On 4d $\mathcal{N}$=3 theories

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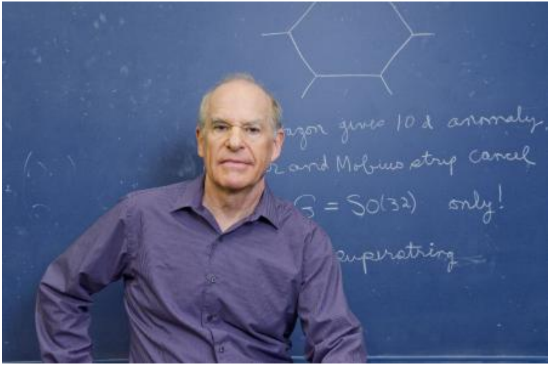
JHS 75, November 2016, Caltech

- It is a great honor to speak at this wonderful occasion celebrating the 75th birthday of John, the founding father of our field.



- This occasion reminded me of a quote of another great man:

- Confucius.

- The quote is this:

  子曰：吾十有五而志於學，三十而立，四十而不惑，五十而知天命，六十而耳順，七十而從心所欲不踰矩。

In translation: Confucius said,

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At 15, I set my heart on learning.

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At 50, I knew the command of Heaven.

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At 50, I knew the command of Heaven.

At 60, I heard it with a compliant ear.

In translation: Confucius said,

At 15, I set my heart on learning.

At 30, I took my stand.

At 40, I was without confusion.

At 50, I knew the command of Heaven.

At 60, I heard it with a compliant ear.

At 70, I follow the desires of my heart and yet
do not overstep the bounds.

(Analects 2:4, translated by Robert Eno.)

Anyway, I'd like to talk about something closely related to John's work, and I recalled the influential paper:

SUPERSYMMETRIC YANG-MILLS THEORIES *

Lars BRINK ** and John H. SCHWARZ
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Received 22 December 1976

Yang-Mills theories with simple supersymmetry are constructed in 2, 4, 6, and 10 dimensions, and it is argued that these are essentially the only cases possible. The method of dimensional reduction is then applied to obtain various Yang-Mills theories with extended supersymmetry in two and four dimensions. It is found that all possible four-dimensional Yang-Mills theories with extended supersymmetry are obtained in this way.

where he constructed $\mathcal{N}=1, 2, 4$ super Yang-Mills in a uniform fashion:

1. Introduction

Three types of supersymmetric Yang-Mills theories in four dimensions are known. In the first one that was found [1] the infinitesimal parameter of the supersymmetry transformation is a Majorana spinor ("simple" supersymmetry). In the second one [2] it is a Dirac spinor ("complex" supersymmetry). In the third case it consists of four Majorana (or Weyl) spinors [3]. This last model was obtained recently by applying the method of dimensional reduction to a supersymmetric Yang-Mills theory in ten-dimensional space-time.

The goal of this paper is to classify all the possible supersymmetric Yang-Mills theories in both two and four dimensions. The interest in four dimensions is obvious, of course, as one of these schemes may be part of a correct theory. The two-dimen-

- $\mathcal{N}=1,2,4$ super Yang-Mills. Why no $\mathcal{N}=3$?
- It was often said that $\mathcal{N}=3 \Rightarrow \mathcal{N}=4$, but that was for **Lagrangian field theories**.
- In the last several years, we learned that **non**-Lagrangian theories are everywhere.

- Indeed, in [Gacía-Etxebarria and Regalado, 1512.06434], genuine $\mathcal{N}=3$ theories were constructed **using F-theory**.

- I was very excited and I wrote two follow-ups,
  [1602.01503 with Nishinaka] and
  [1602.08638 with Aharony],
  and I'd like to report on these subjects.

- $\exists$ several papers since then,
  including [Gacía-Etxebarria and Regalado, 1611.05769]
  that appeared just two days ago!

