Some theoretical comments on multi-channel Kondo effect

an ongoing work with two fantastic collaborators

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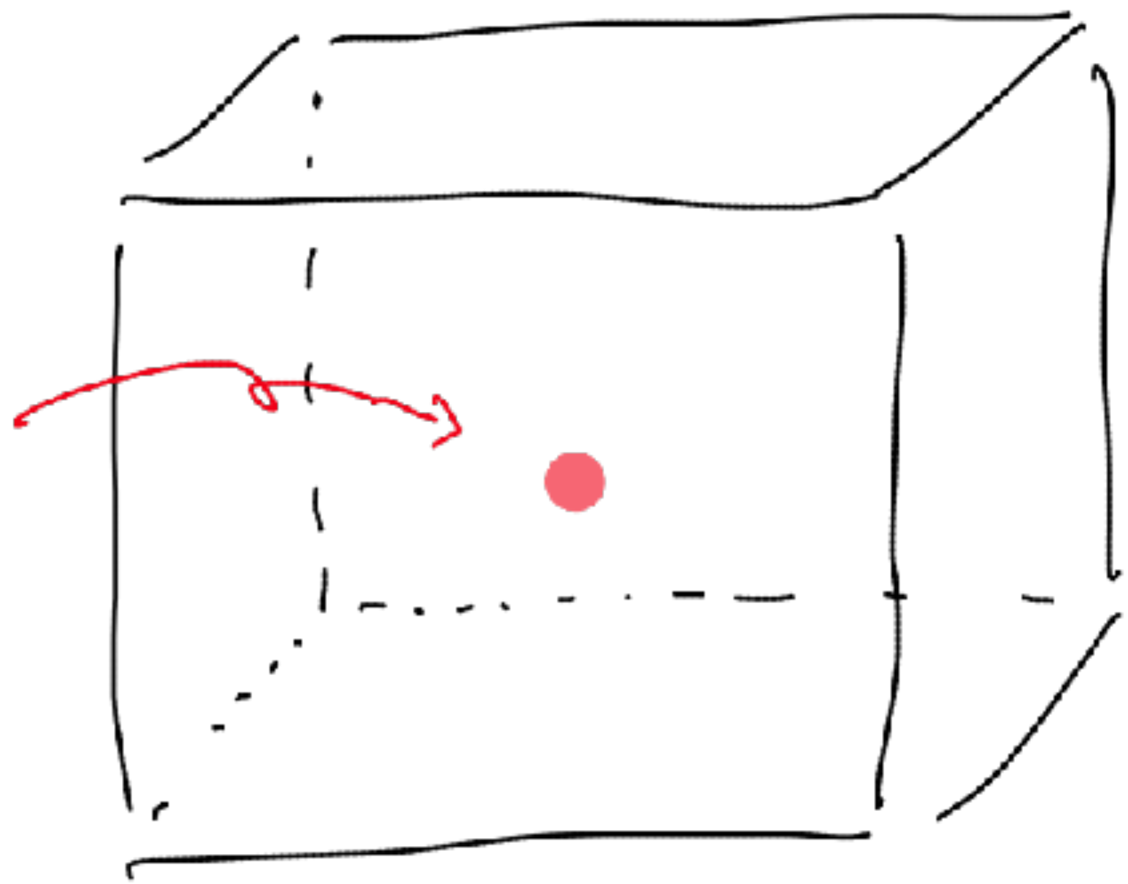
(Hongo)

But why me?
I'm a string theorist.

Why is a string theorist giving a talk on the Kondo effect?

In the Kondo effect,

magnetic impurity



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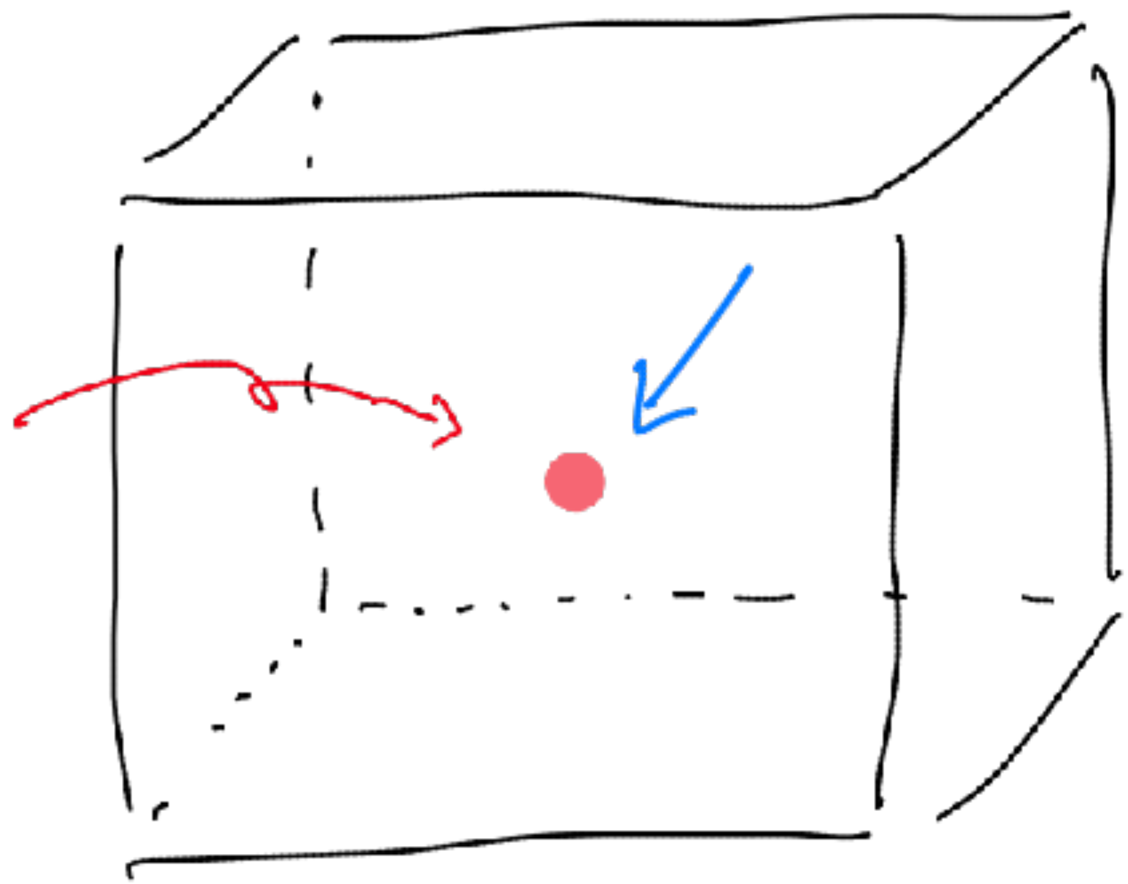

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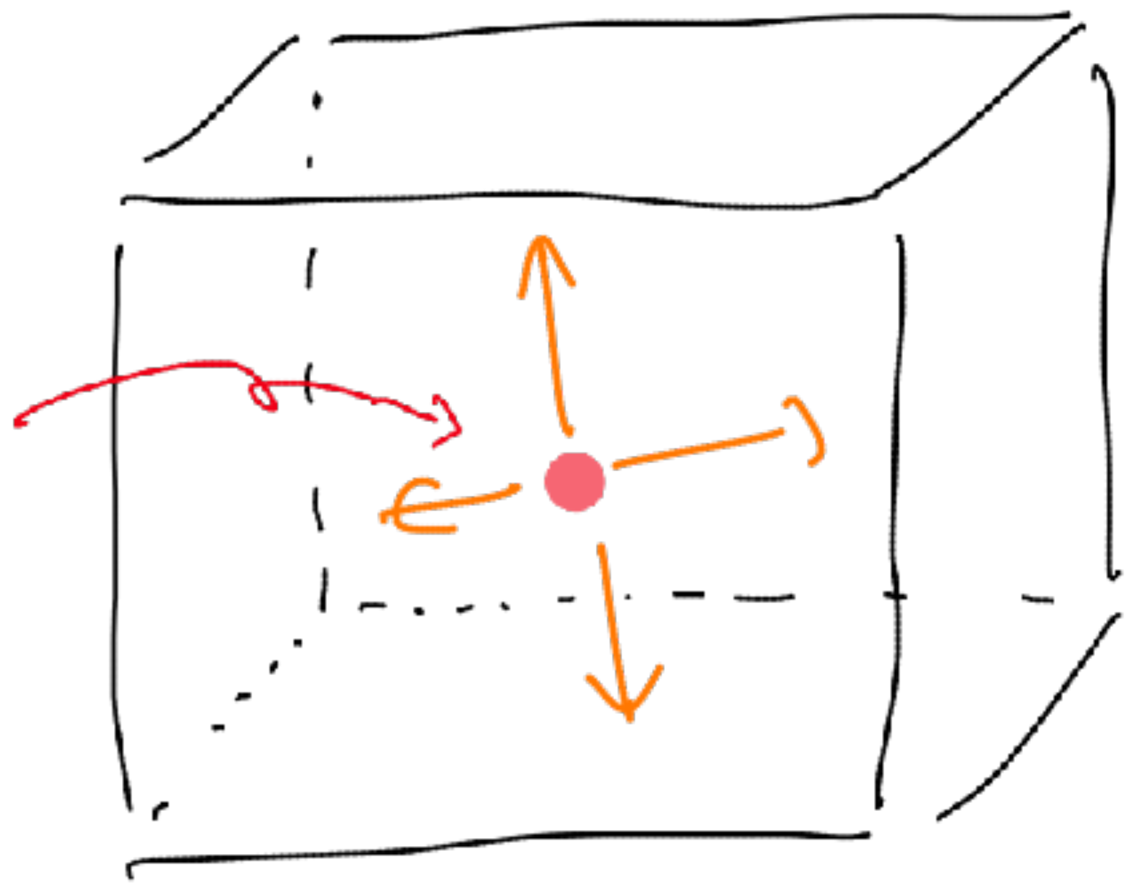
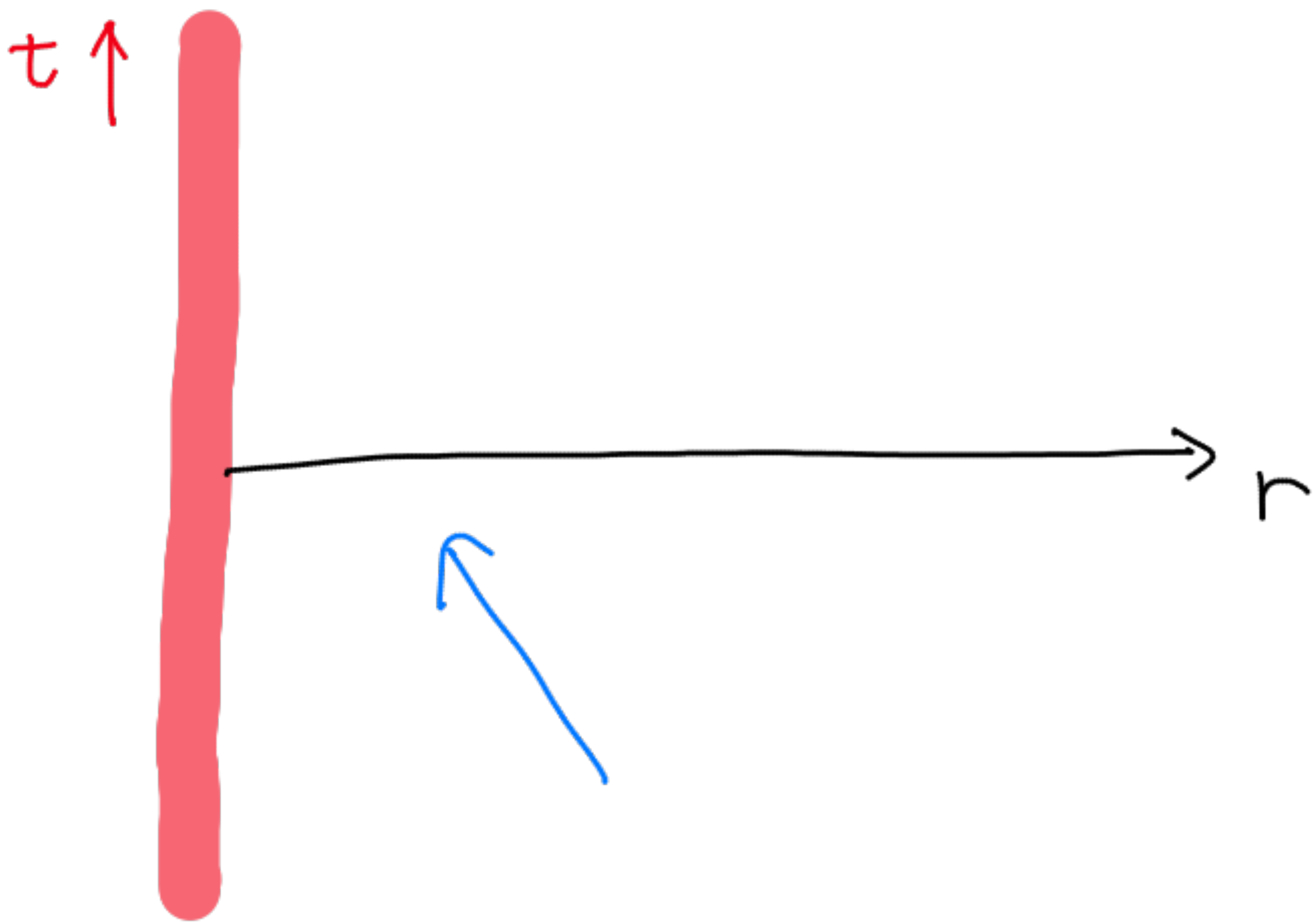


Taking S-wave approximation ...



we get a 1+1d scattering problem of 'relativistic' fermions by a boundary.

