbol{H_y}$, $\boldsymbol{H_z}$, still complicated.
	$\downarrow$	
Equation**s** of motion:		complicated.

In textbooks on quantum field theories, it's often said:

Lagrangian:		$S = \int$ (something simple) d^4x
	$\downarrow$	
Hamiltonian**s**:		H_t, H_x, H_y, H_z, still complicated.
	$\downarrow$	
Equation**s** of motion:		complicated.

Richard Feynman even got a Nobel Prize for coming up with a Lagrangian framework for quantum field theories.

But in the last several years, we learned that

(Lagrangian: not available)

Hamiltonians: H_t, H_x, H_y, H_z, complicated.

$\downarrow$

Equations of motion: complicated.

for many quantum field theories.

So, why did all the textbooks (including mine) mistakenly state that any $\mathcal{N}=3$ quantum field theory automatically has $\mathcal{N}=4$ supersymmetry?

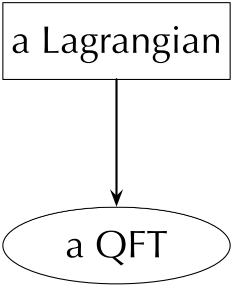
Well, if you assume the existence of a **Lagrangian**,

- you write down all possible $\mathcal{N}=2$ supersymmetric **Lagrangians**
- look for any theory that has more than $\mathcal{N}=2$ in the list
- you only find those with $\mathcal{N}=4$, never with $\mathcal{N}=3$.

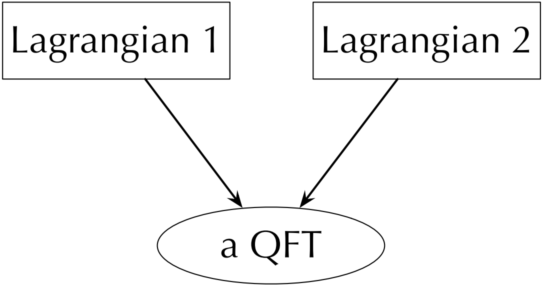
But there can be $\mathcal{N}=3$ theories **without Lagrangians**,
and indeed there are.

Let’s revisit the main issue from a slightly different perspective.

This is the old but standard way to approach a QFT:

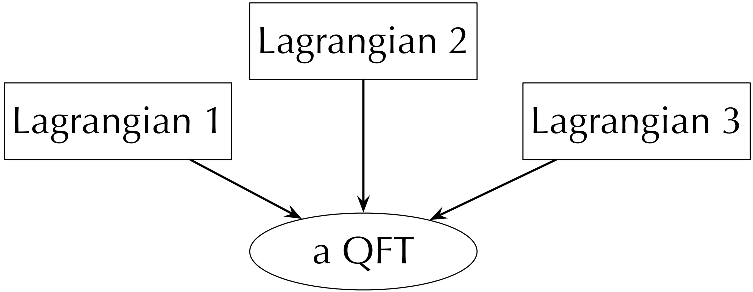


From about 20 years ago it is common to have a situation



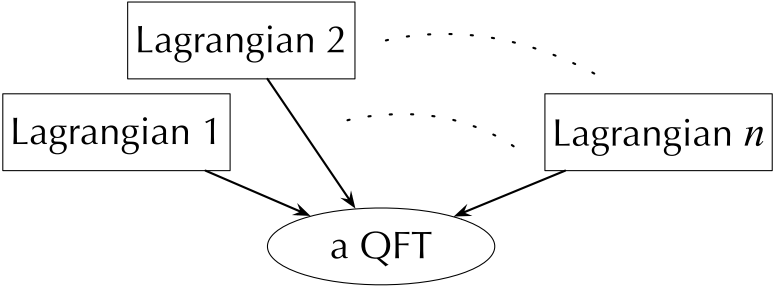
This has been called a **duality**.

There can be more than two:



Some call this a **triality**.