Review of the old argument

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F-theory construction, part I

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F-theory construction, part II

- Analysis of susy multiplets can be found in [Wess-Bagger].
- (spin $\leq \mathbf{1}$ & CPT-inv & $\mathcal{N}=\mathbf{3}$) implies $\mathcal{N}=\mathbf{4}$.
- They only analyzed the multiplets, not interactions.
- At this point $\exists$ possibility of genuine $\mathcal{N}=\mathbf{3}$ interactions.

- Assume $\mathcal{N}=\mathbf{2}$.
  All possible Lagrangians up to lowest derivatives easily written.
  One goes over the list, and finds $\mathcal{N} > \mathbf{2}$ implies $\mathcal{N}=\mathbf{4}$.

- Anyway, this says nothing about non-Lagrangian theories!

- [Aharony-Evtikhiev 1512.03524] studied 4d $\mathcal{N}=3$ **superconformal** multiplets.

- They found, for example, an $\mathcal{N}=3$ preserving **marginal deformation** is in the same multiplet with another supercharge. $\rightarrow$ **the theory enhances** to $\mathcal{N}=4$.
- Put it differently, genuine $\mathcal{N}=3$ theories are intrinsically **isolated**. (see also [Cordova-Dumitrescu-Intriligator, 1602.01217]).

- Field-theoretical construction sounds very difficult.

- But there's an easy stringy construction!

Review of the old argument

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F-theory construction, part I

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F-theory construction, part II

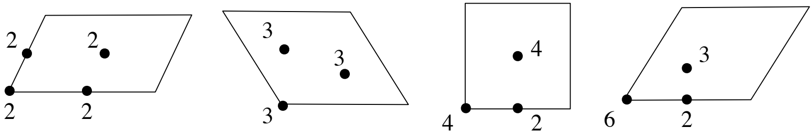
- Let's construct 4d $\mathcal{N}=\mathbf{3}$ theory.
- We need to keep $\mathbf{3/4}$ of SUSY.
- Do you know any other system that preserves $\mathbf{3/4}$ of SUSY?

- Let's construct 4d $\mathcal{N}=3$ theory.
- We need to keep $3/4$ of SUSY.
- Do you know any other system that preserves $3/4$ of SUSY?

- **The ABJM theory** !
- M-theory on $\mathbb{C}^4/\mathbb{Z}_k$, probed by M2s.
- 3d $\mathcal{N}=8$ when $k = 1, 2$, 3d $\mathcal{N}=6$ when $k \geq 3$.

- We can imitate the construction in 4d.
- Just need to consider F-theory on $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_k$, probed by D3s.

- T^2 can have $\mathbb{Z}_k$ action only for $k = 1, 2, 3, 4, 6$.
- For $k = 1, 2$,
  T^2 can be arbitrary.
- For $k = 3, 4, 6$,
  T^2 needs to have a particular shape:



$k = 2$ $k = 3$ $k = 4$ $k = 6$

where the numbers show the orders of the fixed points.

- Consider F-theory on $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_k$, probed by N D3s.
- For $k = 1$, this gives 4d $\mathcal{N}{=}4$ $\mathbf{U}(N)$ SYM.
- The shape of T^2 is arbitrary, and gives τ.

- Consider F-theory on $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_k$, probed by N D3s.
- For $k = 2$, this $\mathbb{Z}_2$ quotient is the standard orientifold 3-planes.
- Has **variants**:

$$\mathbf{O3^-}, \quad \widetilde{\mathbf{O3}}^-, \quad \mathbf{O3^+}, \quad \widetilde{\mathbf{O3}}^+.$$

- They give rise to 4d $\mathcal{N}=4$ SYM with gauge groups

$$\mathbf{SO}(2N), \quad \mathbf{SO}(2N+1), \quad \mathbf{USp}(2N), \quad \mathbf{USp}(2N),$$

respectively.

- The shape of T^2 is arbitrary, and gives τ.

- Consider F-theory on $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_k$.
- For $k = 3, 4, 6$, we have a **something-fold** that generalizes **orientifolds**.