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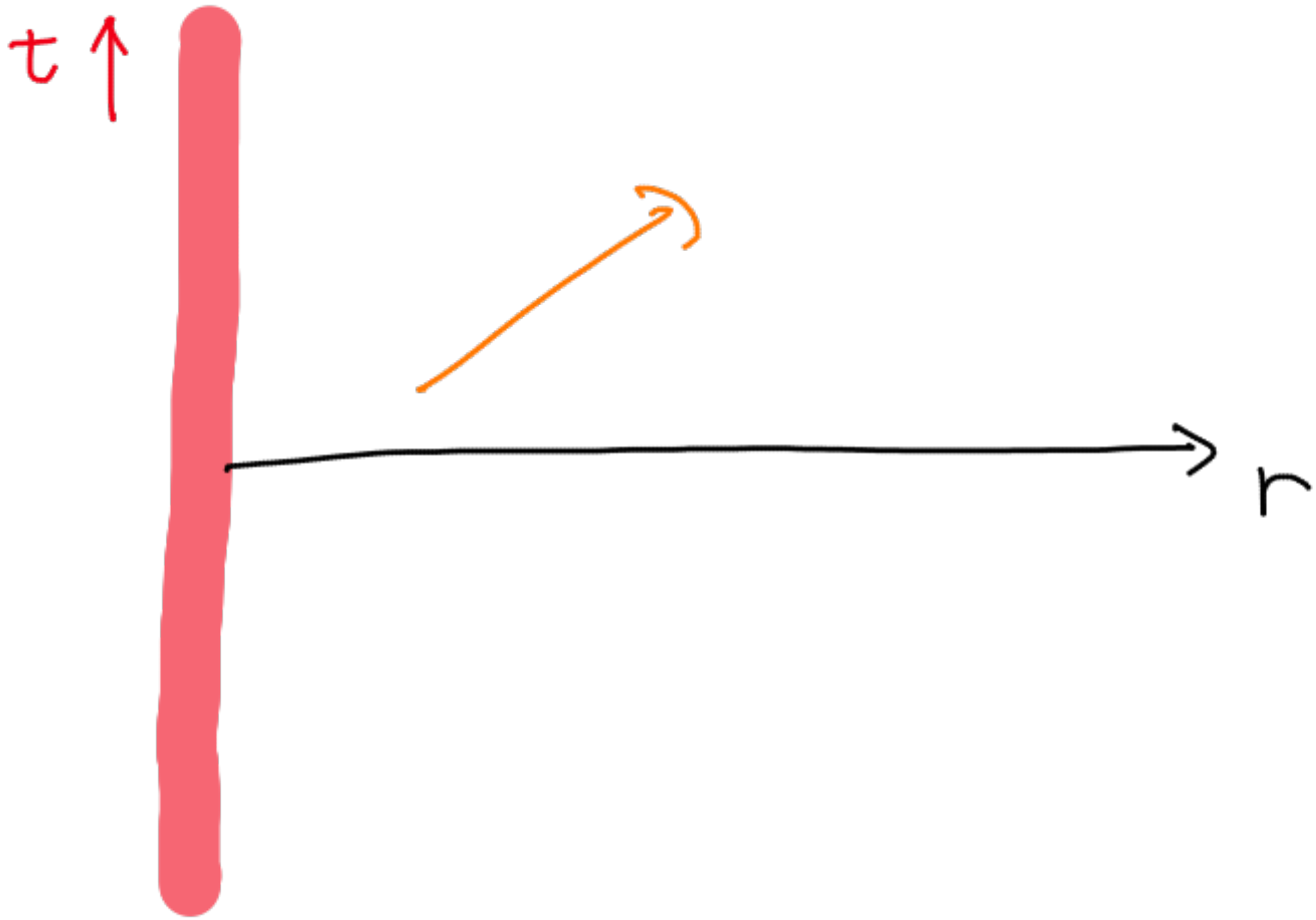
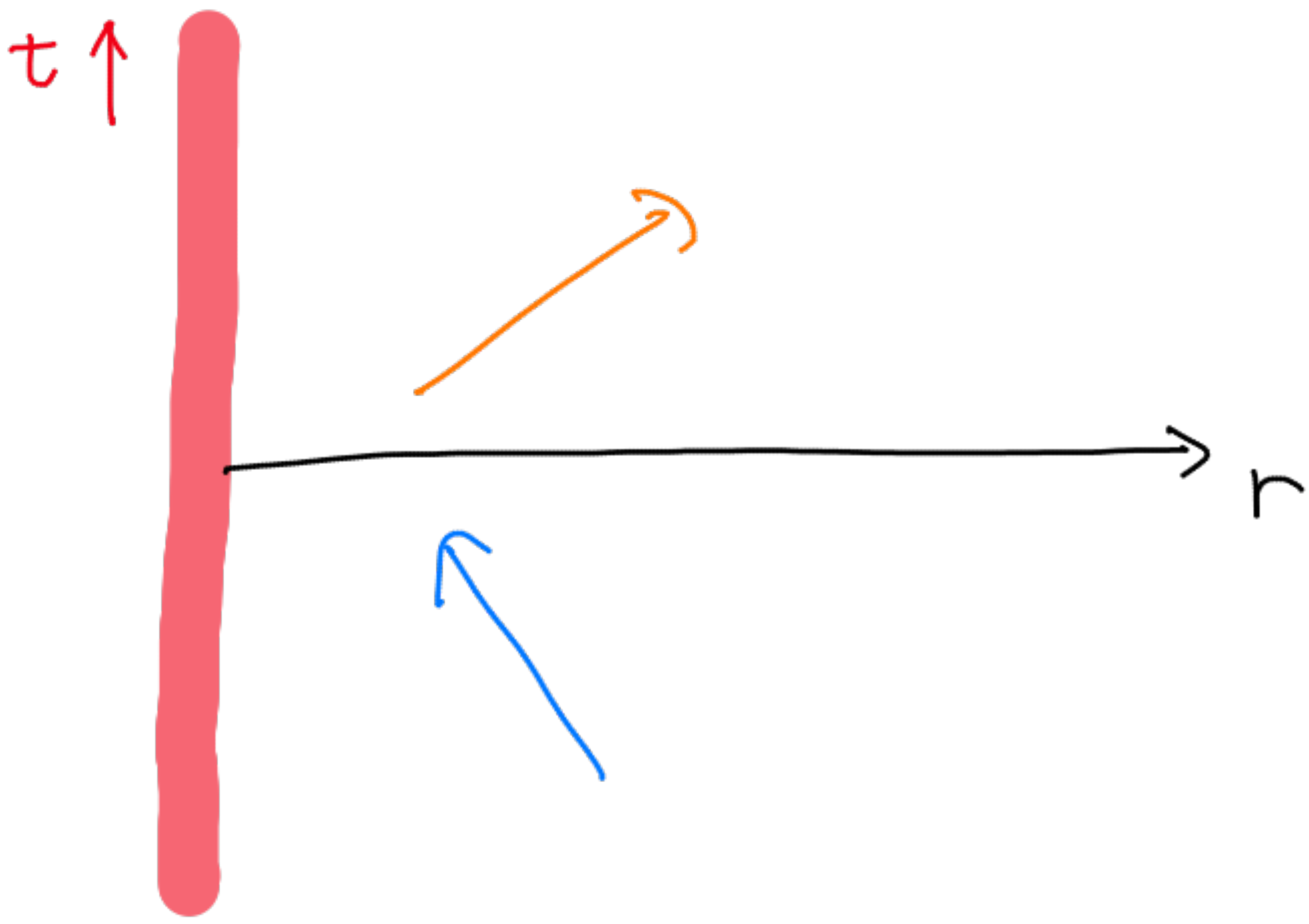


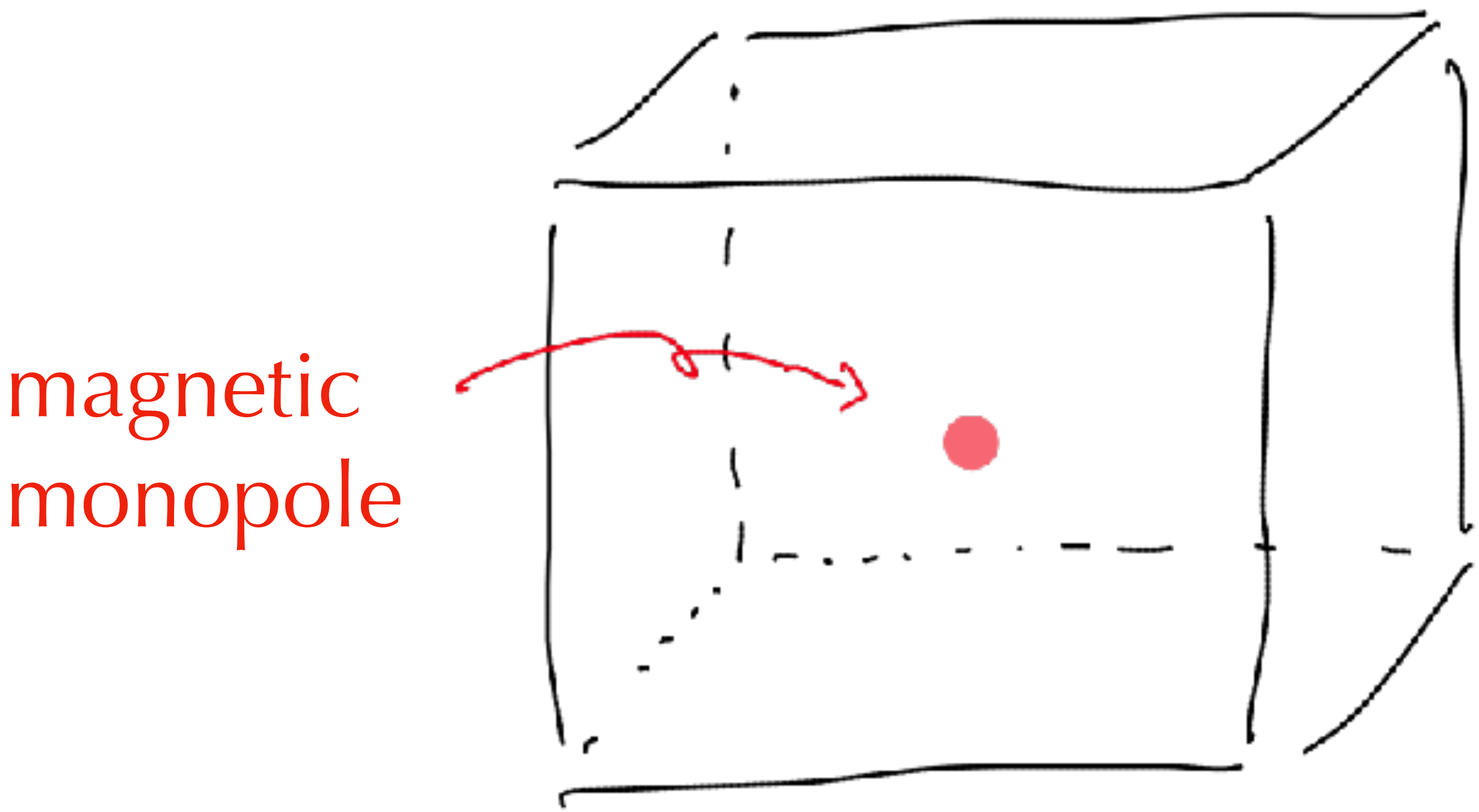
we get a 1+1d scattering problem of 'relativistic' fermions by a boundary.

The boundary has impurity degrees of freedom.



The question is what happens in the low-energy limit.

In hep-th, when analyzing the Callan-Rubakov effect,



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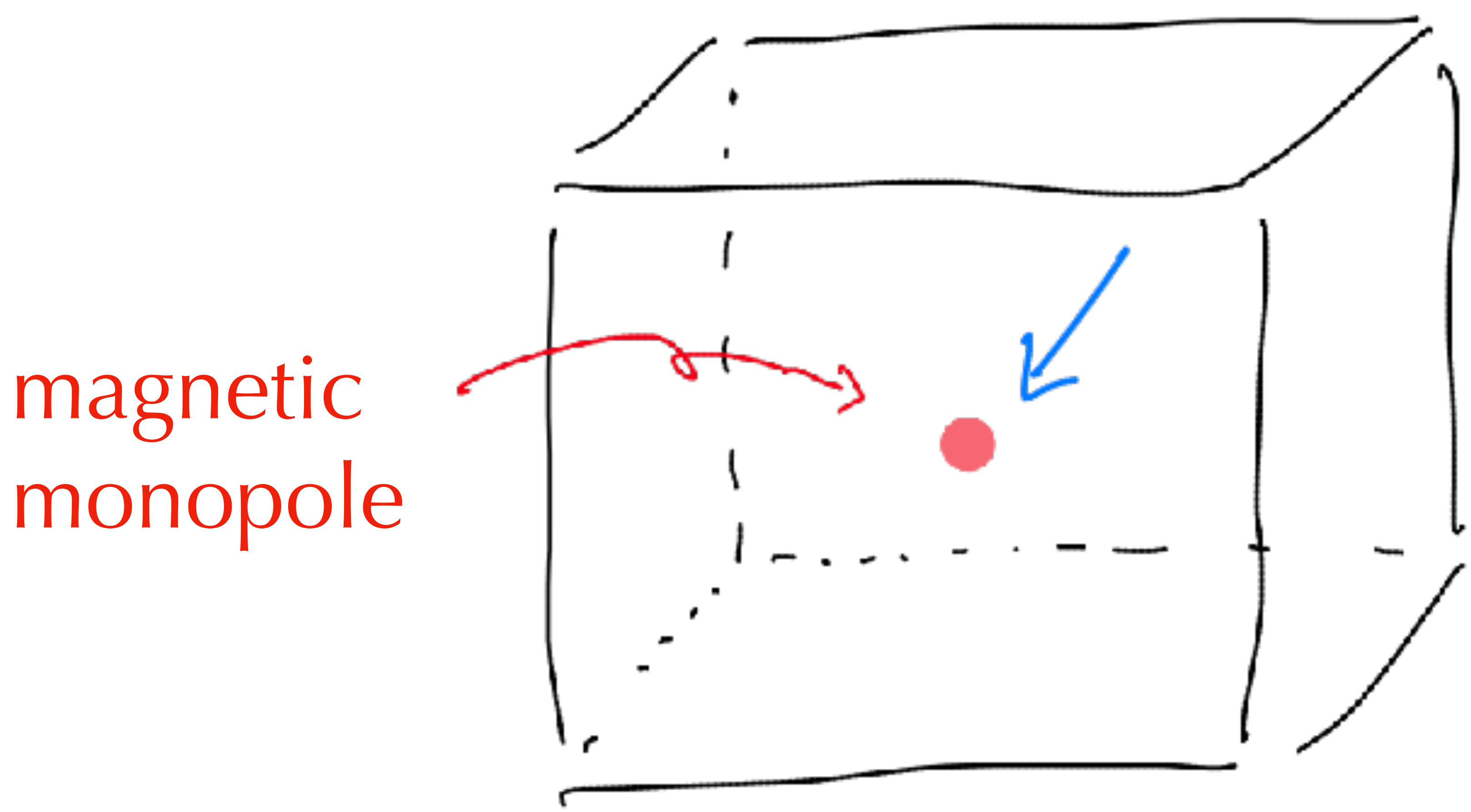
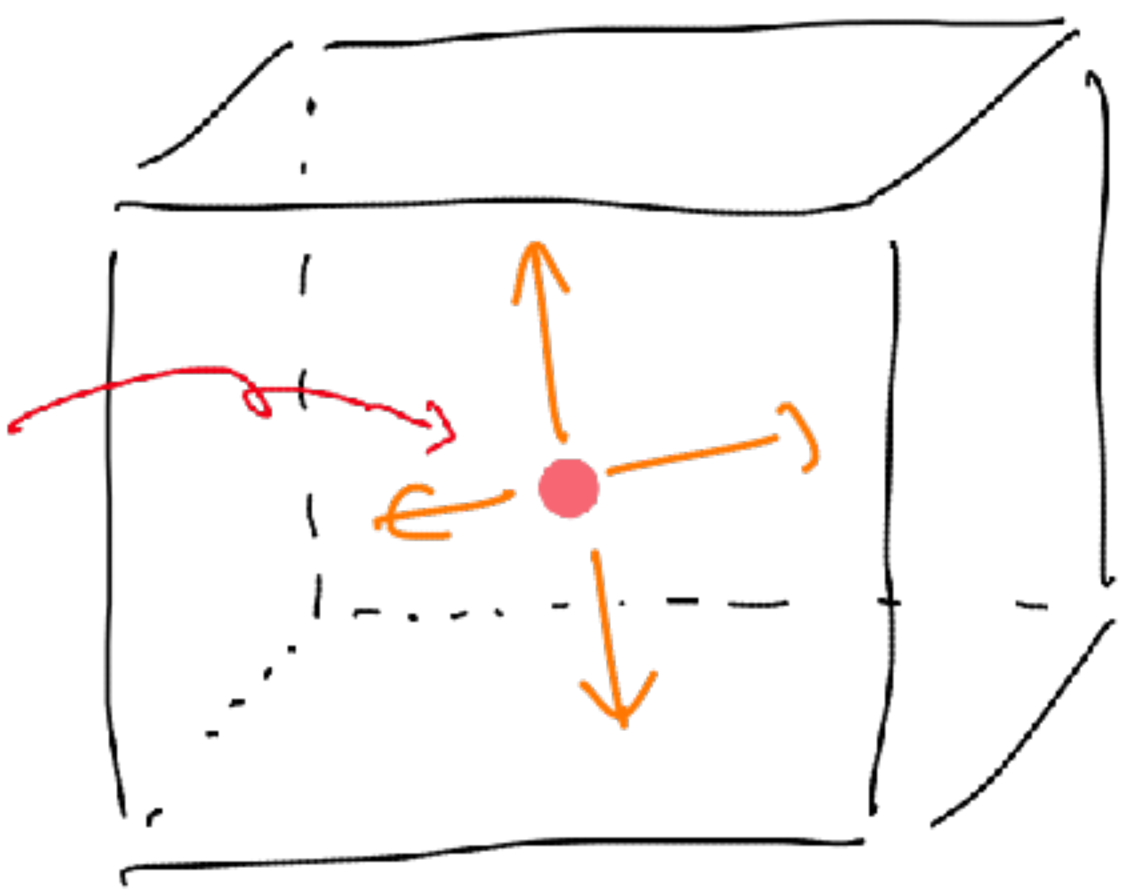