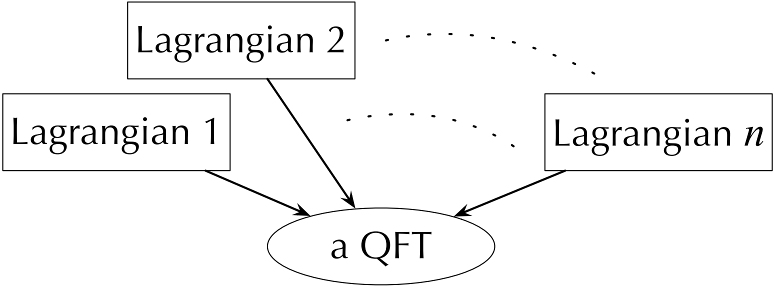
There can be many:



where $\boldsymbol{n}$ is a natural number.

It can be called an $\boldsymbol{n}$-ality, but people just call it duality.

There can be many:

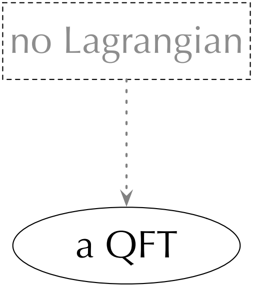


where n is a natural number.

It can be called an n-ality, but people just call it duality.

And now you remember that **natural numbers start from 0**.

So we can also have this situation:

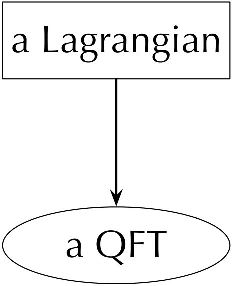


Having no Lagrangian is an example of the $\boldsymbol{n}$-ality where $\boldsymbol{n = 0}$.

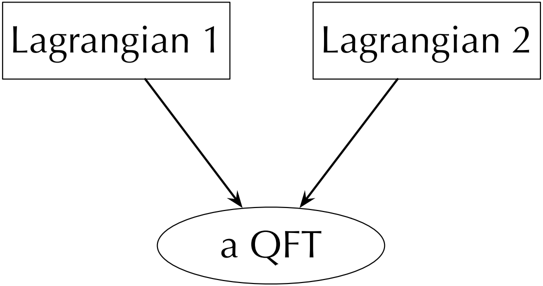
no Lagrangian

a QFT

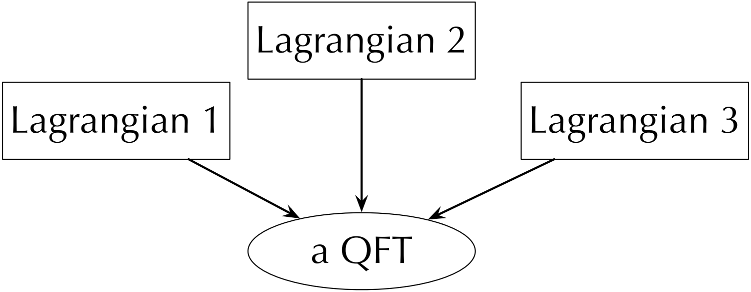
0-ality



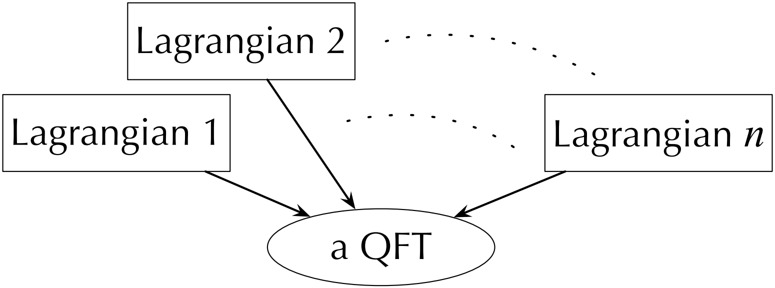
1-ality



Duality



Triality



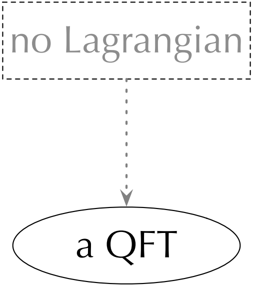
$\boldsymbol{n}$-ality

no Lagrangian

↓

a QFT

0-ality



Non-Lagrangian theory

Let me summarize:

Basically, **all the textbooks on quantum field theories out there** use an old framework that is **simply too narrow**, in that **it assumes the existence of a Lagrangian**.

This is a serious issue, because when you try to come up e.g. with a theory beyond the Standard Model, people habitually start by writing a Lagrangian…but **that might be putting too strong an assumption.**

We need to do something.