- Will have **variants**.

- Probing this setup with N D3-branes, we have genuine $\mathcal{N}=3$ theory, labeled by k, N, and **another label for variants**.

- The shape of T^2 is **fixed, with no marginal parameter**.

- This is basically the content of [García-Etxebarria and Regalado].

- We would like to understand the variants,
  and to get some idea of the field theoretical properties.

Review of the old argument

$\downarrow$

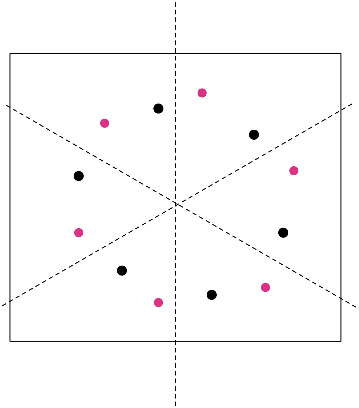
F-theory construction, part I

$\downarrow$

F-theory construction, part II

- How to see the variants of **something-folds** for $\boldsymbol{k = 3, 4, 6}$, analogous to the distinction of $\mathbf{SO(2N)}$ and $\mathbf{USp(2N)}$ for $\boldsymbol{k = 2}$?
- To study this issue, consider D3-branes on $\mathbb{C}/\mathbb{Z}_k \subset \mathbb{C}^3/\mathbb{Z}_k$.
- This corresponds to restricting attention to the $\boldsymbol{\mathcal{N}=2}$ Coulomb branch.

- Say we have N points on $\mathbb{C}/\mathbb{Z}_k \subset \mathbb{C}^3/\mathbb{Z}_k$:



- Coordinates $z_1, \dots, z_N \in \mathbb{C}$, with kN images.
  Coordinates of the "Cartan" of the "gauge group".

- Invariance under

$$z_i \leftrightarrow z_j$$
$$z_i \mapsto \gamma z_i \quad (\gamma^k = 1),$$

  the "Weyl group".
- "Weyl-invariant" combinations are sym. poly. of ${z_i}^k$, with dimension

$$k, \quad 2k, \quad 3k, \quad \cdots \quad , Nk.$$

- Taking $k = 2$, this gives the gauge invariants of $\mathbf{SO}(2N + 1)$ or $\mathbf{USp}(2N)$, but **not** those of $\mathbf{SO}(2N)$.

- Weyl group of $\mathbf{SO}(2N+1)$ is

$$\begin{aligned} z_i &\leftrightarrow z_j \\ z_i &\mapsto -z_i \quad \text{fixing others} \end{aligned}$$

- Weyl group of $\mathbf{SO}(2N)$ is

$$\begin{aligned} z_i &\leftrightarrow z_j \\ (z_i, z_j) &\mapsto (-z_i, -z_j) \quad \text{fixing others} \end{aligned}$$

  a subgroup of the above group.

- The top-dimension invariant of the former, the **determinant**

$$(z_1 \dots z_N)^2$$

  is the square of the top-dimension invariant of the latter, the **Pfaffian**

$$(z_1 \dots z_N).$$

- Similarly, the group

$$z_i \leftrightarrow z_j$$
$$z_i \mapsto \gamma z_i \quad (\gamma^k = 1),$$

has subgroups for $k = p\ell$:

$$z_i \leftrightarrow z_j$$
$$z_i \mapsto \gamma^p z_i \qquad \text{fixing others}$$
$$(z_i, z_j) \mapsto (\gamma z_i, \gamma^{-1} z_j) \quad \text{fixing others}$$

- The top-dimension invariant of the former, the "**determinant**"

$$(z_1 \dots z_N)^k$$

is the p-th power of
the top-dimension invariant of the latter, the "**Pfaffian**"

$$(z_1 \dots z_N)^\ell$$

- So the **variants of those something-fold would be labeled** by $\ell | k$.
- The question is which ℓ is realized in F-theory:

$k=2$	$\ell=1$	$\ell=2$			
$k=3$	$\ell=1$		$\ell=3$		
$k=4$	$\ell=1$	$\ell=2$		$\ell=4$	
$k=6$	$\ell=1$	$\ell=2$	$\ell=3$		$\ell=6$

- We found that those marked in **green do not exist**.
  How do we see that?