In hep-th, when analyzing the Callan-Rubakov effect,

magnetic
monopole



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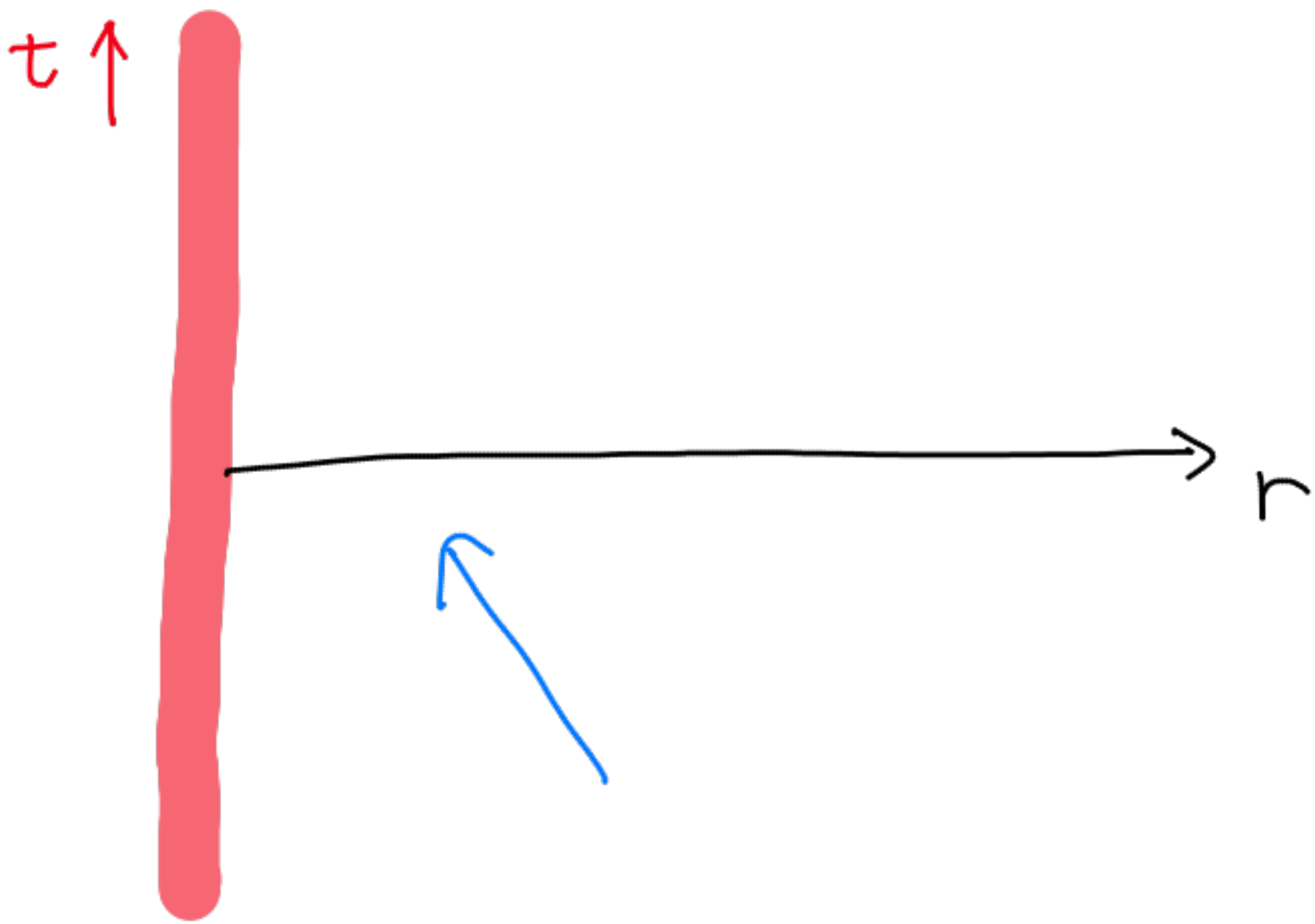
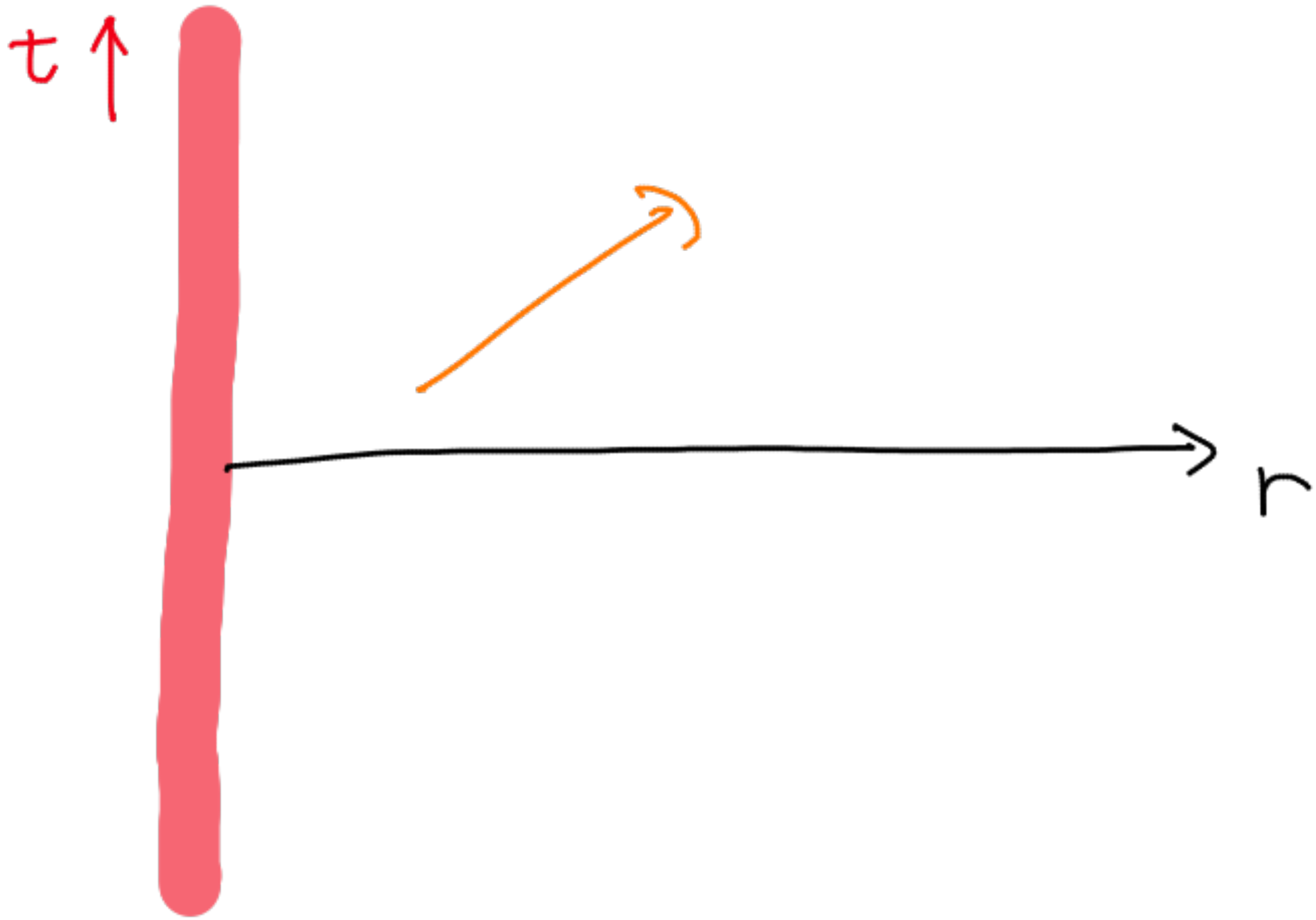


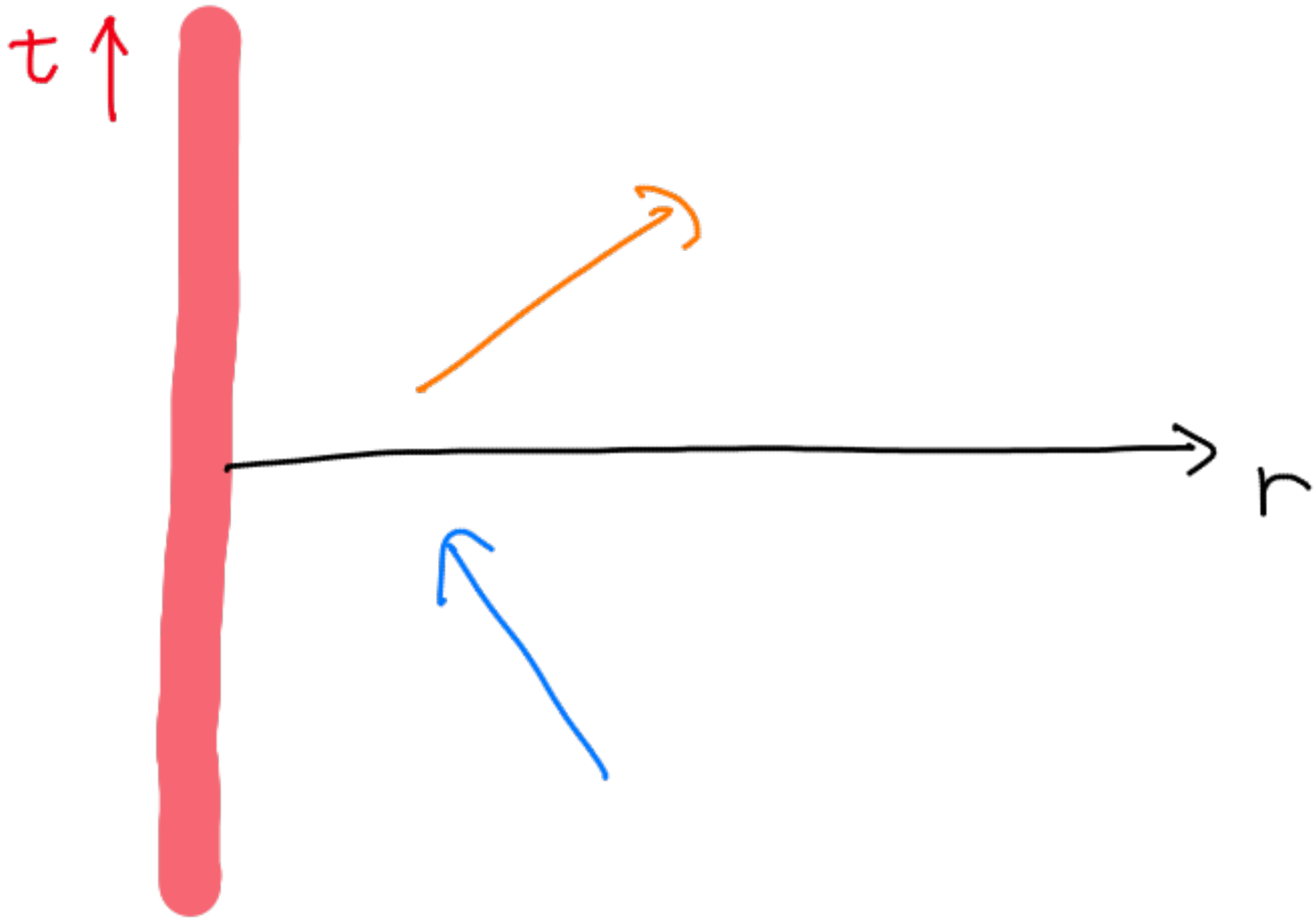
we get a 1+1d scattering problem of relativistic fermions by a boundary.

Taking S-wave approximation ...



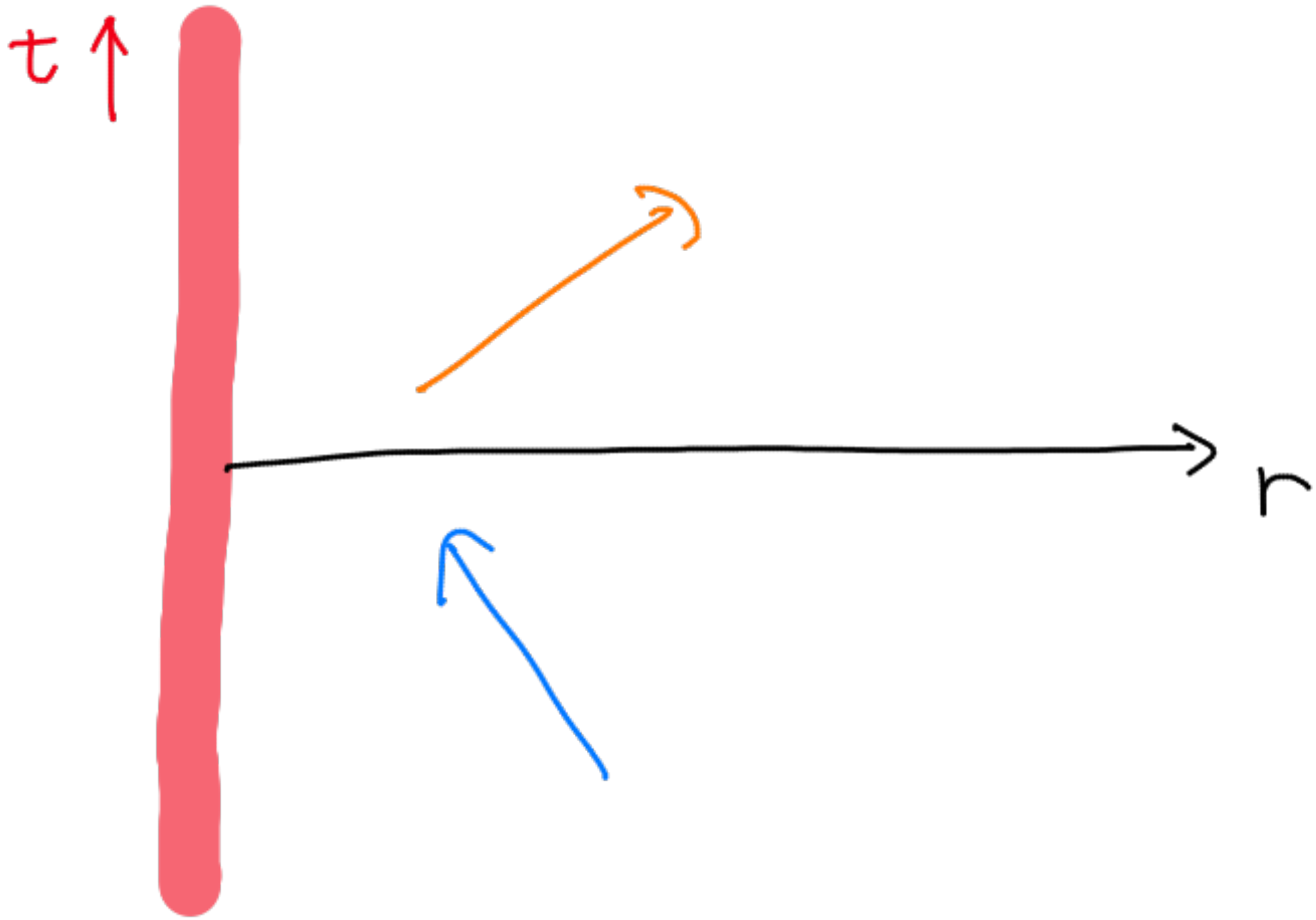
we get a 1+1d scattering problem of relativistic fermions by a boundary.

The boundary has internal degrees of freedom of the magnetic monopole.



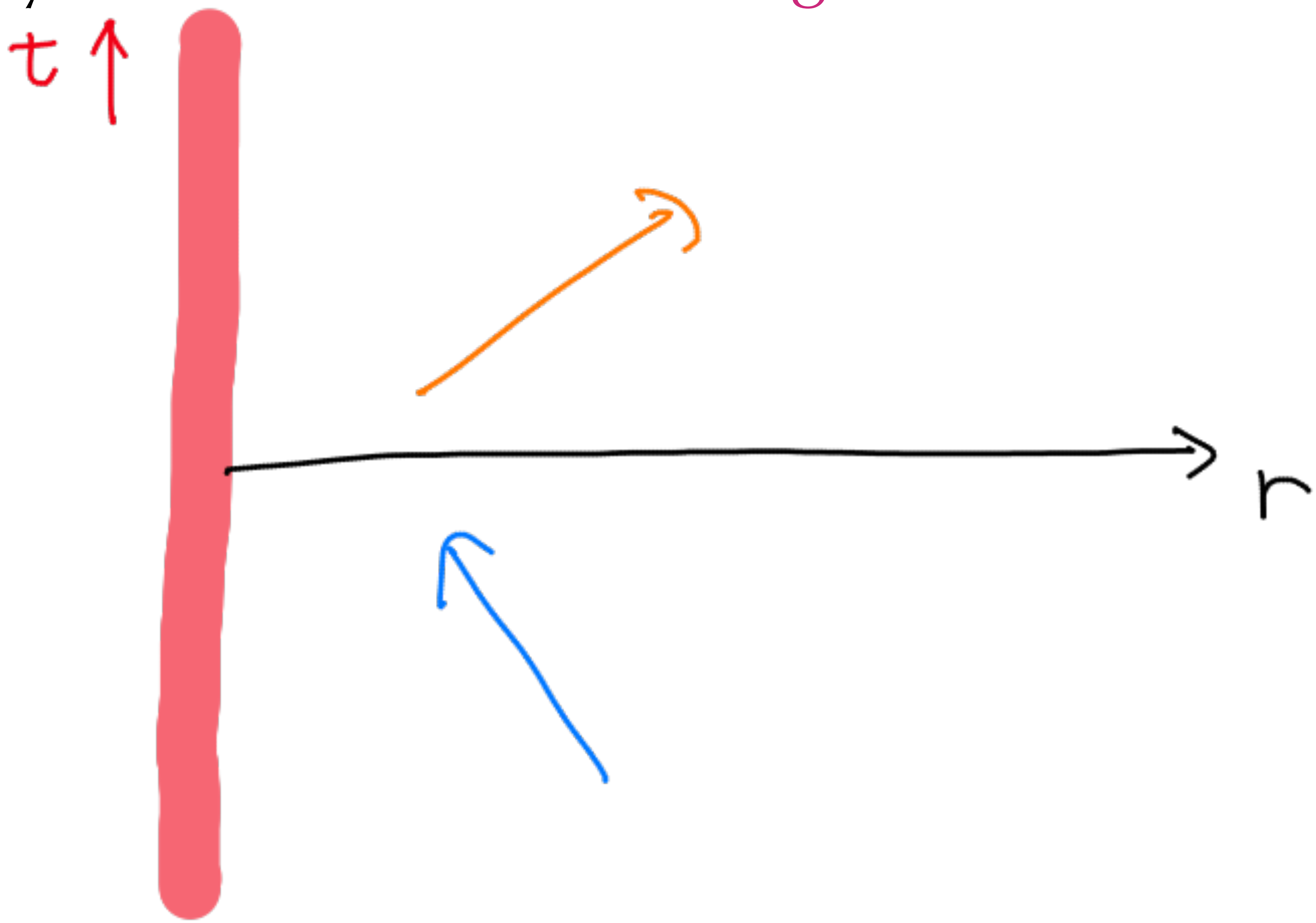
The question is what happens in the low-energy limit.

For two-channel Kondo effect and
the Callan-Rubakov effect,



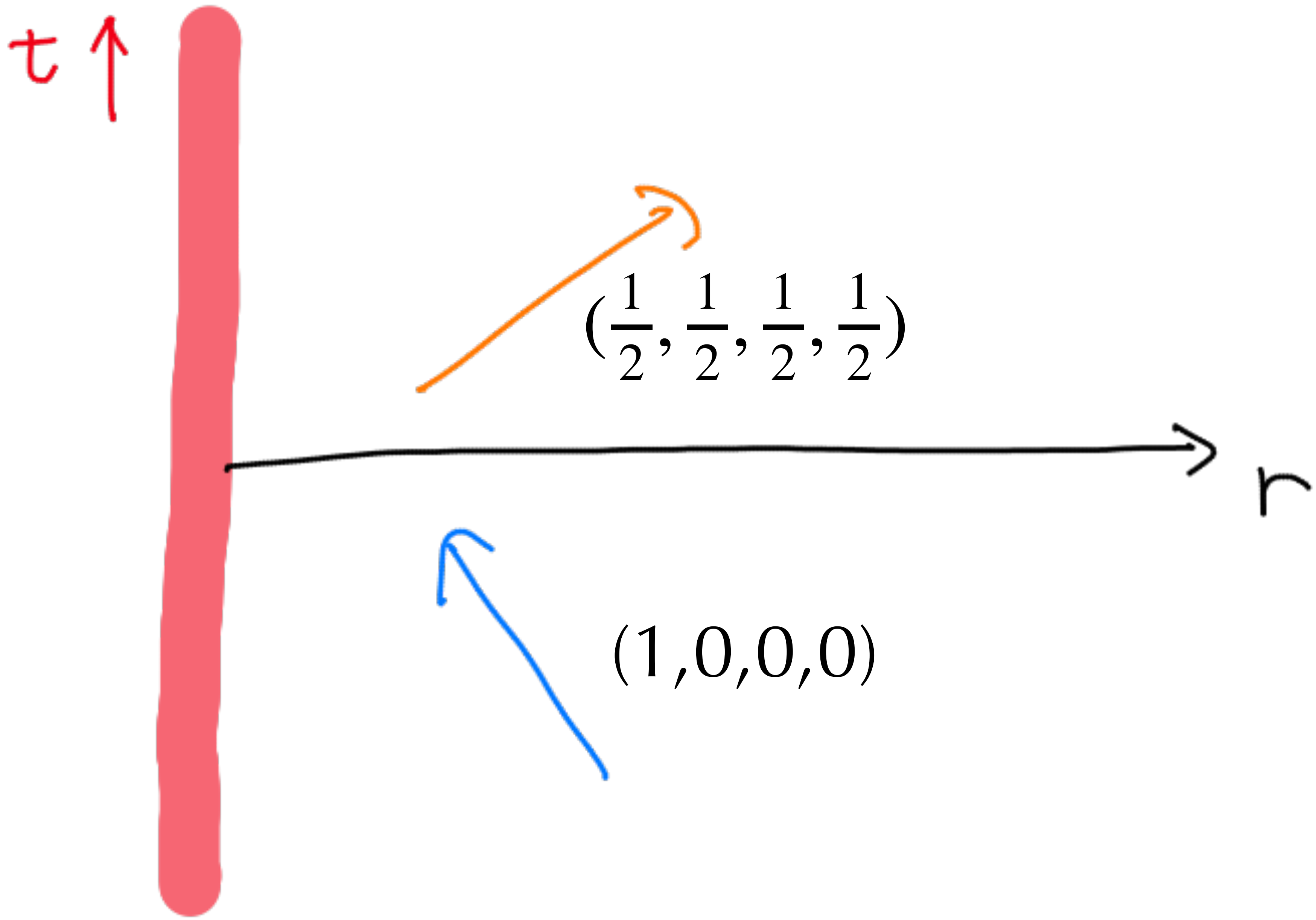
we get a 1+1d fermion scattering problem
by essentially the same boundary condition.

The resulting boundary condition
in the low-energy limit was found using CFT methods
by Maldacena and Ludwig in [cond-mat/9502109].

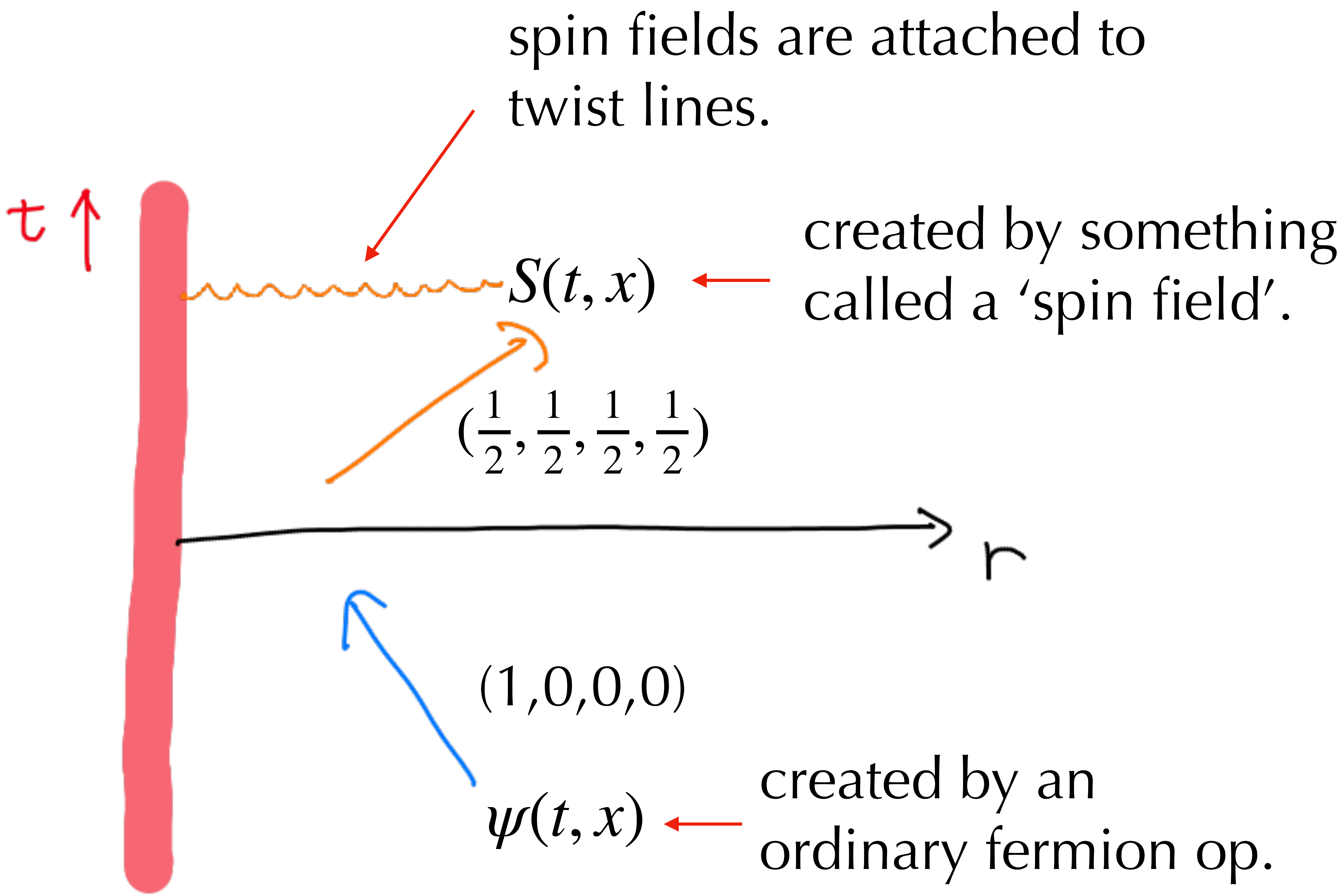


(You might not know who Maldacena is,
but he later became somewhat famous in hep-th,
by similarly considering a bulk-boundary problem.)

The bulk has four complex fermions, $\psi_{1,2,3,4}$,
with four U(1) charges, e.g. ψ_1 has charge (1,0,0,0).

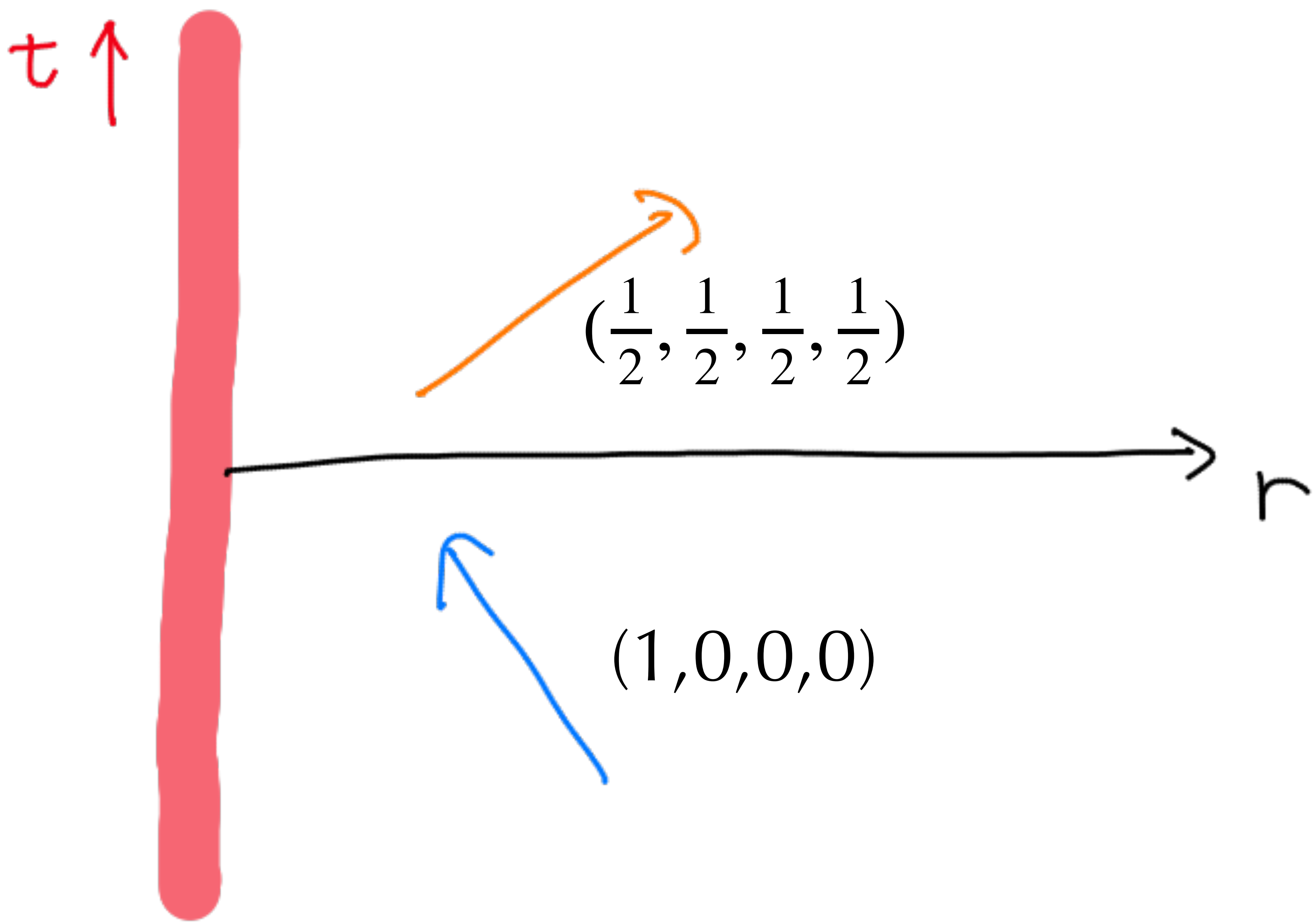


M-L found that
the reflected excitation has charge $(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2})$.


spin fields are attached to twist lines.
t
$S(t, x)$
created by something called a 'spin field'.
$(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$
r
(1,0,0,0)
$\psi(t, x)$
created by an ordinary fermion op.