- One way to see this is as follows.

- Consider the holographic dual $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_k$.

- The operator $(z_1 \cdots z_N)^\ell$ corresponds to a D3-brane wrapped around $S^3/\mathbb{Z}_k \subset S^5/\mathbb{Z}_k$ ℓ times.
- Nontrivial flux $\in H^3(S^5/\mathbb{Z}_k, (\mathbb{Z} \oplus \mathbb{Z})_\rho)$ obstructs this.

- For $k = 2$, studied in [Witten, hep-th/9805112].

- For $k = 6$, $H^3 = \mathbb{Z}_1$, so there's no nontrivial flux to start with. So $\ell = 1$ always allowed.

- For $k = 3$, $H^3 = \mathbb{Z}_3$. When the flux is zero, $\ell = 1$. When the flux is nonzero, only $\ell = 3$ allowed.

- For $k = 4$, $H^3 = \mathbb{Z}_2$. When the flux is zero, $\ell = 1$. When the flux is nonzero, only $\ell = 4$ allowed, after a careful consideration.

- Another way to see this is as follows.
- Consider the holographic dual $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_k$.
- It's easily seen that

$$a \sim c \sim \frac{k(N+\epsilon)^2}{4} + O(N^0)$$

where ϵ is the intrinsic D3-charge of the something-fold.

- Meanwhile, we know

$$a = c, \qquad 2a - c = \sum_u \frac{2\Delta(u) - 1}{4}$$

with $\Delta(u)$'s given by

$$k, \quad 2k, \quad \cdots \quad , (N-1)k; \quad \ell k$$

- Comparing with

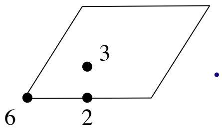
$$a \sim c \sim \frac{k(N+\epsilon)^2}{4} + O(N^0)$$

one finds

$$\epsilon = \frac{2\ell - k - 1}{12}.$$

- This intrinsic D3-charge of the something-fold can be computed independently, by compactifying the system on $\boldsymbol{S^1}$ and use M-theory.

- Take e.g. $\boldsymbol{k = 6}$. We have $\mathbb{R}^3 \times (\mathbb{C}^3 \times \boldsymbol{T^2})/\mathbb{Z}_6$.
- The $\mathbb{Z}_6$ action on $\boldsymbol{T^2}$ is given by



- So there are $\mathbb{C}^4/\mathbb{Z}_2$, $\mathbb{C}^4/\mathbb{Z}_3$, $\mathbb{C}^4/\mathbb{Z}_6$ singularities.

- A $\mathbb{C}^4/\mathbb{Z}_s$ singularity is known to have M2-charge

$$-\frac{1}{24}(s-\frac{1}{s}).$$

- We have

$$-\frac{1}{24}\left[(2-\frac{1}{2})+(3-\frac{1}{3})+(6-\frac{1}{6})\right]=-\frac{5}{12}.$$

- This agrees with

$$\epsilon=\frac{2\ell-k-1}{12}$$

with $\ell=1$, $k=6$.

- All other examples work out nicely.

Conclusions

- Genuine $\mathcal{N}=3$ theories exist!
- Some properties were understood,
  a and c, 2d chiral algebra, variants.
- Many properties still to be determined,
  e.g. Schur indices and more general superconformal indices.

- The quotients of [Gacía-Etxebarria and Regalado] can be generalized to $\mathcal{N}=2$.
  $\Rightarrow$ a lot of novel $\mathcal{N}=2$ SCFTs so far unknown.

- Many things to do!

Happy 75th birthday, John!