All these were known in 1995.

Aren't we satisfied?


t
$(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2})$
r
(1,0,0,0)

- This is a free theory of four fermions $\psi_{1,2,3,4}$, with charges (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1).

- The reflected excitation has charge $(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2})$.

- There is no such thing in the Fock space.

- Continuum description is an approximation in cond-mat, so you might not need to worry that there are something extra.
- But it is more pressing on the hep-th side, because these fermions are quarks and leptons, and are supposed to be elementary and fundamental.
- Do we really have something in addition to what we see in the Fock space, even in a free theory?
- If so, have we been teaching something wrong in the QFT class?

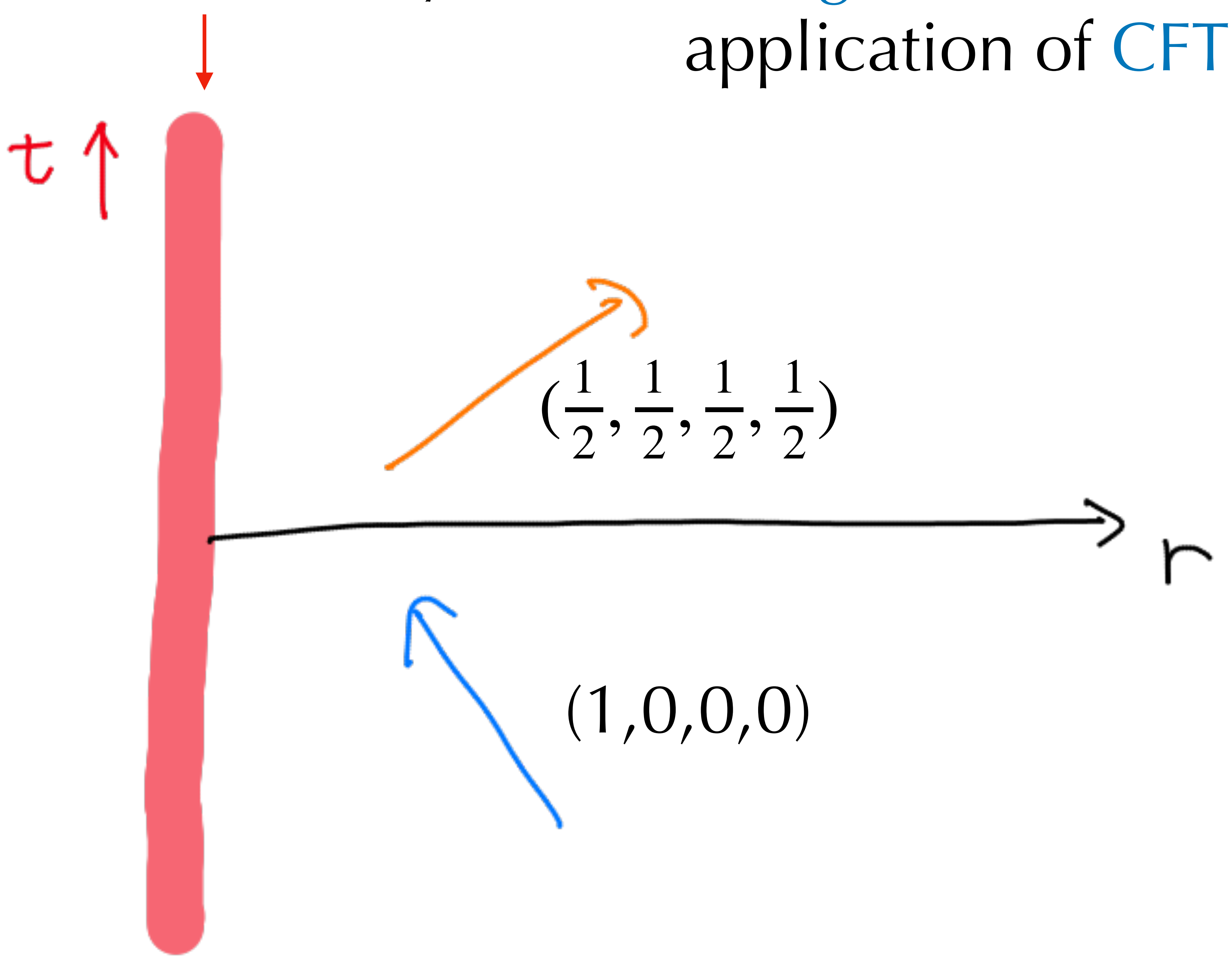
- This question has been taken up again by hep-th people from about 2020, under the name of Callan-Rubakov problem and/or monopole-fermion problem, but never as a Kondo problem …

- This is the context in which I got interested in this issue. So it was a purely 'gedanken' question.

- But while preparing this talk, I also learned that there are proposals to measure exotic reflected excitations in the two-channel Kondo effect experimentally. [1710.03030] [2302.02295]

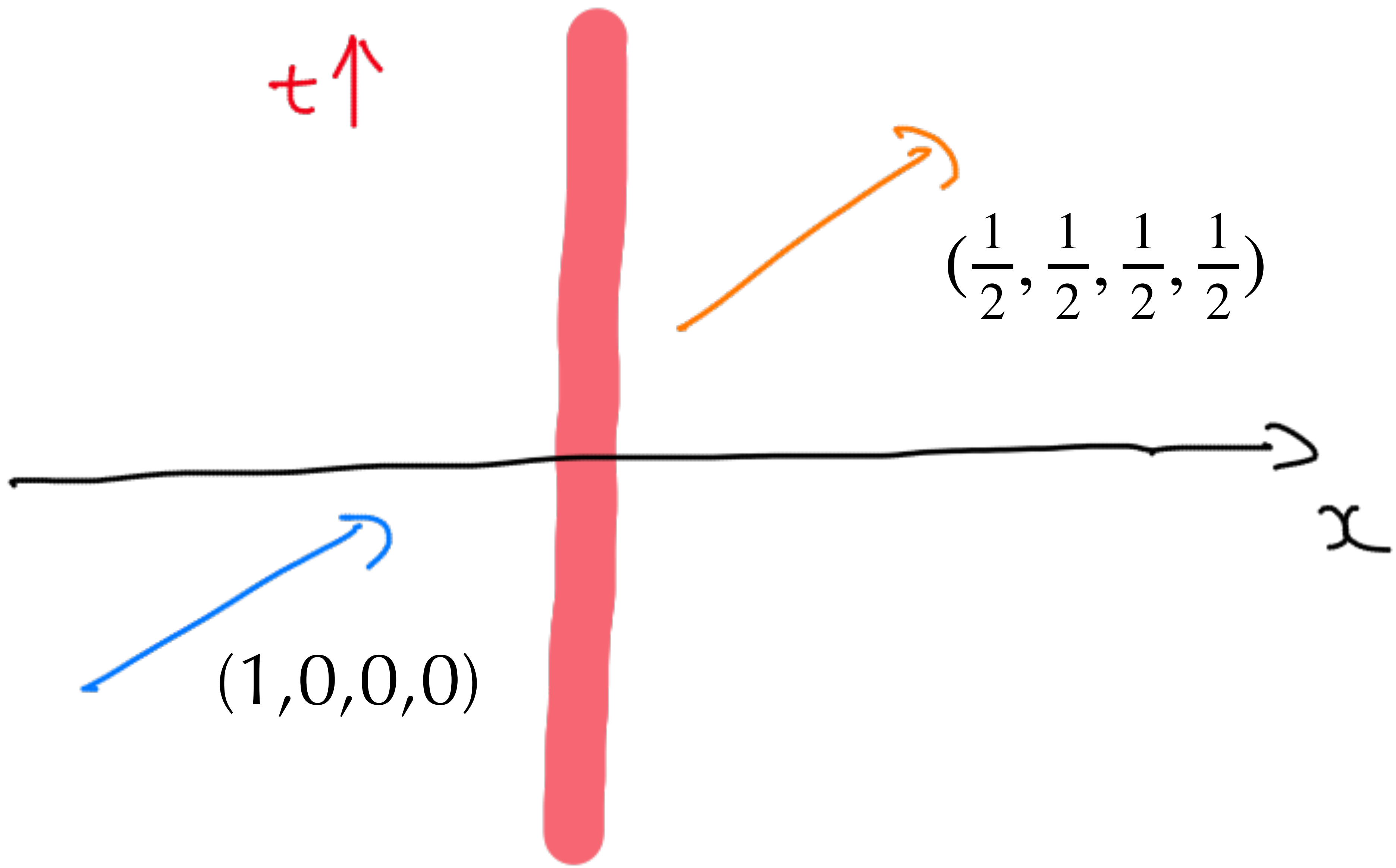
Let me talk about our small update. Recall

Conformal boundary ← Being conformal allowed application of CFT methods.



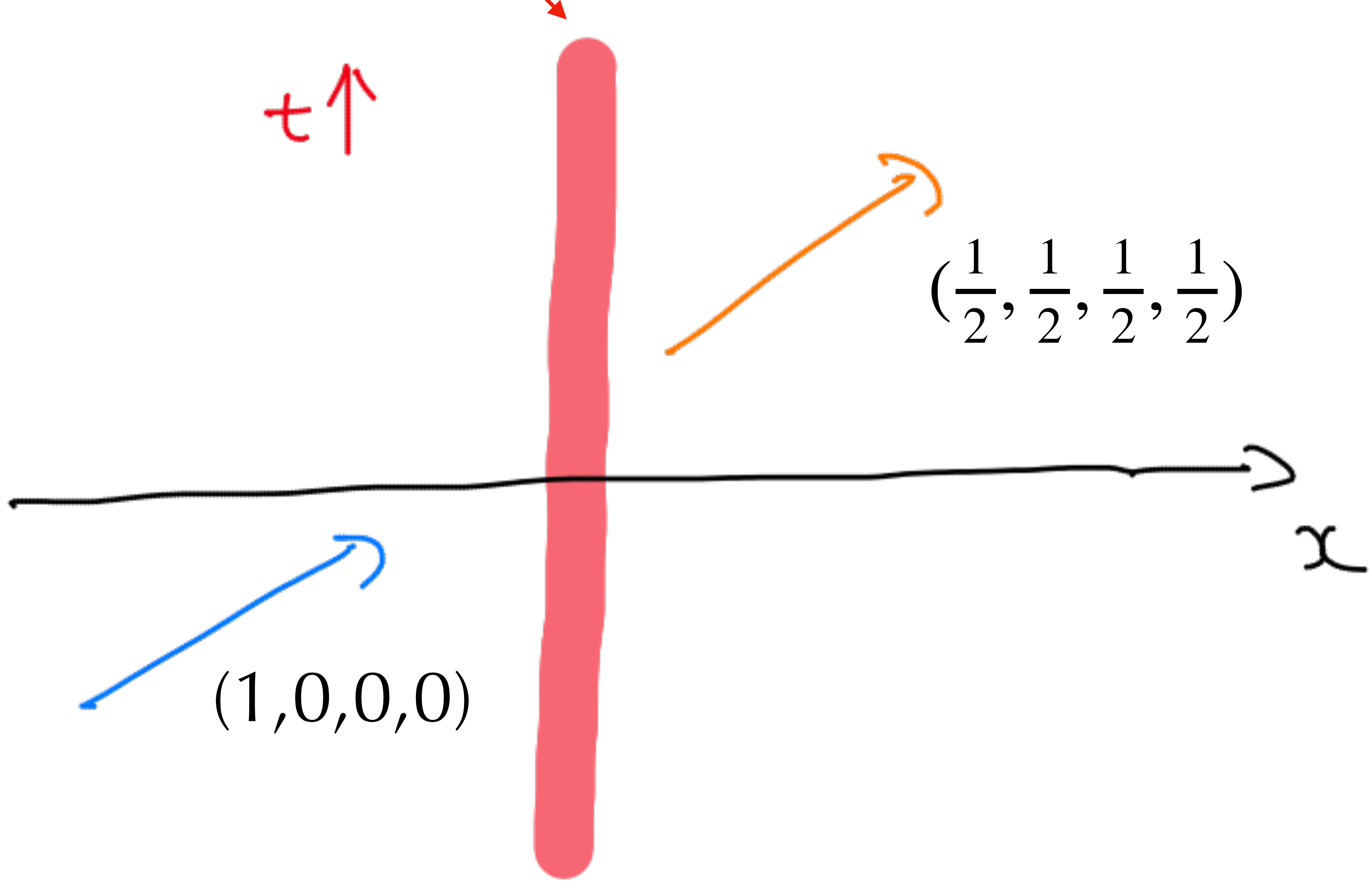
Let's unfold it!

(Applying unfolding to the Callan-Rubakov / Kondo effect was originally due to [Polchinski 1984].)



Let's unfold it!

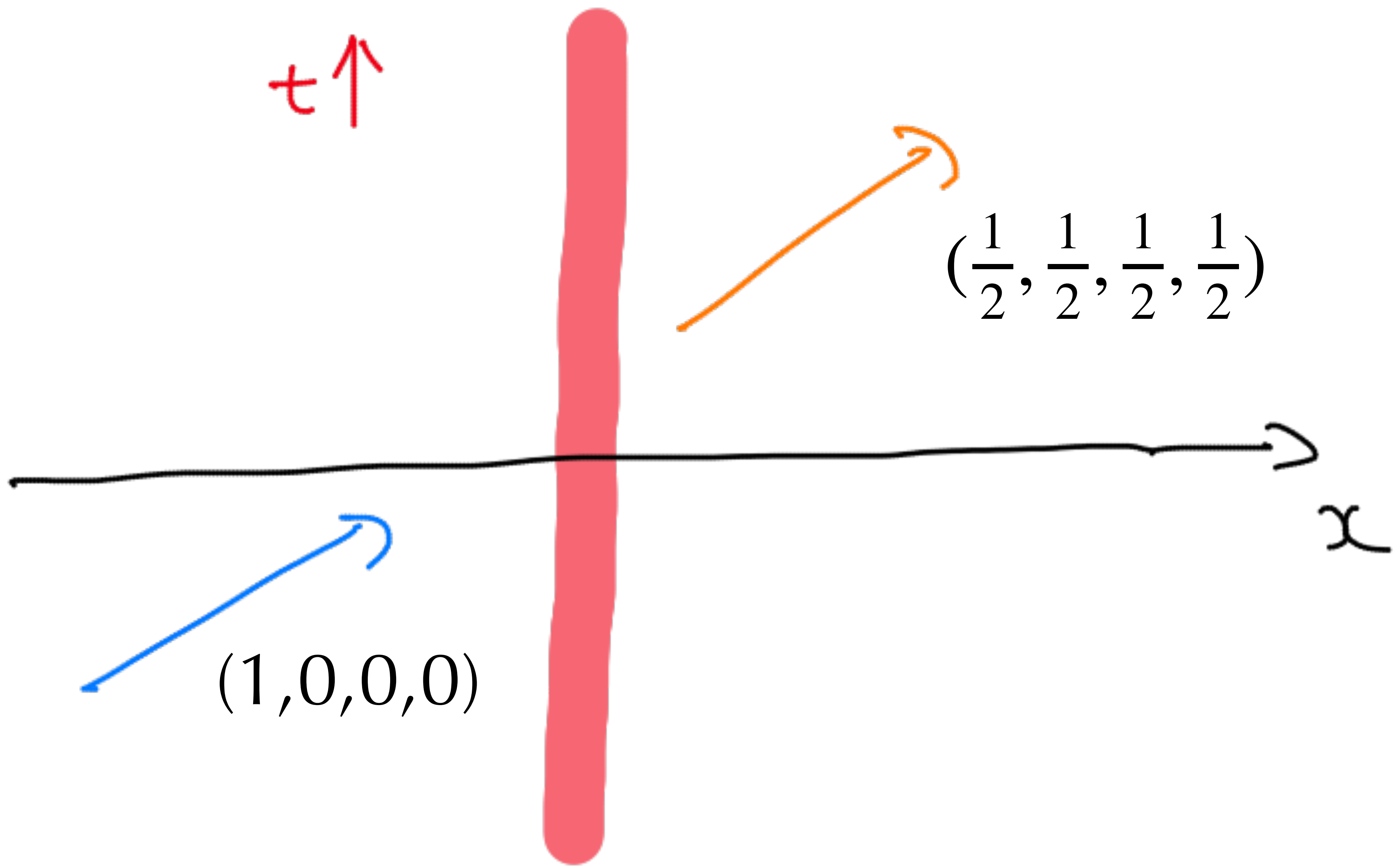
Topological domain wall
(not just conformal)



Let's unfold it!

Being topological allows application of topological methods.

Topological domain wall
(not just conformal)



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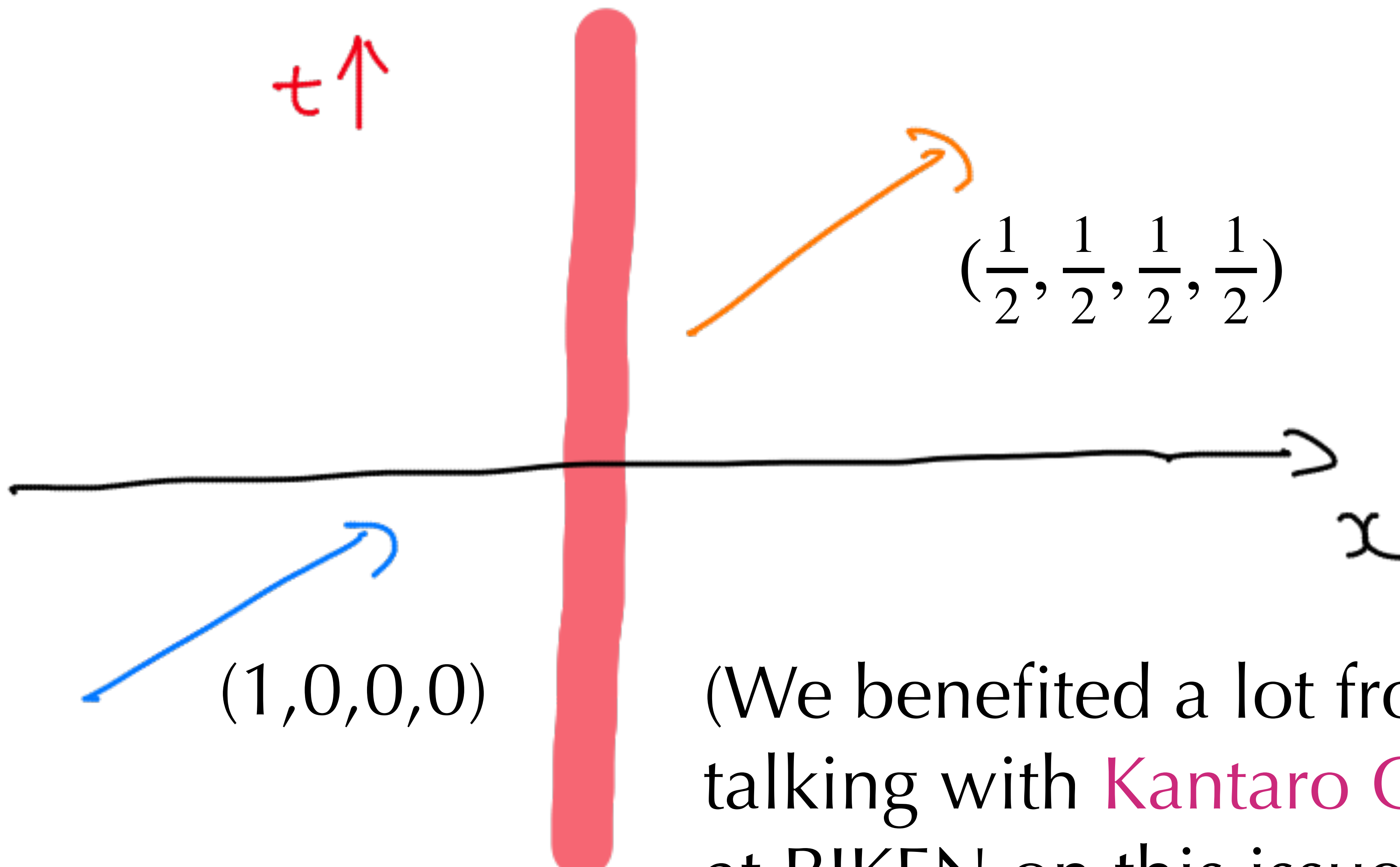

(We benefited a lot from talking with Kantaro Ohmori at RIKEN on this issue.)

Let's unfold it!

Being topological allows application of topological methods.

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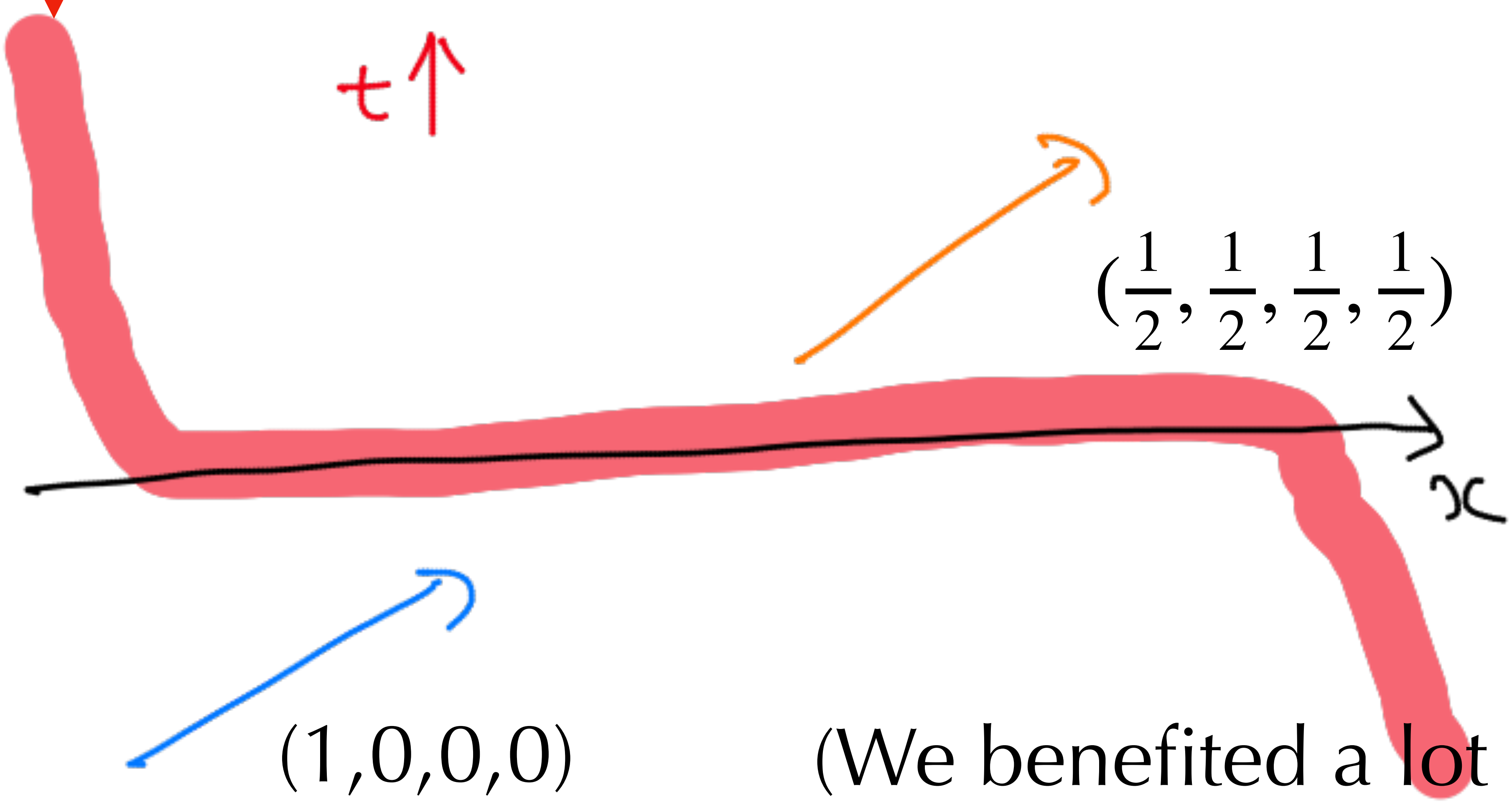

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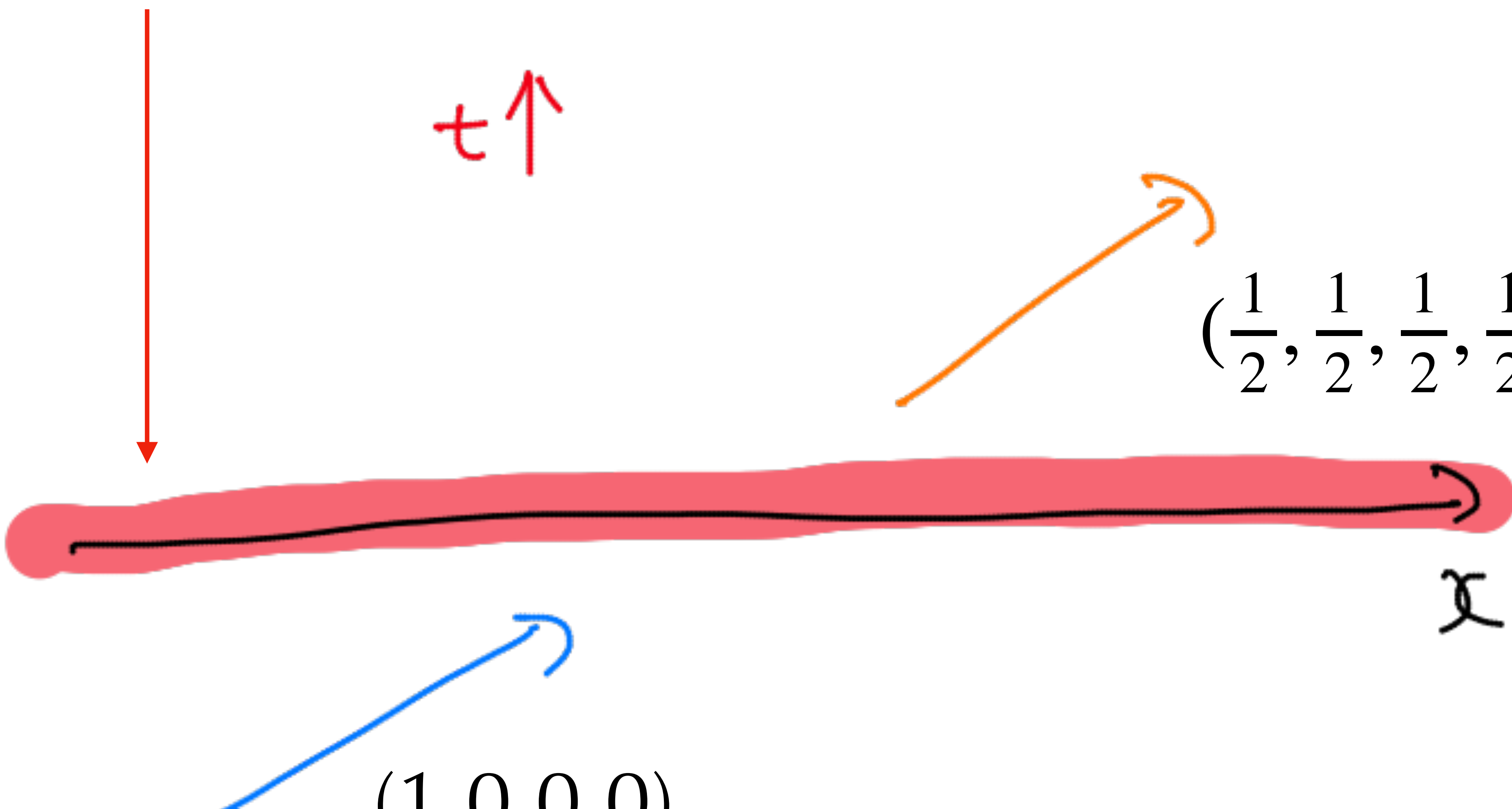
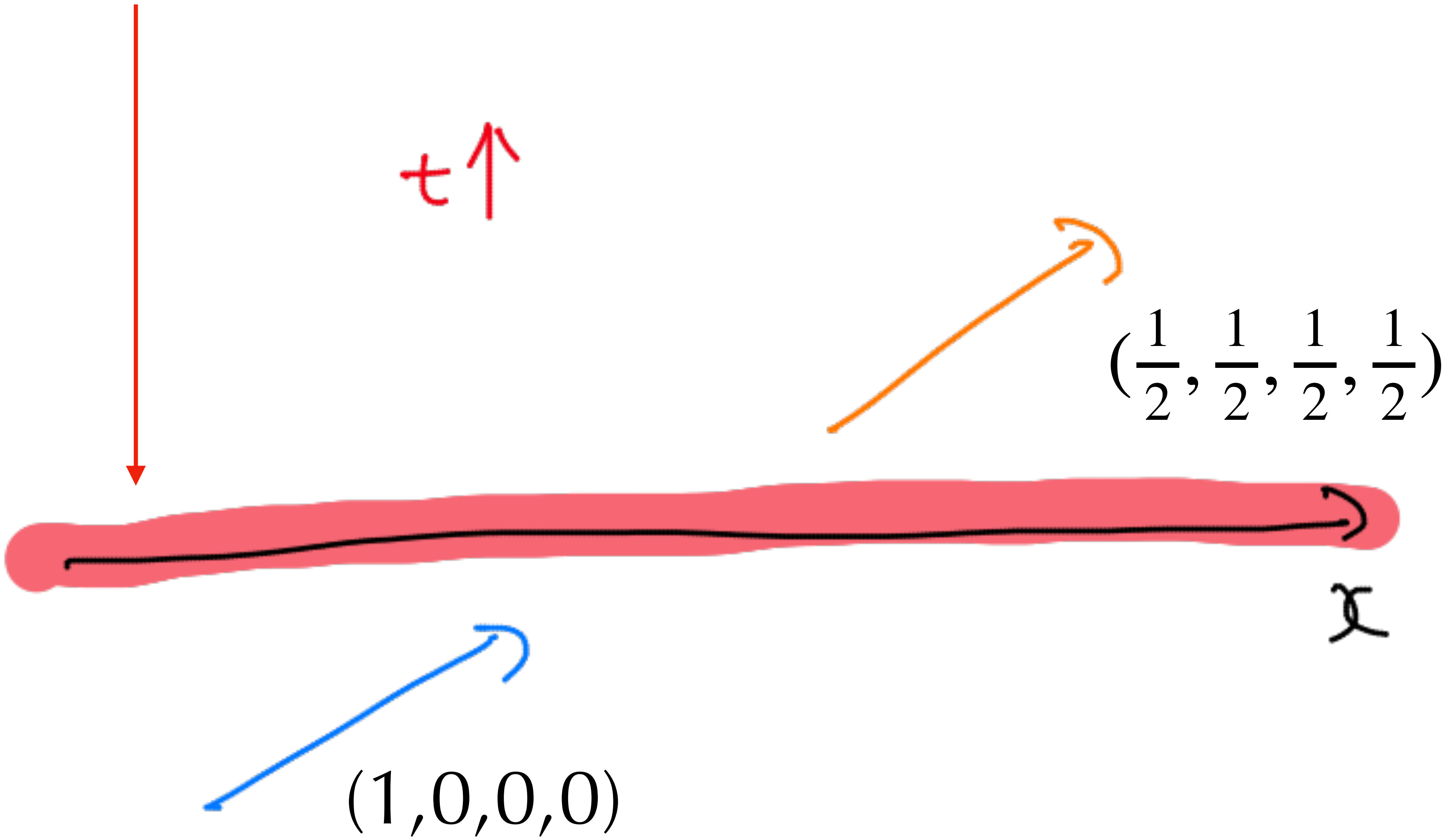


$(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2})$

(1,0,0,0)

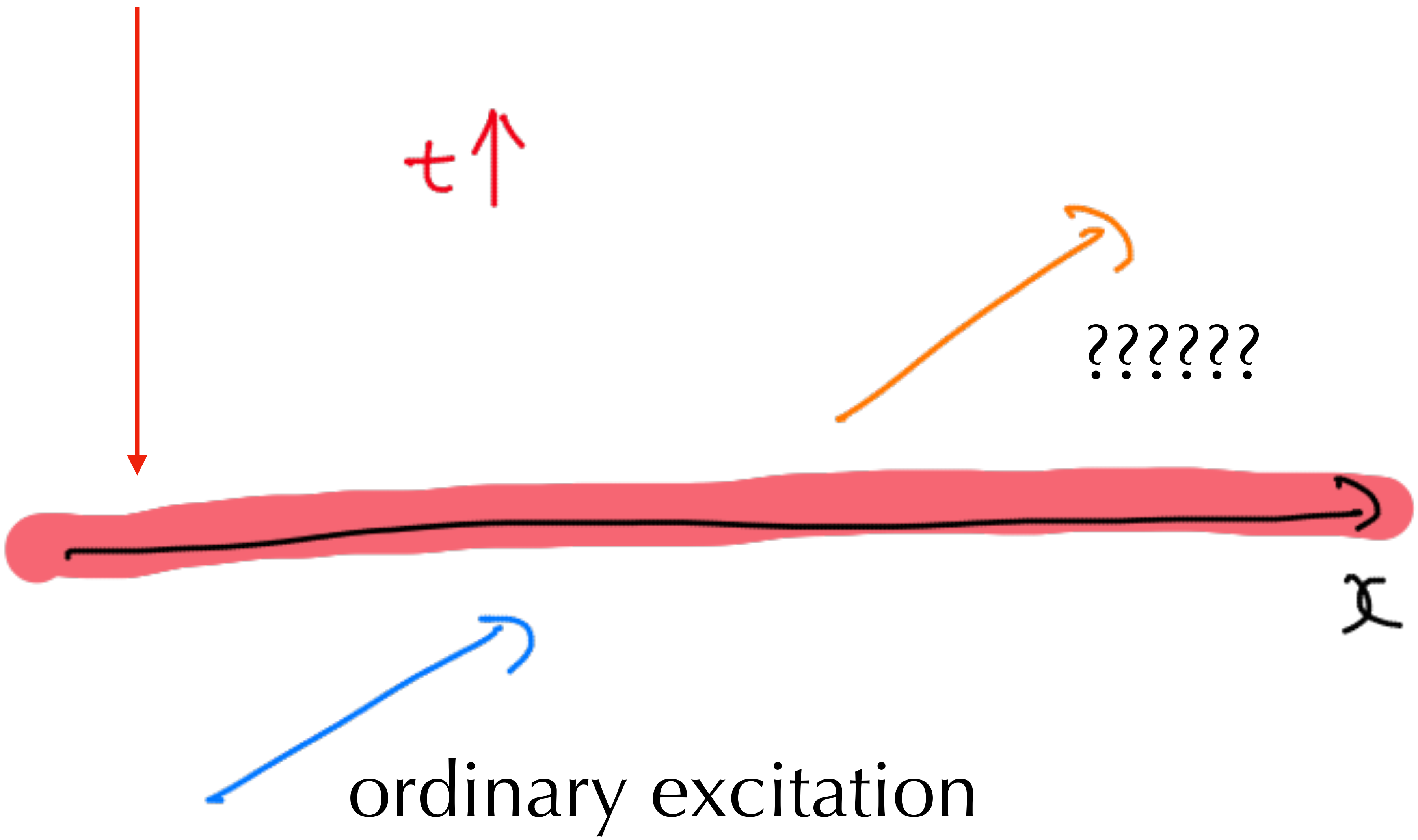
Let's unfold it!

Action of a generalized symmetry operation



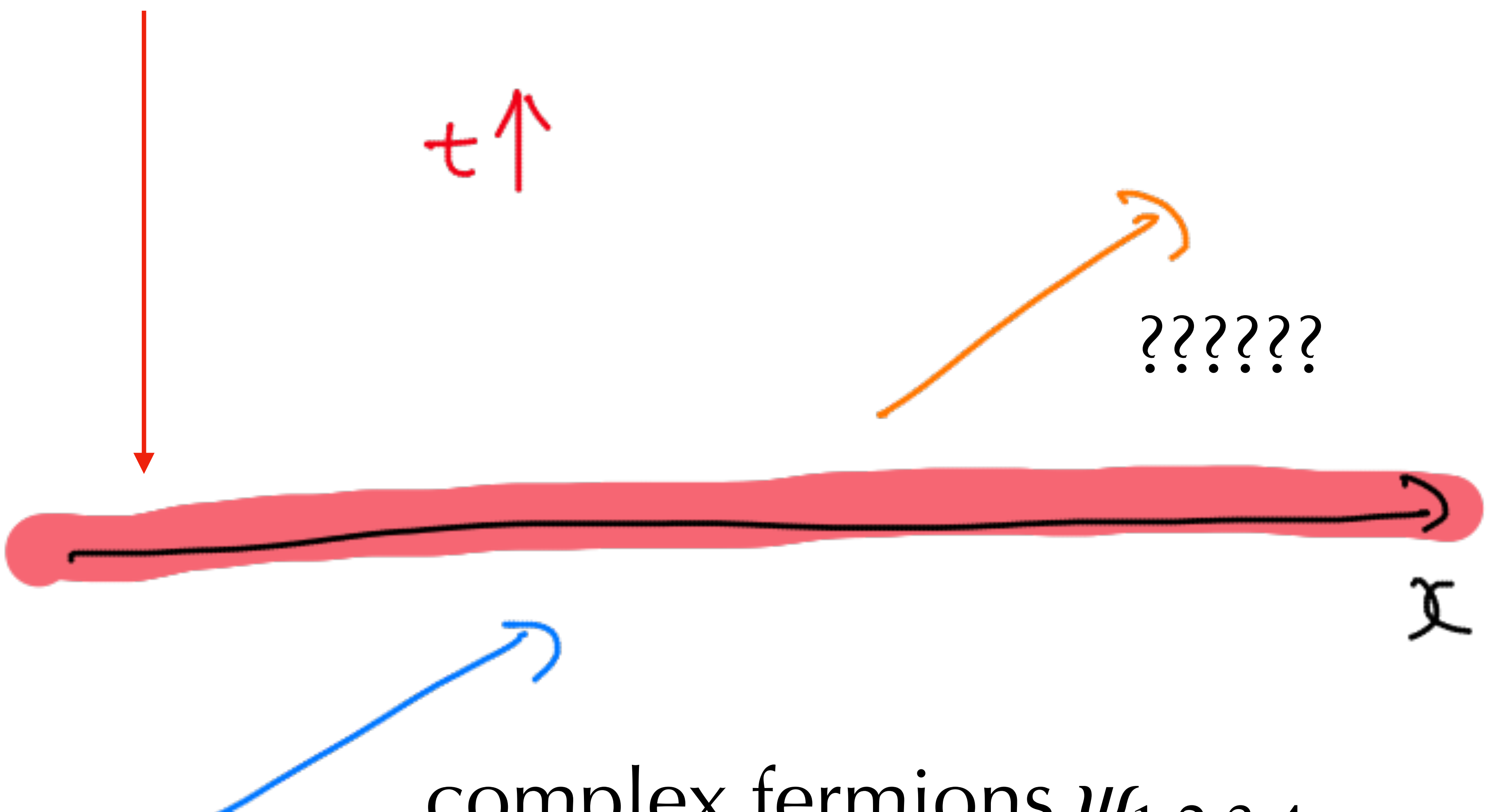
So it's an instance of a more general question:

What happens to an excitation when it is acted on by a generalized symmetry operation?



In the case of two-channel Kondo / Callan-Rubakov / Maldacena-Ludwig question,

Action of a generalized symmetry operation

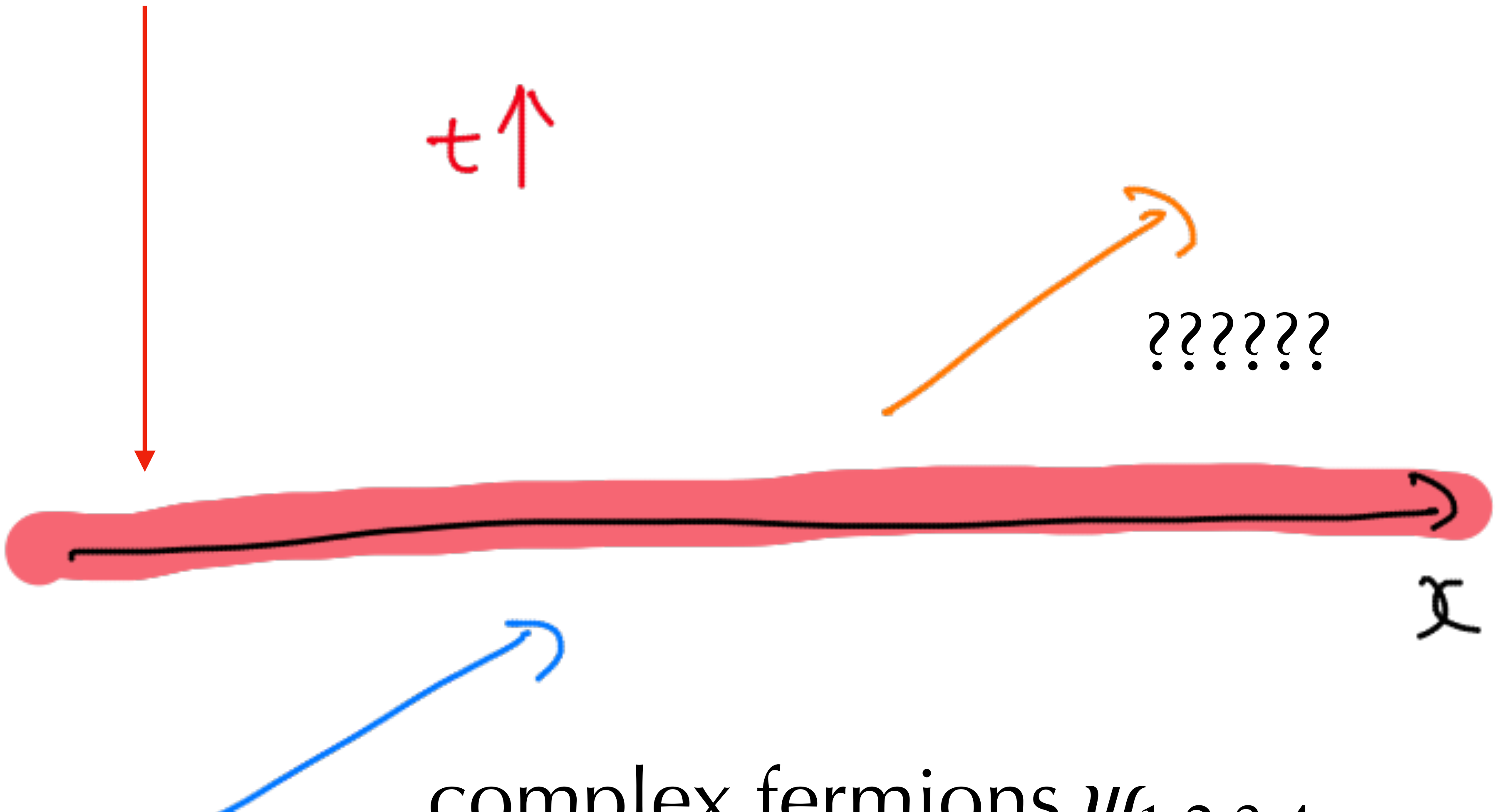


complex fermions $\psi_{1,2,3,4}$

$\sim$ $so(8)_1$ current algebra

In the case of two-channel Kondo / Callan-Rubakov / Maldacena-Ludwig question,

Action of a $\mathbb{Z}_2$ outer-automorphism of $so(8)_1$ current algebra



complex fermions $\psi_{1,2,3,4}$

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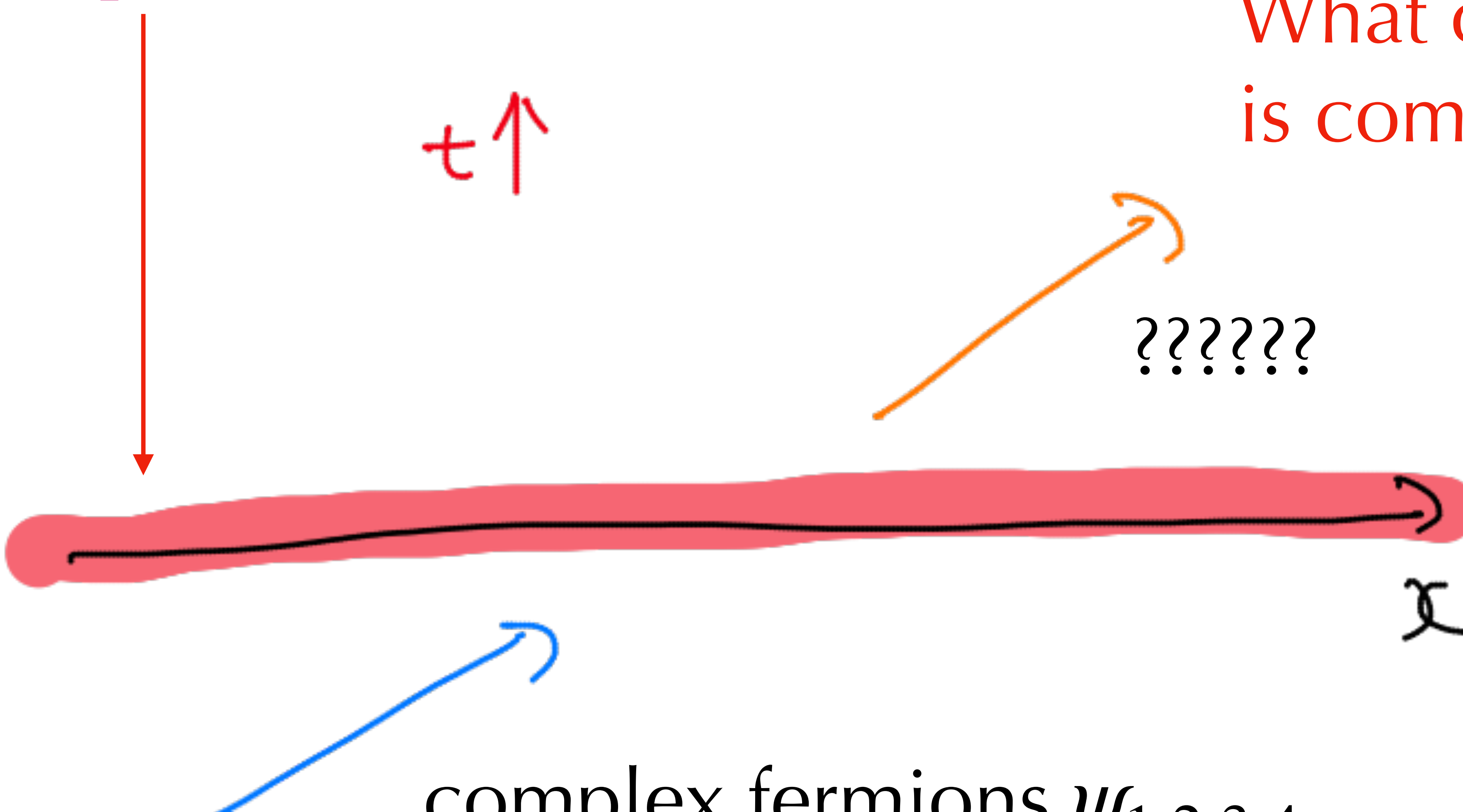
In the case of two-channel Kondo / Callan-Rubakov / Maldacena-Ludwig question,

Action of a $\mathbb{Z}_2$ outer-automorphism of $so(8)_1$ current algebra

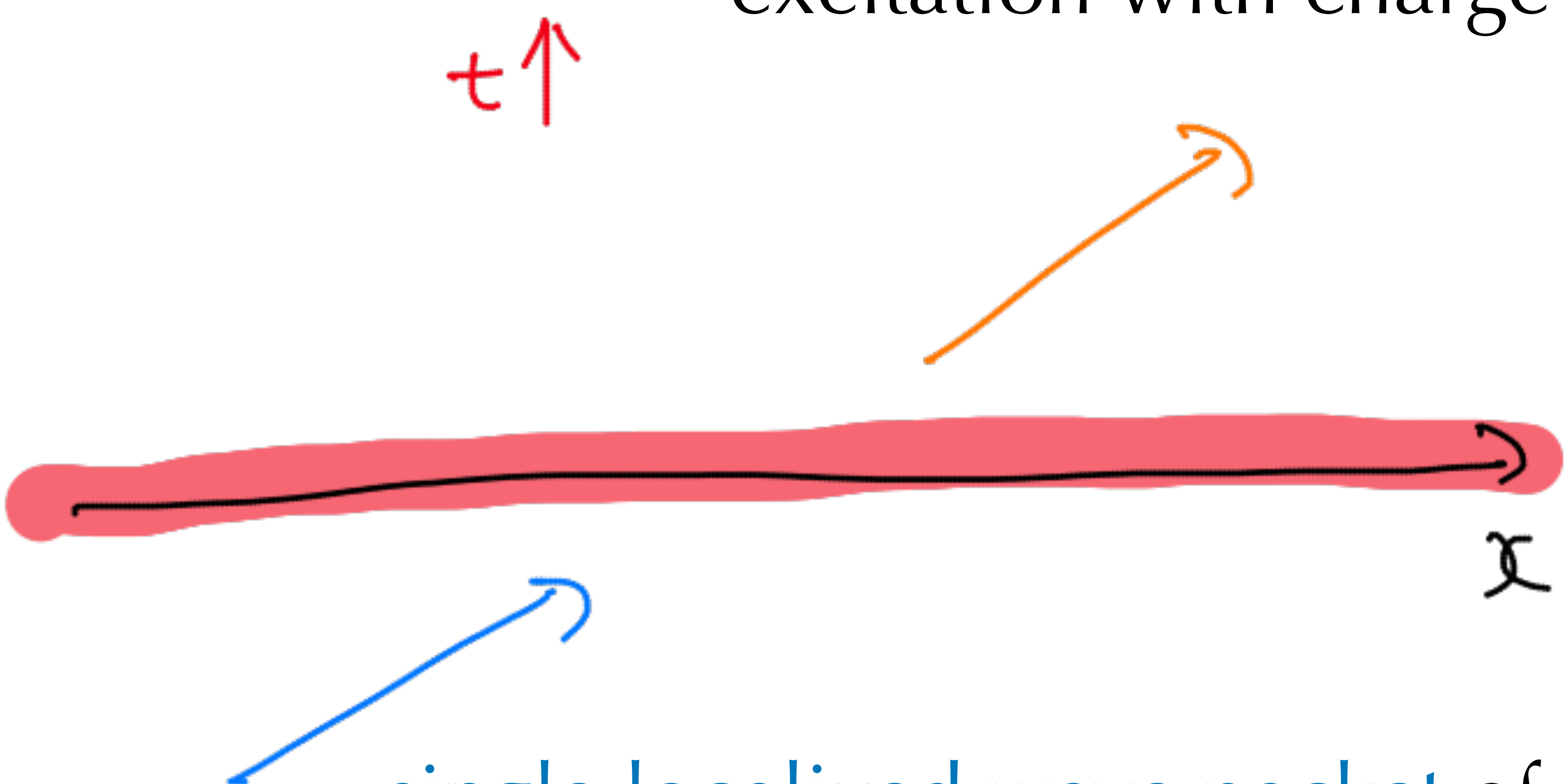
What comes out is computable!



complex fermions $\psi_{1,2,3,4}$ ~ $so(8)_1$ current algebra

In the case of two-channel Kondo / Callan-Rubakov / Maldacena-Ludwig question,

single localized wave packet of excitation with charge $(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2})$



single localized wave packet of fermion ψ_1 with charge (1,0,0,0)

This means that in a free theory of
four complex fermions $\psi_{1,2,3,4}$ with integer charges

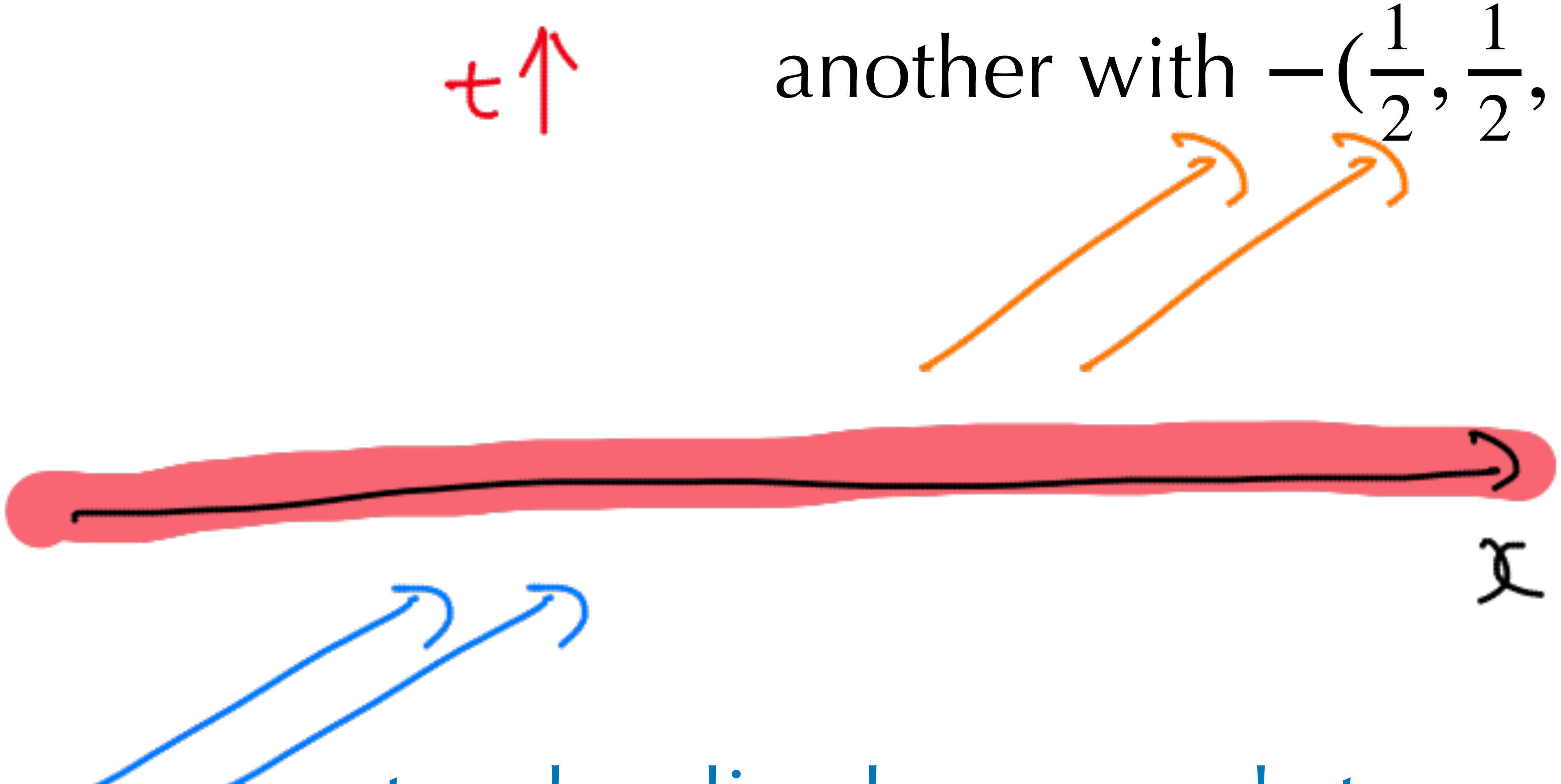
$$(1,0,0,0),\ (0,1,0,0),\ (0,0,1,0),\ (0,0,0,1),$$

there is a localized particle excitation with charge

$$\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}\right).$$

So, there does exist something physical outside of Fock space, and what I've been teaching in QFT classes was wrong!

two localized wave packets,
one with $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ and
another with $-(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$



two localized wave packets,
one with ψ_1 and another with $\overline{\psi}_1$

can be rewritten as a coherent state of pairs

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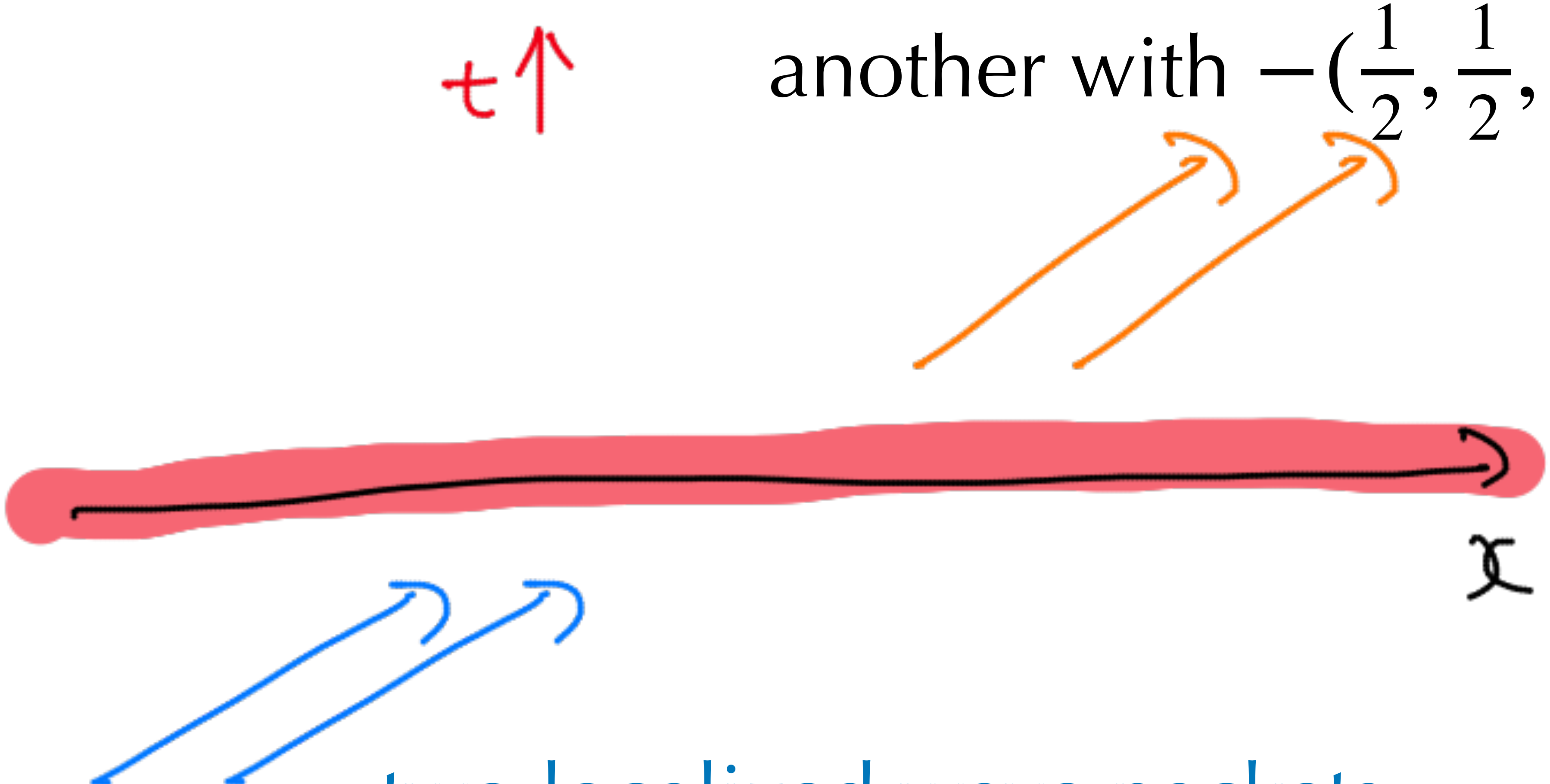

two localized wave packets,
one with ψ_1 and another with $\overline{\psi}_1$

For example,

$$\Big(1 + (x_0 - y_0)\psi_1(x_0)\overline{\psi_1}(y_0)\Big)\,|0\rangle$$

becomes

$$\exp\left(\iint_{x_0}^{y_0} \frac{dxdy}{x-y}\left(\sqrt{\frac{(x-x_0)(y-y_0)}{(x-y_0)(y-x_0)}} - 1\right)\sum_i \psi_i(x)\overline{\psi_i}(y)\right)|\,0\,\rangle.$$

We're trying to extract some physics
from this explicit expression…

In a free theory of four complex fermions $\psi_{1,2,3,4}$ with integer charges

$$(1,0,0,0),\ (0,1,0,0),\ (0,0,1,0),\ (0,0,0,1),$$

there are localized particle excitations with charge

$$(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2})$$

outside of the ordinary Fock space, but two of them can be rewritten as a coherent state of the $\psi\overline{\psi}$ pairs.

Executive summary

- Two-channel Kondo = Callan-Rubakov.
- Even in a free theory, strange excitations can exist.
- Oshikawa-san, please have fun in the US!